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The final report summarizes in its first part the most recent scientific achievements. They are documented in the listed and enclosed copies of publications and drafts of papers. In the second part the recent academic activities are reported, and the third part gives and outlook of planned activities based on the work supported by this grant.

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Final Report

on the Project

Theoretical Studies on Microwave Plasma Electronics and Microhollow Cathode Discharges

Grant No: F49620-98-1-0065

submitted to

Dr. Robert J. Barker

Air Force Office of Scientific Research
801 N. Randolph St.
Arlington, VA 22203-1977

by

Liu Shenggang

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and

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$$\left.\begin{aligned}\bar{E}_\perp &= \frac{j\omega\mu_0}{k^2 - k_z^2}\left[-\nabla_\perp H_z \times \bar{e}_z + \bar{J}_\perp\right]\\ \bar{H}_\perp &= \frac{-jk_z}{k^2 - k_z^2}\left[\nabla_\perp H_z + \bar{J}_\perp \times \bar{e}_z\right]\end{aligned}\right\} \tag{33}$$

for the TE modes.

For the case of a plasma fill, when $B_0 \to \infty$, (22) becomes:

$$\nabla_\perp^2 E_z + \left(k^2 - k_z^2\right)\left(1 - \omega_p^2 / \omega^2\right) E_z = j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0}\rho \tag{34}$$

while when $B_0 \to 0$, we have:

$$\nabla_\perp^2 E_z + \left[k^2\left(1 - \omega_p^2 / \omega^2\right) - k_z^2\right] E_z = j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0\left(1 - \omega_p^2 / \omega^2\right)}\rho \tag{35}$$

B. Interaction equations expressed in terms of the transverse fields:

The beam-wave interaction equations (22) and (23) can also be expressed in terms of the transverse field components as follows:

$$\begin{aligned}&\nabla_\perp^2 \bar{E}_\perp + \frac{k^2\left(\varepsilon_1^2 - \varepsilon_2^2\right) - k_z^2 \varepsilon_3}{\varepsilon_1} \bar{E}_\perp = \\ &\frac{jk_z\omega\mu_0\varepsilon_2}{\varepsilon_1}\bar{H}_\perp + k_z\omega\mu_0\frac{\left(\varepsilon_3 - \varepsilon_1\right)}{\varepsilon_1}\bar{e}_z \times \bar{H}_\perp + j\omega\mu_0\bar{J}_\perp - \frac{\omega\mu_0\varepsilon_2}{\varepsilon_1}\bar{e}_z \times \bar{J}_\perp + \frac{\nabla_\perp\rho}{\varepsilon_0\varepsilon_1}\end{aligned} \tag{36}$$

and:

$$\nabla_\perp^2 \bar{H}_\perp + \left(k^2\varepsilon_3 - k_z^2\right)\bar{H}_\perp = -jk_z\omega\varepsilon_0\varepsilon_2\bar{E}_\perp + k_z\omega\varepsilon_0\left(\varepsilon_3 - \varepsilon_1\right)\bar{e}_z \times \bar{E}_\perp - \left(\nabla \times \bar{J}\right)_\perp \tag{37}$$

Then E_z and H_z can be written as:

$$E_z = -\frac{j}{\omega\varepsilon_0\varepsilon_3}\left[\left(\nabla \times \bar{H}_\perp\right)\cdot\bar{e}_z - J_z\right] \tag{38}$$

$$H_z = \frac{j}{\omega\mu_0}\left(\nabla \times \bar{E}_\perp\right)\cdot\bar{e}_z \tag{39}$$

Summary

Since January of 1998 four progress reports have been submitted, and are enclosed in this final report:

Progress Report I	Jan. 1998 - July. 1998;
Progress Report II	July. 1998 - Dec. 1998;
Progress Report III	Jan. 1999 - Jun. 1999.
Progress Report IV	Aug. 1999 - Jan. 2000.

The Final Report summarizes in its first part the most recent scientific achievements. They are documented in the listed and enclosed copies of publications and drafts of papers. In the second part the recent academic activities are reported, and the third part gives an outlook of planned activities based on the work supported by this grant.

I. Publications

1. The paper by Liu Shenggang, Robert J. Barker et al. "A New Hybrid Ion-Channel Maser Instability" has been accepted for publication in IEEE Trans. Plasma Science after minor revisions.

2. The two-part paper by Liu Shenggang, Robert J. Barker et al. "Basic Theoretical Formulations on Plasma Microwave Electronics" needs to be revised. In particular some numerical calculations need to be checked.

3. The following papers are ready for submission to scientific Journals:
   A. Liu Shenggang; Robert J.Barker et al. "A New Kind of Waves Propagating Along Magnetized Plasma Waveguide"
   B. Liu Shenggang; Robert J.Barker et al. "Electromagnetic Wave Pumped Ion-Channel Free Electron Laser".

4. The following new papers have been drafted:
   A. Liu Shenggang; Robert J.Barker et al. "Dispersion Characteristics of PASOTRON slow wave structure with ion-channel taken into account";
   B. Liu Shenggang; Robert J.Barker et al. "A New Kind of Electron Beam-Wave Interactions in Magnetized Plasma Waveguide",
   C. Liu Shenggang; Robert J.Barker et al. "Linearized Field Theory of Electron Beam-Wave Interactions in Magnetized Plasma Waveguide",
   D. Liu Shenggang; Robert J.Barker et al. "Theoretical and Experimental Study of PASOTRON",
   E. Liu Shenggang; Robert J.Barker et al. "Theory of Ion-Channel Formulations in Plasma by Electron Beam".

5. Revisions are made and computer calculations are carried out for the following papers:
   A. Liu Shenggang; Robert J.Barker et al. "Basic Theoretical Formulations on Plasma Microwave Electronics", Part I and Part II;
   B. Liu Shenggang; Robert J.Barker and Dennis M. Manos "Theoretical Study of the Waveguide and Resonator for Plasma Microwave Excited Eximer Laser",

II. Academic Activities

1. The final review of the proposal on "Basic Research on Elecromagnetic Field Effects on Biological Cell/Cells and their Applications" by Liu Shenggang et al. in the program "Fundamental Research Project" was held on Dec.7th 1999. Prof. Liu was successful in obtaining a grant over 4 Million Chinese Yuan against strong competition. He plans to use this money to establish the "International Research Lab" according to the agreement signed by him and the Dean of College of Engineering and technology at Old Dominion University, Dr. Swart.

2. Prof. Kristiansen of Texas Technical University was invited to visit UESTC and to present several lectures.

3. Mr. Bookbander, the officer in charge of literature and education at the United States General Consulate in Chengdu, was invited to visit UESTC and present a lecture. His lecture was very well received and was excellent for the students, both undergraduate and graduate students.

4. Prof. Liu was attending the Sino-Hong Kong Seminar on "The Tendency of the Development of Electronic and Information Science and Technology in the 21. Century", Hong Kong University, Dec. 1-3, 1999. He was invited to present a talk on "Electromagnetic Wave Pumped Ion-Channel Free Electron Laser" This paper had been presented at the 24th International Conference on IRMMW in Monterey, CA, USA, September 1999.

III. Preparatory Work on the 25th Intern. Conference on IRMMW

to be held in Beijing, China, Sept.12-16, 2000. This International Conference will be held jointly with the 2nd Pacific-Asian Conference on Microwave and Millimeter Waves. Prof. Liu is the Conference Chairman for these two conferences. He is in the process of:

A. trying to obtain the sponsorship by IEEE. The 2nd Pacific-Asian Conference is sponsored by IEEE, but the 25th conference not.
B. organizing the committees for both conferences;
C. organizing the Plenary Talks and Invited Talks for both conferences;
D. preparing the final call for papers for both conferences.

IV. Proposed Research Work for the Period from January 2000 to December 2001

1. Almost all the material published in Russian in the Plasma Microwave Electronics area has been collected. This material is very valuable, since it provides information on what has been done and how it has been done in Russia. From this literature it can be concluded that the work carried on by Prof. Liu and coworkers on the "Electron Beam-Wave Interactions in Magnetized Plasma Waveguide" is very important. All research done in Russia was based on the approximation of "the external magnetic field is infinitely high or equals to zero". Therefore the work of Prof. Liu and coworkers is really valuable and creative; but still much work needs to be done.

2. A number of the manuscripts written in the previous grant period should be completed using computer calculations. In order to accelerate the calculation work and obtain the results as soon as possible, two young scientists who are familiar with this work, will be invited as Visiting Scholars to join the group of Prof. Liu, here in the United States. They will focus on computer calculations required to complete the manuscripts that have been drafted so far.

The proposed work will broaden and deepen the knowledge in "Plasma Microwave Electronics", a special area of science and technology.

Progress Reports I - IV

on the Project

Theoretical Studies on Microwave Plasma Electronics and Microhollow Cathode Discharges

Grant No: F49620-98-1-0065

Progress Report IV

Professor Liu Shenggang
Fellow IEEE

Department of Applied Science
The College of William and Mary

September1, 1999

Progress Report IV

Professor Liu Shenggang
Fellow IEEE

Department of Applied Science
The College of William and Mary

Since Jan.1998 I have presented three progress reports for the following periods of time:

Progress Report I, Jan. 1998---July. 1998;
Progress Report II, July. 1998---Dec. 1998;
Progress Report III, Jan. 1999---Jun. 1999.

A final report should be presented in March 2000 as required. Therefore, this Progress Report will be the Summary of the final report for the time period from Jun. 1999 to Sept. 1999 only about two months. I am very pleased that a lot of significant achievements have been obtained. In this report I would like to include only the Academic achievements, mainly papers, and some recent important academic activities. Others will be given in the Final Report.

I.Papers:

A. The following papers have been submitted for publication:

(1)."Electromagnetic Characteristics of a Spherical Biological Cell",
By Liu Shenggang; Robert J. Barker; Karl H.Schoenbach; et al.

(2)."Basic Theoretical Formulations on Plasma Microwave Electronics", Part A.
By Liu Shenggang; Robert J. Barker, et al.

(3)."Basic Theoretical Formulations on Plasma Microwave Electronics", Part B.
By Liu Shenggang; Robert J. Barker et al.

(4)."A New Hybrid Ion-Channel Maser Instability",
By Liu Shenggang; Robert J. Barker et al.

(5)."A New Kind of Waves Propagating Along Magnetized Plasma Waveguide", By Liu Shenggang; Robert J.Barker; et al.

(5)."Electromagnetic Wave Pumped Ion-channel Free Electron Laser", By Liu Shenggang; Robert J.Barker; et al.

B. The following papers have been presented at conferences:

(1). "Electromagnetic Characteristics of A Spherical Biological Cell", By Liu Shenggang; Robert J. Barker; Karl H.Schoenbach, et.al. Accepted for presentation at 98 FEL International Conference, Jefferson National Lab, New Port News, USA, Sept. 1998,

(2). "A New Hybrid Ion-Channel Maser Instability", By Liu Shenggang; Robert J. Barker, et al. presented at the 23rd International Conference on IRMMW, Essex University, UK, Sept. 1998,

(3). "Basic Theoretical Formulations on Plasma Microwave Electronics", By Liu Shenggang; Robert J. Barker, et.al. Invited talk, Presented at PERSE, Hong Kong, Feb. 1998, Invited talk at 4th International Conference on FIRMMW, Beijing, Aug.1998.

(4)."Electromagnetic Wave Pumped Ion-channel Free Electron Laser", By Liu Shenggang; Robert J. Barker, et al. Accepted to present at 4th Pacific Asian FEL Conference, Korea, Jun. 1999.

(5)."Electromagnetic Wave Pumped Ion-Channel Free Electron Laser", to be presented at 24th International Conference on IRMMW, Monterey, USA, Sept. 1999,

(6)."A New Kind of Waves Propagating Along Magnetized Plasma Waveguide", By Liu Shenggang; Robert J. Barker; et al. to be presented at 24th International Conference on IRMMW, Monterey, USA, Sept. 1999,

(8). "Basic Theoretical Formulations on Plasma Microwave Electronics" Part A and Part B, part of these papers will be involved in the Plenary Talk, "An Examination of Plasma Microwave Electronics", at 24th International Conference on IRMMW, By Dr. Robert J. Barker. Monterey, CA USA, Sept.5-10,1999.

C. Drafts of Papers:

The following drafts of papers have been worked out:

(1)."Theoretical Study on Micro-Hollow Cathode Discharge", By Liu

Shenggang; Karl H. Schoenbach; Robert J.Barker; et al.

(2)." Theory of Wave Propagation of Plasma Filled Helix and Helical Waveguide", By Liu Shenggang; Robert J. Barker; et al.

(3)." Nonlinear Theory of Plasma Filled Gyromonotron", By Liu Shenggang; Robert J. Barker; et al.

(4)."Theory of Waveguide System for Microwave Plasma Excited Excimer Laser",

(5)."Theory of Waveguide Cavity for Microwave Plasma Excited Excimer Laser",

(6)."Theory of Electromagnetic Pumped Free Electron Laser in a Cylindrical Waveguide", By Liu Shenggang; Robert J. Barkert; et al.

(7)."Theoretical Study on Ion-Channel RWO-FEL", By Liu Shenggang; Robert J. Barker; et al.

(8)."General Theory of Ion-Channel Free Electron Laser", By Liu Shenggang; Robert J. Barker; et al.

II. Recent Academic Activities:

The following recent academic activities are very important:

(1).Signed an Agreement with the Dean of Engineering College of ODU, Prof. S Swart, for establishment of the International Research Cooperative Laboratories between ODU and UESTC;

(2).Attending the First International Symposium on "Nonthermal Medical/Biological Treatment Using Electromagnetic Fields and Ionized Gases", held in Norfolk, US, April, 1999;

(3). Attending the 24th International Conference on IRMMW, to be held in Monterey, CA US, Sept.5-7, 1999,

(4), Attending the 5th Workshop on ECH Transmimtion Lines, to be held in Monterey, CA, US, Sept.1-3, 1999,

(5). Joining the work of the K.J.Button Prize Committee (I am the member of the committee),

(6). Joining the work of International FEL Prize Committee (I am the member of the committee),

(7).The Preparatory Work on the 25th International Conference on IRMMW, to be held in Beijing, P.R.China, (I am honored to be the Conference Chairman),

(8). The work on the Application of the National Key Project of the Fundamental Research, supported by the State Ministry of P.R.C. We have passed the first two debat and will take the Semi-Final debat This work if also of significant, and get the support from Dr. Robert

J.Barker, Prof . Karl H. Schoenbach and Prof. Dennis Manos. It is a big grant, each item will get tens million Chinese Yuan. So, the competition is very strong, We are making efforts to apply one item on "Fundamental Research on the Mechanism and Applications of Electromagnetic Effects on Biological Cell/Cells".

At the end of the report, I would like to take the opportunity to extend my sincere appreciation to Dr. Robert J. Barker, Prof. Karl H. Schoenbach and Prof. Dennis Manos for their kind and warm support, help and cooperation. My sincere thanks should also be given to all the people who give me kind help and support.

Progress Report III

(Jan. 1999—June 1999)

Professor Liu Shenggang
Fellow IEEE

Department of Applied Science
The College of William and Mary

Progress Report III

(Jan. 1999—Jun. 1999)

Professor Liu Shenggang
Fellow IEEE

Department of Applied Science
The College of William and Mary

The research work and the related activities carried on during this period of time (Jan.1999—Jun.1999) are reported here.

I. Research and Academic Work

A. Papers

1. The following manuscripts of the papers have been finished and submitted to the 24th International Conference on IRMMW to be held in Monterey, CA, USA, Sept. 1999.

   (1). Basic Theoretical Formulations of Microwave Plasma Electronics, PartA, and Part B.

   Liu, Shenggang, Robert J. Barker, etc.

   (2). Theory of Electromagnetic Wave Pumped Ion-Channel Free Electron Laser.

   Liu Shenggang, Robert J.Barker, Gao Hong etc.

   (3). A New Kind of Waves Propagating Along Magnetized Plasma Waveguide

   Liu Shenggang, Robert J. Barker, Yan Yang etc.

   The first one is for an invited talk, the speaker is Dr. Robert J. Barker; and the rest two are for contributed papers, the speaker is Liu Shenggang.

2. The manuscripts of the following papers have been submitted for publication:

(1). Basic Theoretical Formulations of Microwave Plasma Electronics, Part A,

(2). Basic Theoretical Formulations of Microwave Plasma Electronics, Part B.;

(3). Electromagnetic Wave Pumped Ion-Channel Free Electron Laser,

(4). A New Kind of Waves Propagating Along magnetized Plasma waveguide,

(5). Electromagnetic Characteristics of A spherical Biological Cell.

Liu Shenggang; Robert J. Barker; Karl H. Schoenbach, etc.

All the drafts of the above papers have been carefully revised.

3. The following drafts of new papers have been worked out:

(1). Theory of Rectangular Magnetized Plasma Filled Waveguide,

Liu Shenggang; Robert J. Barker, etc

(2). Theoretical Study of Waveguide System For Microwave Plasma Excited Excimer Laser

Liu Shenggang; Robert J. Barker; Dennis Manos etc.

(3). Theory of Electromagnetic Pumped FEL in A Cylindrical Magnetized Plasma Waveguide, Part A.

Liu Shenggang; Robert J. Barker, etc

4. A new cooperation project, " Microwave Plasma Excited Excimer Laser" with Prof. Dennis Manos in CWM started. It is a very important new area, and it may create a new area of Microwave Plasma Electronics.

5. Attending the "First International Symposium on Nonthermal Medical/Biological Treatment Using Electromagnetic Fields and Ionized Gases"(ElectroMed'99), April 11^{th}-14^{th}, 1999, Norfolk, VA. I was honored to serve as the co-chairman, the member of Program Committee and the member of Panel. I also chaired two sessions of the symposium.

I will also serve for the Special Issue of IEEE Trans. PL, as one of the Editors. The Symposium was initiated by Dr. Robert J. Barker, Prof. Karl H. Schoenbach and myself. The Symposium was very very successful indeed. Most organizing and preparatory works were carried on by Prof. Karl H. Schoenbach and Dr. Robert J. Barker.

6. Meeting with Dr. Robert J. Barker,

Taking the opportunity of attending the symposium, I have had a very nice and efficient meeting with Dr. Robert J. Barker in April 13th, afternoon. The following items were discussed:

(1) Submitting papers for publication.

We agreed that since we have obtained a lot of achievements of our cooperation, a number of papers have been finished, it is the time now to submit them for publication,

(2) Th new drafts of papers

(3) The revising work of these drafts of our new papers.

(4) The organizing work of the 25th International Conference on IRMMW to be held in Cina in the year 2000. I am honored to be the conference chairman, and Dr, Robert J. Barker is the standing member of the International Committee of the conference and also will be invited to serve as a member of program committee. So, I expect and I am sure that I can get strong support and help from Dr. Robert J. Barker. I cordially invited Dr. Barker to attend the preparatory meeting to be held in Aug, 1999. Dr. Robert J. Barker accepted the invitation with pleasure.

(5) . A new area of Microwave Plasma Electronics.

I have transferred from ODU to CWM since last Dec. Prof. Dennis Manos invited me to join the project of Microwave Plasma Excited Excimer Laser. I have found that it is a very important new area with very bright perspectives. And after discussion we agreed that the research work in this project may create a new area of Microwave Plasma Electronics.

(6) Since we have obtained a lot of achievements, and we will get more. I would like to suggest that it is natural, on the basis of our work, to write a book (Monograph book). We will discuss this in more detail later

II. Organizing and Preparatory work on 25th International Conference on IRMMW

The 25th International Conference on IRMMW, together with the 5th International Conference on FIRMMW will be held in China in the year 2000, and will be followed by the Asian-Pacific MMW Conference. I am honored to be the conference chairman of all these conferences. So it is a very hard job. A preparatory committee has been organized and we have had meeting twice. The third meeting will be held in May. A lot of preparatory work has been carried out.

(1). Successfully getting the universities in Hong Kong to be involved in. In March, I was invited to pay a visit to Hong Kong, I met the President and vice president of the City University of Hong Kong and the Head of the Department of ECE of Hong Kong University. I am very pleased that all of them give me very strong support and are willing to be involved in the preparatory work of the conference.

(2). I also get the strong support from the CIE (Chinese Institute of Electronics, I am the vice president of CIE). Most administrative preparatory work will be carried on by CIE.

(3). The change of the conference location.

Because the Southeast University is going to hold another International Conference, the president of the university does not want to hold the 25th IRMMW conference. After detailed discussion, a preliminary decision has been made that the 25th IRMMW, together with the 5th FIRMMW conference will be held in Beijing. The final decision will be made at the committee meeting in May.

(4). Next we should start the organization of committees of the conferences.

In all these works, I expect and I am sure that I will get very strong support and help from Dr. Robert J. Barker. I will do my best to make the conference a real successful one.

III. Seminar at Old Dominion University

I have prepared three topics for the seminar that I am required to give at ODU:

(1).Brief Introduction to UESTC,

(2). Research Activities in the Research Institute of High Energy Electronics, UESTC,

(3). Higher Education in China.

It seems that the last one is most interesting for ODU. I presented the seminar at ODU, March 26th afternoon. Totally, there were more than 80 audience, graduate students and some faculty members. There were a number of audience asking questions after my talk. It seems that the seminar was very successful.

IV. Work on the Second Application of the "Key National Project on Basioc Research of Science and Technology"

Last year I organized a team to apply the project on " Basic Research on Electromagnetic Fields/Waves Effects on Biological Cell/Cells and Their Applications". Unfortunately, we did not succeed. This year we decided to try again with strong confidence. Dr. Robert J. Barker and Prof. Karl H. Schoenbach and Prof. Denis Manos give us very strong support. And I am very pleased to say that the "First Symposium on Nonthermal Medical/Biological Treatment Using Electromagnetic Field and Ionized Gases" may bring very strong influences on our application. Besides, I have reorganized our team, and make it stronger. However, the competition is also very very strong! It seems that I always have very heavy load. I will do my best.

At the end of the report, I would like to extend my sincere appreciation to Dr. Robert J. Barker, Prof. Karl H. Schoenbach and Prof. Denis Manos for their kind and warm support, help and cooperation. My sincere thanks should also be given to all the people who give us kind help and support. I and my wife, Mrs. Jiang Chenqi, are really enjoyed very much indeed the work and staying here in USA.

Progress Report II

(July 1998—December 1998)

Professor Liu Shenggang
Fellow IEEE

Department of Electric and Computer Engineering
Old Dominion University (ODU)

Progress Report II

(July 1998—December 1998)

Professor Liu Shenggang
Fellow IEEE

Department of Electric and computer Engineering
Old Dominion University (ODU)

A lot of significant achievement have been obtained during this period of time. The research work and related activities done during the time period of July 1998 through December 1998 are reported in the report.

I. Academic work

A. Papers

1. The following papers have been revised:

(1). Basic theoretical formulations of microwave plasma electronics, Part A,

(2). Basic theoretical formulations of microwave plasma electronics, Part B,

The original four-part paper has been revised into two-part paper. Now it becomes the final draft of a new two-part paper.

(3). A new hybrid ion-channel maser instability,

The final draft of the paper.

(4). Basic Theory of Micro-hollow Cathode Discharge,

Start the numerical calculations.

2. Drafts of new papers:

(4). A new kind of waves in a waveguide filled with magnetized plasma. The draft for final revising.

(5). Wave propagation along a plasma helix and a helical waveguide filled with plasma(first draft).

(6). Nonlinear theory of Gyromonotron filled with plasma(first draft).

(7). Theory of Electromagnetic wave pumped Free Electron Laser(first draft).

All the above works are carried on in cooperation with Dr. Robert J. Barker.

B. Meeting and Discussion with Dr. Robert J. Barker.

Taking the opportunity of Dr. R.J.Barker&rsquos visit to ODU, we have two time meeting and discussion on our scientific cooperation, to revise our papers, to exchange our new ideas on our field and to discuss the drafts of our new drafts. It is very productive. I enjoyed very much all these meetings and discussions.

C. Preparatory work on the1st International Symposium on NONTHERMAL MEDICAL/BIOLOGICAL TREATMENTS USING ELECTROMAGNETIC FIELDS AND IONIZED GASES (co-chairman),

1. Join the preparatory work directed by Prof. Karl H. Schoenbach and Dr. Robert J. Barker.
2. Organizing papers and attendees from China.
3. Guest Editor for microwave effects

D. Preparatory work on the "25th International Conference on IRMM Waves" to be held in China in the year 2000, I am honored to be the conference chairman.

1. Organizing the committees,
2. Discussion with Chinese colleagues about the location of the conference,
3. Discussion with the scientists in all over the world on the nomination of plenary talks and invited talks,
4. Others.

I will do my best to make this conference a successful and fruitful one.

II. Organizing and heading the joint delegation of UESTC and TCL to visit USA.

UESTC (University of Electronic Science and Technology of China) is one of the key universities in China and TCL Holdings Co., Ltd. is the third largest company in electronic industry in China. We have invited to visit three universities (Stanford, ODU and New York Polytech), one National lab(Jefferson Lab) and 11 companies(Intel, Cardence, Synopsis, Laucent etc). This visit was very successful, it greatly promote the metual understanding and friendship between China and US. As a results, I have signed an agreement with the president of ODU, Prof. Koch, and an agreement with the New York Polytech Univ. has also prepared and will be signed soon.

III. Lectures to be given at Old Dominion University

Three lectures have been prepared for giving at ODU:

1. The research Activities in the Research Institute of High Energy Electronics in UESTC.
2. Introduction to the Higher Education Structure in China,
3. Introduction to the UESTC.

IV. Transfer from Old Dominion University to College of William & Merry

I have very smoothly transferred from the Old Dominion University to the College of William & Mary. Both the ODU and CWM are warmly welcome me and very kind and friend to me.

V. The consultant work on the Millimeter Wave systems for Old Dominion University

The lab of Prof. Karl H. Schoenbach of ODU wants to establish MM wave systems for research work, I have given consultant.

VI. Work on the application of the Fundamental Research Project of China on &ldquo Basic Research on the Effects of Electromagnetic Field on Bilogical Ceil/Cells&rdquo.

1. Organizing the application team, 5 institutions are involved: University of Electronic Science and Technology of China; Research Institute of Electronics of Chinese Academy of Sciences(CAS); Research Institute of Biophysics of CAS; Southwest China Medical University; Sichuan Agriculture University and Fudan University.

2. Preparatory meeting.

I have organized four meetings to discuss the draft of our report.

3. Drafting the application report.

A 80 pages report (in Chinese) has been worked out and has been put forward to the State Ministry of Science and Technology (SMST) .

According to the project, there are 40 items, and 15 of them have been determined last year. Unfortunately we failed. Now we are going to try again. I have visited the authority of State Ministry of Science and Technology, he promised to give us support. Since the grant is very big (60 Million Yuan per item), the competition is very strong. During the procedure of our application, I got very strong support and encourage from Dr. Robert J. Barker and Prof. Karl H. Schoenbach. It seems that their supports are very important and effective.

Appendix:

I. The abstracts of papers(here the abstracts of the final drafts are given):

1. Basic theoretical formulations of microwave electronics, Part A,
2. Basic theoretical formulations of microwave electronics, Part B,
3. A new hybrid ion-channel maser instability,
4. A new kind of waves in magnetized plasma waveguide.

II. Cooperative Agreement between University of Electronic Science and Technology of China and Old Dominion University

III. The consultant material of MM Wave systems for Old Dominion University

Appendix I.1

Basic theoretical formulations of plasma microwave electronics

Part A. General theoretical formulations of electron beam-wave

Interactions in magnetized plasma waveguide

Liu Shenggang, Fellow IEEE; R.J.Barker , Fellow IEEE,et. al.

Abstract. Basic theoretical formulations for electron beam-wave interactions in a plasma-filled waveguide immersed in a finite magnetic field are presented in this two-part paper. The general interaction and dispersion equations of the longitudinal and transverse interactions in both ammoth and corrugated magnetized plasma-filled waveguides are formulated. The resultant equation are then applied to examine the specifics of plasma Cherenkov radiation, plasma-filled travelling-wave-tube/backward-wave-oscillator (TWT/BWO), the plasma-filled electron cyclotron resonance maser (ECRM) and many types of beam-wave interactions including those involving ion-channels. Some possible new interactions in magnetized plasma filled waveguide that do not appear in previously published papers are proposed. A detailed discussion and analysis of the physics of the important role of plasma background are given. It is pointed out that in a magnetized plasma-filled waveguide, there are many interesting features of beam-wave interactions, two of them being most essential, one is that the transverse interactions are always accompanied by the longitudinal interactions. The other is that the magnetized plasma itself is strongly involved in the interaction mechanisms via an additional component of field. The paper consists of two parts. Part A presents general theoretical formulations of electron beam-wave interactions in magnetized plasma waveguide using only a fluid model for both the plasma and beam. Part B extends the analysis of the interaction by retaining a fluid treatment for the plasma-fill but substituting a kinetic theory treatment for the electron beam. It continues further to include a detailed treatment of the physical effects of the ion-channel that is formed in the plasma by an intense electron beam. In both parts of the paper, sample numerical calculations are presented in order to illustrate the physics.

Appendix I.2

Basic Theoretical Formulations of Microwave Plasma Electronics
Part B. Kinetic Theory of Electron Beam-Wave Interactions in a Magnetized Plasma Waveguide

Liu Shenggang, Fellow IEEE; R.J.Barker , Fellow IEEE,et. al.

Abstract. Based on the theory given in Part A of this paper, the kinetic theory of Electron Beam-Wave interactions in a magnetized plasma-filled waveguide is presented in this part. The dispersion equations of longitudinal and transverse interactions, both in smooth and corrugated waveguides, are derived by using kinetic theory, including the kinetic theory of plasma filled electron cyclotron resonance master (ECRM), a Cherenkov device and a TWT, BWO etc., a combination of Cherenkov-Cyclotron resonance are discussed. It is interesting to note that in a magnetized plasma waveguide, the transverse interaction (ECRM, for example) is always coupled with a longitudinal interaction.

Appendix I.3

A New Hybrid Ion-Channel Maser Instability

Liu Shenggang, Fellow IEEE; R.J.Barker , Fellow IEEE,et. al.

Abstract. A new hybrid maser instability has been found for the case of electron beam-wave interaction in a plasma-filled waveguide with ion-channel taken into account for Bo=0. A complete linear formulation and numerical calculations are presented. Some important and interesting features of this new hybrid instability are indicated.

Appendix I.4

A New Kind of Waves in Magnetized Plasma Waveguide

Liu Shenggang, Fellow IEEE; R.J.Barker , Fellow IEEE,et. al.

I. Abstract. A new kind of waves that can propagate along magnetized plasma waveguide has been found in the paper. This new kind of waves did not appear in published papers. It is indicated in the paper that this new kind of waves has very special characteristics: it contains a quasi-static field components and it exists in a sets of discrete frequencies. Both analytical theory and numerical calculations approve the existence of this kind of waves. The importance of this new kind of waves in both physics and mathematics is indicated in the paper.

Progress Report

(Jan. 1998—July 1998)

Professor Liu Shenggang

Department of Electric and Computer Engineering
Old Dominion University (ODU)

Progress Report

(Jan. 1998—July 1998)

Professor Liu Shenggang

Department of Electric and computer Engineering
Old Dominion University (ODU)

I have arrived at ODU on Jan. 21st 1998. I met Prof. K.H.Schoenbach immediately and discussed with him my research work at ODU. The three directions along which my research work has proceeded are:

I. Plasma Microwave Electronics
II. Microhollow Cathode Discharge
III. Electromagnetic Field Effect on Biological Cell/cells
IV. References

In the period which is covered by this report wide range of topics has been addressed. Some of the many achievements are very important. The work done during that period of time can be described as follows.

I. Plasma Microwave Electronics

The following achievements have been obtained:

a. A four-part paper "Basic Theoretical Formulations of Plasma Microwave Electronics" has been finished:

Part I "General Theoretical Formulations of Electron Beam-Wave Interactions in Magnetized Plasma Waveguide"

Part II. "Kinetic Theory of Electron Beam-Wave Interactions in Magnetized Plasma Waveguide"

Part III. "Theoretical Study on Electron Beam-Wave Interactions in Magnetized Plasma Waveguide with Ion-Channel Taken into Account"

Part IV. "Numerical Calculations"
It is a very important paper. In fact it consists of four papers. This is an invited keynote paper at the GA of APFA'98 and APPTC'98 (The First General Assembly of Assian Plasma and Fusion Association joint with the Third Asian Pacific Plasma Theory Conference), Sept. 21-25, 1998,

Beijing. This paper was also invited to be presented at other two Int. Conferences, but I was not able to attend.

b. A paper " A New Hybrid Ion-Channel Maser Instability" has been finished. This paper was presented at the 23rd International conference on Infrared and Millimeter Waves held in UK. Essex University, Sept. 7-12, 1998.
In this paper a new interaction mechanism has been proposed and studied.

Any one of these papers has been considered an important contribution to the field of research by the audience.

c. Preparatory work on the 25th International Conference to be held in the year 2000, where I am honored to be the conference chairman. A report has been made by me at the International Committee of the 23rd Int. Conf. held in the UK.

II. Microhollow Cathode Discharge

a. Collected and read a large number of materials on the "Microhollow Cathode Discharge" and its applications. It provides a good background for further study.

b. The draft of a paper "Theoretical Study on Micro-Hollow Cathode Discharge" has been prepared. The analytical theory has been finished.

III. Electromagnetic Field Effects on Biological Cell/Cells

a. Collected and read a large number of materials; it provides a good background for further study

b. A paper on " Electromagnetic Properties of Spherical Biological Cell" has been drafted and was accepted by the 20h FEL International Conference held at Thomson Jefferson National Lab, Williamsburg, Sept. 17-22,1998.

c. Joint the preparatory work for the "First International Symposium on Nonthermal Medical/Biological Treatments Using Electromagnetic Fields and Ionized Gases", including attending the preparatory meeting and drafting some materials, organized by Prof. Schoenbach, and Dr. Barker. I will be the Co-Chair of the Symposium.

During this period of time, Dr. Robert J. Barker, Prof. Karl H. Schoenbach, take very good care of me. I got very strong support from the University, the president, the department heads, and the Dean of the College of Engineering. The International Office has done a lot for me. I would like to take the opportunity to extend my sincere appreciation to all of them. I really enjoy my stay here together with my wife, Mrs. Chenqi Jiang.

I am sure that, based on the already obtained achievements, my research work in the remaining time will be very successful and fruitful. And my visit and stay here will certainly and greatly promote the scientific exchange and the friendship between China and US, in particular, between ODU and my university UESTC.

IV. References

1. **Basic Theoretical Formulations of Microwave Plasma Electronics**

   Part I: General Theoretical Formulations of Electron Beam-Wave Interaction In Magnetized Waveguide; Liu Shenggang**, *Senior Member, IEEE*, R. J. Barker*** *Fellow, IEEE*, Zhu Dajun**, and Yan Yang**

   Part II : Kinetic Theory of Electron Beam-Wave Interaction in Magnetized Waveguide; Liu Shenggang, Senior Menber, IEEE, R.J. Barker, Fellow, IEEE, Zhu Dajun, and Yan Yang

   Part III. Theoretical Study on Electron Beam-Wave Interaction in Magnetized Waveguide with the Ion-Channel Taken into Account; Liu Shenggang, R. J. Barker, Yan Yang , Zhu Dajun and Deng Jimdong

   Part IV. Numerical Calculations; Liu Shenggang, R.J. Barker, Yan Yang, Zhu Dajun and Gao Hong .

   Presented at the GA of APFA'98 & APPTC'98, as an invited keynote talk. Sept. 21-25, 1998, Beijing, China.

   The paper was invited to be presented at other Int. Conf. but I and othe authers were not able to attend.

2. **A New Hybrid Ion-Channel Maser Instabilty;** Liu Shenggang, R.J. Barker, Gao Hong, Zhu Dajun and Yan Yang. Presented at the 23rd International conference on IRMMW, Sept. 7-12, 1998, UK, Essex Univ.

3. **Electromagnetic Properties of A Spherical biological Cell;** Liu Shenggang, R.J.Barker, and Yu Guofen. Accepted for presentation at 20th FEL International Conference, Aug. 17-22, 1998, Thomas Jefferson National Lab, Williamsburg, USA. (I was not able to attend)

Liu Shenggang, Zhu Dajun,Guo Hong, Deng Jingdong , Yu Guofen and Yan Yang are with theUniversity of Electronic Science and Technology of China, Chengdu 610054, Sichuan, Peoples Republic Of China. Liu Shenggang is on sabbatical leave at ODU.

R.J. Barker is with the U. S. Air Force Office of Scientific Research, Washington, DC, USA.

Manuscripts
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List of Attached Manuscripts

1. Shenggang Liu, Robert J. Barker, Zhu Dajun, Yan Yang, and Gao Hong, "Basic Theoretical Formulations of Plasma Microwave Electronics Part I: A Fluid Model Analysis of Electron Beam-Wave Interactions".

2. Shenggang Liu, Robert J. Barker, Yan Yang, and Zhu Dajun, "Basic Theoretical Formulation of Plasma Microwave Electronics Part II: Kinetic Theory of Electron Beam-Wave Interactions".

3. Shenggang Liu, Robert J. Barker, Zhu Dajun, Yan Yang, and Gao Hong, "Basic Theoretical Formulations of Microwave Plasma Electronics" .

4. Robert J. Barker and Shenggang Liu, "A New Hybrid Ion-Channel Maser Instability".

5. Shenggang Liu, Robert J. Barker, Karl H. Schoenbach, Guofen Yu, and Chaoyu Liu, "Electromagnetic Characteristics of an Individual Spherical Biological Cell".

6. Shenggang Liu, Robert J. Barker, Yan Yang, and Zhu Dajun, "A New Type of Waves in Magnetized Plasma Waveguide".

7. Shenggang Liu, Robert J. Barker, Gao Hong, and Yan Yang, "Electromagnetic Wave Pumped Ion-Channel Free-Electron Laser".

8. Shenggang Liu, Zhu Dajun, Yan Yang, and Robert J. Barker, "Dispersion Characters of Wave Propagation Along a Plasma Waveguide in a Finite Magnetic Field".

9. Robert J. Barker and Shenggang Liu, "An Examination of Plasma Microwave Electronics".

10. Karl H. Schoenbach, Robert J. Barker, and Shenggang Liu, "Nonthermal Medical/Biological Treatments Using Electromagnetic Fields and Ionized Gases".

11. Shenggang Liu and Robert J. Barker, "Electromagnetic Wave Field Excited by a Single Moving Charged Particle in Plasma".

12. Shenggang Liu, et al, "Theory of Waveguide System for Microwave Plasma Excited Excimer Laser".

13. Shenggang Liu, et al, "Theoretical Study on Micro-Hollow Cathode Discharge".

14. Shenggang Liu, et al, "Microwave Cavity Excitation".

15. Shenggang Liu, "Higher Education in P. R. China," presentation to graduate students and Faculty in the Electrical and Computer Engineering Department of ODU (viewgraphs).

1. **Shenggang Liu, Robert J. Barker, Zhu Dajun, Yan Yang, and Gao Hong, "Basic Theoretical Formulations of Plasma Microwave Electronics Part I: A Fluid Model Analysis of Electron Beam-Wave Interactions".**

8

Submited for publication in IEEE Trans. on plasma Science.

Basic Theoretical Formulations of Plasma Microwave Electronics[+]

Part I: A Fluid Model Analysis of Electron Beam-Wave Interactions

Liu Shenggang[**], *Fellow, IEEE*, Robert J. Barker[***], *Fellow, IEEE*, Zhu Dajun[**], Yan Yang[**], and Gao Hong[**]

Key Words: plasma microwave electronics, BWO, gyrotron, plasma-fill, fluid model

Abstract-- **Basic theoretical formulations for electron beam-wave interactions in a plasma-filled waveguide immersed in a finite magnetic field are presented in this two-part paper. The general interaction and dispersion equations of the longitudinal and transverse interactions in both smooth and corrugated magnetized plasma-filled waveguides are formulated. These are then applied to examine plasma Cherenkov radiation, the plasma-filled travelling-wave-tube/backward-wave-oscillator (TWT/BWO), the plasma-filled electron cyclotron resonance maser (ECRM) and other beam-wave interactions including those involving ion-channels. Some possible new interactions in a magnetized plasma-filled waveguide (MPW) are proposed. A detailed discussion and analysis of the important physical role of the plasma background are given. Many interesting features of beam-wave interactions in an MPW (magnetized plasma-filled waveguide) are pointed out, ~~two~~ three of them being most essential. One is that transverse interactions are always accompanied by longitudinal interactions. The second other is that the magnetized plasma itself is strongly involved in the interaction mechanisms via an additional component of the field. This paper consists of two parts. Part I presents general theoretical**

The last one is that the plasma filled ECRM should operate at high cyclotron harmon...

[+] This paper was presented in part at the 21st Int. Conf. on IR/MM Waves. Berlin, Germany, 1996, and also at the 22nd Int. Conf. on IR/MM Waves, Wintergreen, Virginia, USA, 1997.

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formulations of electron beam-wave interactions in an MPW using a fluid model for *both* the plasma and the beam. It also includes a detailed treatment of the physical effects of the ion channel that is formed in the plasma by an intense electron beam. Part II extends the analyses by retaining a fluid treatment for the plasma-fill but substituting a kinetic theory treatment for the electron beam. In both parts of the paper, sample numerical calculations are presented in order to illustrate the physics. The theory presented in both part I and part II the paper is based on the "given field" approach which has been being widely used successfully in science and technology, in particular in Microwave Electronics.

I. Introduction

The goal of Microwave Electronics is to create improved microwave devices that provide higher output power with higher efficiency. There have been great achievements in this area in recent years. Gigawatt output power levels have been reached in pulsed relativistic microwave devices [1], [2] and megawatts in continuous (CW) operation (e.g. - using a gyrotron) [3], [4]. Further enhancement of output power and efficiency now faces some fundamental limits. One of the most important limits is the space charge effect due to mutual repulsion of the electrons that limits the maximum current density due to the beam "blow-up" instability [5].

Space charge also causes the potential at the center of the electron beam to be lower than that on the outer surface of the beam. This effect may thus also cause the defocusing of the beam, degrading the beam quality. For particularly intense beams and/or long beam paths this can even cause the beam to strike the surface of the vacuum envelope, decreasing the beam transmission. These and other effects of space charge seriously perturb beam-wave interactions and degrade the performance and behaviour of microwave devices.

The most promising approach for increasing the net beam current density is by introducing a background plasma fill. By this means, the beam space charge can be fully or partially neutralized.

The maximum current limit of the beam is then determined by the Pierce instability [5]. Plasma space charge neutralization may thus provide more than a five-fold increase of the beam current.

It has already been shown that a plasma fill can dramatically enhance the output power and efficiency of some microwave devices, such as the relativistic BWO and plasma Cherenkov radiation devices [6]-[10]. Plasma filling has even been attempted in Free Electron Lasers (FELs) [11]-[13]. Even more intriguing, plasma filling has permitted the development of some entirely new plasma-based microwave devices (e.g., the PASOTRON [14] and the Plasma Wave Tube [15]) that eliminate the need for an axial magnetic field.

These studies also show that a plasma fill has some fundamental effects on beam-wave interactions. When an electron beam is injected into a plasma, an ion-channel is formed which improves beam transmission. This is the so-called "channelling effect" of plasma that improves beam quality. The plasma may also provide a focusing force on electrons to reduce transverse diffusion in the beam, further improving beam quality [14]. The resultant ion-channel gives rise to its own beam-wave interactions [16]-[18]. Those as well as a new combined instability mechanism are presented and analysed in Section VI of this paper. Computer calculations are also presented for both longitudinal and transverse interactions. These calculations show many interesting characteristics of beam-wave interactions for the case of a plasma fill.

The study of beam-wave interactions in the presence of a plasma fill has formed a new field of science and technology, namely Plasma Microwave Electronics (PME). Almost two decades ago in 1981, L. S. Bogdankevich and his colleagues published a paper entitled "Plasma Microwave Electronics" [8]. Later, in 1992, M. V. Kuzelev et al and Y. Carmel et al. both presented papers on "Relativistic Plasma Microwave Electronics" [9]. These papers give good reviews of the main work and achievements obtained at that time. Although a large number of additional papers have been published more recently, there still remains much work to be done. The fundamental influences of a plasma fill on beam-wave interactions are not yet completely understood. For example, a magnetiz plasma fill couples the TE and TM modes in waveguides and resonators. But this important fact ignored in almost all published papers to-date. Beam-wave interactions are still treated by using T modes or TM modes separately.

The presence of the plasma, in particular when there is a magnetic field, causes importar changes to the wave and wave fields in the waveguide or resonator. The TE and TM modes ar coupled; they cannot exist independently. This coupling generates the hybrid HE and EH modes [19] [20]. The magnetized plasma fill enables varieties of propagating waves which could not exist in th vacuum case or even in the case of a plasma fill without a magnetic field [21]. It will also be show later in this paper (Section II) that ~~m~~agnetized plasma even produces ~~some~~ additional components o the wave field, completely ~~changing~~ the field ~~pattern. These factors significantly influence beam-wave interactions.~~ The plasma itself is intimately involved in the interactions, making them much more complicated and rich.

The clear importance of space-charge effects leads one to also consider introducing plasma into gyrotrons to increase their output power. To the best of the authors' knowledge, there are only a few papers published on plasma-filled gyrotrons. Reference [22] tried to explain the output power enhancement of a plasma-filled relativistic backward wave oscillator (BWO) through a combination of Cherenkov radiation and the electron cyclotron resonance instability. It will be shown later in this paper that actual electron beam-wave interactions in an MPW, when the ECRM is taken into account, are much more complicated.

The detailed analyses presented in this two-part paper vastly expands upon and generalises upon the authors' previous work in this field [20] which only examined wave propagation in an MPW in the absence of an electron-beam. This paper goes on to demonstrate four major physics issues inherent in PME devices. First, a magnetized plasma fill strongly changes the behavior of wave propagation in a waveguide. Specifically, the TE and TM modes can ~~no longer~~ exist independently; ~~they~~ are replaced by the EH and HE hybrid modes. Also, with ~~there are~~ two eigenvalues and two corresponding eigenfunctions instead of only the one that exists for the vacuum case or for the cases where the applied axial magnetic field, B_0, goes to zero or infinity. Therefore we have:

$$E_z = A_1 J_m(p_1 R) + A_2 J_m(p_2 R) \tag{1}$$

$$H_z = A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R) \tag{2}$$

where the wave factor $\exp\{j(\omega t - k_z z - m\varphi)\}$ is neglected, and the two eigenvalues, p_1 and p_2, are given by:

$$p_{1,2}^2 = \frac{1}{2\varepsilon_1}\left[-k_z^2(\varepsilon_1+\varepsilon_2)+k^2\left(\varepsilon_1\varepsilon_3+\varepsilon_1^2-\varepsilon_2^2\right)\right] \pm \frac{1}{2\varepsilon_1}\cdot \left\{\left[-k_z^2(\varepsilon_3-\varepsilon_1)+k^2\left(\varepsilon_1\varepsilon_3-\varepsilon_1^2+\varepsilon_2^2\right)\right]^2+4k^2k_z^2\varepsilon_2^2\varepsilon_3\right\}^{\frac{1}{2}} \tag{3}$$

$$h_{1,2} = \frac{\left(-k_z^2+k^2\varepsilon_1\right)\varepsilon_3-\varepsilon_1 p_{1,2}^2}{j\omega\mu_0 k_z \varepsilon_2} \tag{4}$$

If the waveguide is completely filled with plasma, we have:

$$A_i = \begin{cases} A & i=1 \\ -\dfrac{J_m(p_2 R_c)}{J_m(p_1 R_c)} \times A & i=2 \end{cases} \tag{5}$$

Therefore, the field pattern, the cut-off frequency (thus the wave number) and the dispersion relations are all changed. It is clear that formulations based on the TE mode or TM mode alone (e.g. – [23]) are insufficient for rigorously dealing with the full contribution of a magnetized plasma fill.

The second major PME physics issue is that the magnetized plasma background strongly changes not only the behavior of wave propagation but also the character of the beam-wave interactions. Since the ~~TE and TM modes are~~ always coupled, E_z and $\vec{E}_\perp$ always exist simultaneously. If the electron beam has both longitudinal and transverse components of motion, it is inherent that the longitudinal (J_z~E_z) and the transverse ($\bar{J}_\perp - \bar{E}_\perp$) interactions in the waveguide are always present together. This implies that Cherenkov type and TWT/BWO type interactions are always accompanied by ECRM and/or gyro-peniotron type interactions.

The third major PME physics issue is that the magnetized background plasma itself is deeply involved in the beam-wave interactions. Moreover since there are varieties of waves that can propagate along an MPW, clearly there must be coupling between the waves through the electron beam. This coupling (e.g.- parametric coupling) may lead to instabilities. This kind of instability has been studied in [24], however, it was there again based only on the TM mode without taking into consideration the fact that the TM and TE modes are always coupled together.

Finally, the fourth major PME physics issue involves the dispersion relations for plasma-filled systems. The dispersion characteristics of wave propagation in a circular cylindrical MPW is shown in Fig.A-1 of Appendix A. This figure shows that there are at least three kinds of waves that can propagate through the waveguide: plasma waves (the Trivelpiece-Gould or T-G modes, in the frequency range $\omega \le \omega_p$); cyclotron waves (in the range $\mathrm{Max}(\omega_p, \omega_c) \le \omega \le \omega_h$); and waveguide waves ($\omega > \omega_h$). All these waves are electromagnetic waves; all of them can interact with the electron beam.

It is very interesting to note that in an MPW some of the cyclotron waves in the frequency range: ($\omega_p < \omega < \omega_c$ or $\omega_c < \omega < \omega_p$) are natural backward waves (negative dispersion). These waves can directly interact with the electron beam since their phase velocity may be less than the speed of light. In particular, we can even use these backward waves to construct a BWO (backward wave oscillator) without a periodic structure. Magnetized plasma filling, therefore, makes beam-wave interactions both more complicated and richer.

Since the cutoff frequencies of waveguide modes are higher than $\omega_b = (\omega_{ce}^2 + \omega_{pe}^2)^{1/2}$, the ECRM in plasma filled waveguide, therefore should operate at higher cyclotron harmonics. This new physics will

Following this introduction, the field equations for an MPW are given in Section II. Section III deals with the general interaction equations for both longitudinal and transverse beam-wave interactions in such a waveguide. The interactions in a periodic system (corrugated MPW) are discussed in section IV. Dispersion equations for both the smooth and corrugated plasma waveguides are given in Section V. Section VI deals with beam-wave interactions with the ion-channel taken account, with sample numerical calculation given in Section VII. Some new interactions that may occur in these systems are given in Section VIII. Detailed discussion and conclusion ~~analysis~~ are given in Section IX. Section X concludes with a summary of the major points. Some detailed derivations are given in several appendices at the end of this paper.

II. Field Equations in an MPW

For the sake of completeness, a brief review of the

In this section, equations for the electromagnetic field components in a magnetized plasma, in both uniform and corrugated waveguides will be given. ~~It is well known~~ [1], [3]-[5] that in a plasma filled waveguide immersed in a finite axial magnetic field the TM and TE modes are always coupled to form the HE and EH hybrid modes. Thus the field expressions are complicated. To simplify the later mathematical manipulations and, in particular, to obviate the influence of the plasma background on the beam-wave interactions, the field components can be split into four parts. The field components produced by the H_z field are defined as the "TE-like" part, while the components produced by E_z are defined as the "TM-like" part. The two additional field parts are associated with the response of the plasma background and are thus defined as the "plasma-produced TE-like" part and the "plasma produced" TM-like part.

According to the fluid model, magnetized plasma can be described by the following permittivity tensor:

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_1 & j\varepsilon_2 & 0 \\ -j\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \tag{6}$$

where:

$$\varepsilon_1 = 1 - \frac{\xi^2}{1-\tau^2} \quad ; \quad \varepsilon_2 = -\frac{\xi^2 \tau}{1-\tau^2} \quad ; \quad \varepsilon_3 = 1 - \xi^2 \tag{7}$$

$$\xi = \frac{\omega_p}{\omega} \quad ; \quad \tau = \frac{\omega_c}{\omega} \tag{8}$$

$$\omega_p = \left(\frac{n_e e^2}{m_0 \varepsilon_0} \right)^{\frac{1}{2}} \quad ; \quad \omega_c = \frac{eB_0}{m_0} \tag{9}$$

ω_p is the electron plasma frequency of the background plasma, and ω_c is the electron cyclotron frequency. Collision effects are neglected.

The field components can then be expressed as follows (See Appendix A for details):

$$\left. \begin{aligned} E_z &= \sum_i A_i J_{mi} \\ H_z &= \sum_i A_i h_i J_{mi} \end{aligned} \right\} \quad (i=1,2) \tag{10}$$

$$\left. \begin{aligned} E_R &= E_{R1} + E_{R2} + E_{R1p} + E_{R2p} \\ E_\varphi &= E_{\varphi 1} + E_{\varphi 2} + E_{\varphi 1p} + E_{\varphi 2p} \\ H_R &= H_{R1} + H_{R2} + H_{R1p} + H_{R2p} \\ H_\varphi &= H_{\varphi 1} + H_{\varphi 2} + H_{\varphi 1p} + H_{\varphi 2p} \end{aligned} \right\} \tag{11}$$

The subscripts "1" and "2" denote the "TE-like" and "TM-like" part, respectively, and "*1p*" and "*2p*" denote the "plasma-produced TE-like" and "plasma-produced TM-like" part, respectively. ~~From~~ The formulae and the field components can found in Appendix A. ~~we get~~:

$$\left.\begin{aligned}
E_{R1} &= \sum_i \frac{A_i}{D} \omega\mu_0 h_i K^2 \frac{m}{R} J_{mi} \\
E_{R2} &= \sum_i \frac{A_i}{D} \left(-jk_z K^2 p_i J'_{mi}\right) \\
E_{R1p} &= \sum_i \frac{A_i}{D} \omega\mu_0 h_i k_g^2 p_i J'_{mi} \\
E_{R2p} &= -\sum_i \frac{A_i}{D} jk_z k_g^2 \frac{m}{R} J_{mi}
\end{aligned}\right\} \tag{12}$$

Move to Appendix A

$$\left.\begin{aligned}
E_{\varphi 1} &= \sum_i \frac{A_i}{D} j\omega\mu_0 h_i K^2 p_i J'_{mi} \\
E_{\varphi 2} &= \sum_i \frac{A_i}{D} k_z K^2 \frac{m}{R} J_{mi} \\
E_{\varphi 1p} &= \sum_i \frac{A_i}{D} j\omega\mu_0 h_i k_g^2 \frac{m}{R} J_{mi} \\
E_{\varphi 2p} &= \sum_i \frac{A_i}{D} k_z k_g^2 p_i J'_{mi}
\end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned}
B_{R1} &= \sum_i \frac{A_i \mu_0}{D} \left(-jk_z h_i K^2 p_i J'_{mi}\right) \\
B_{R2} &= \sum_i \frac{A_i \mu_0}{D} \left(-\omega\varepsilon_0 \varepsilon_1 K^2 \frac{m}{R} J_{mi}\right) \\
B_{R1p} &= \sum_i \frac{A_i \mu_0}{D} \left(-jk_z h_i k_g^2 \frac{m}{R} J_{mi}\right) \\
B_{R2p} &= \sum_i \frac{A_i \mu_0}{D} \left(-\omega\varepsilon_0\right)\varepsilon_2 \left(k_z^2 p_i J'_{mi} - k_g^2 \frac{m}{R} J_{mi}\right)
\end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned}
B_{\varphi 1} &= \sum_i \frac{A_i \mu_0}{D} k_z h_i K^2 \frac{m}{R} J_{mi} \\
B_{\varphi 2} &= \sum_i \frac{A_i \mu_0}{D} \left(-j\omega\varepsilon_0 \varepsilon_1 K^2 p_i J'_{mi}\right) \\
B_{\varphi 1p} &= \sum_i \frac{A_i \mu_0}{D} k_z h_i k_g^2 p_i J'_{mi} \\
B_{\varphi 2p} &= \sum_i \frac{A_i \mu_0}{D} j\omega\varepsilon_0 \varepsilon_2 \left(k_g^2 p_i J'_{mi} - k_z^2 \frac{m}{R} J_{mi}\right)
\end{aligned}\right\} \tag{15}$$

where $J_{mi} = J_m(p_i R)$, $J'_{mi} = J'_m(p_i R)$ and the wave factor, $\exp\{j\omega t - jk_z z - jm\varphi\}$, has been neglected. Other parameters are defined in Appendix A, (A-7).

For the corrugated waveguide, the fields are expanded into spatial harmonics to obtain:

$$\left.\begin{aligned} E_z &= \sum_s E_{z,s} e^{j\{\omega t - k_{z,s} z - m\varphi\}} \\ H_z &= \sum_s H_{z,s} e^{j\{\omega t - k_{z,s} z - m\varphi\}} \end{aligned}\right\} \tag{16}$$

and similarly for the E_R, E_φ, B_R and B_φ components; where:

$$k_{z,s} = k_z + \frac{2\pi s}{L_p} \qquad (s = 0,\ \pm 1,\ \pm 2,\ \ldots) \tag{17}$$

We can then get all the field components ($E_{R1,s}$, $E_{R2,s}$, $E_{R1p,s}$, $E_{R2p,s}$, $E_{\varphi 1,s}$, $E_{\varphi 2,s}$, $E_{\varphi 1p,s}$, $E_{\varphi 2p,s}$, $B_{R1,s}$, $B_{R2,s}$, $B_{R1p,s}$, $B_{R2p,s}$, $B_{\varphi 1,s}$, $B_{\varphi 2,s}$, $B_{\varphi 1p,s}$, and $B_{\varphi 2p,s}$) (see Appendix A) ~~by means of replacing (A_i, h_i, K^2, p_i, and J_{mi}, J'_{mi}) by ($A_{i,s}$, $h_{i,s}$,~~ K_s^2, $p_{i,s}$, and $J_{mi,s}$, $J'_{mi,s}$), respectively, where $J_{mi,s} = J_m(p_{i,s}R)$ and $J'_{mi,s} = J'_m(p_{i,s}R)$ and:

$$A_{i,s} = \begin{cases} A_s & i = 1 \\ -\dfrac{J_m(p_{2,s}R_c)}{J_m(p_{1,s}R_c)} \times A_s & i = 2 \end{cases} \tag{18}$$

$$p_{i,s}^2 = \frac{1}{2\varepsilon_1}\left[-k_{z,s}^2(\varepsilon_1 + \varepsilon_2) + k^2\left(\varepsilon_1\varepsilon_3 + \varepsilon_1^2 - \varepsilon_2^2\right)\right] + (-1)^{i+1}\frac{1}{2\varepsilon_1} \cdot \left\{\left[-k_{z,s}^2(\varepsilon_3 - \varepsilon_1) + k^2\left(\varepsilon_1\varepsilon_3 - \varepsilon_1^2 + \varepsilon_2^2\right)\right]^2 + 4k^2 k_{z,s}^2 \varepsilon_2^2 \varepsilon_3\right\}^{\frac{1}{2}} \tag{19}$$

$$h_{i,s} = \frac{\left(-k_{z,s}^2 + k^2\varepsilon_1\right)\varepsilon_3 - \varepsilon_1 p_{i,s}^2}{j\omega\mu_0 k_{z,s}\varepsilon_2} \tag{20}$$

Note that all the plasma-produced field parts are proportional to $k_g^2 = k^2\varepsilon_2$ (see Appendix A). Therefore, when the plasma is absent, or the magnetic field $B_0 \to 0$ or $B_0 \to \infty$, we have $\varepsilon_2 = 0$ and these field parts vanish. For the vacuum case ($\varepsilon_2=0$, $\varepsilon_1=\varepsilon_3=1$) we have independent TE and TM modes, and the TE-like and TM-like fields automatically become the fields of the TE and TM modes, respectively.

III. General Equations Governing Electron Beam-wave Interactions in a Smooth-Walled MPW

Maxwell's equations can be written as:

$$\left.\begin{aligned} &\nabla \times \bar{E} = -j\omega\mu_0 \bar{H} \\ &\nabla \times \bar{H} = j\omega\varepsilon_0 \bar{D} + \bar{J} \\ &\nabla \cdot \bar{D} = \rho \\ &\bar{D} = \varepsilon_0 \bar{\varepsilon} \cdot \bar{E} \\ &\nabla \cdot \bar{B} = 0 \end{aligned}\right\} \tag{21}$$

From (21), we can obtain the following two sets of equations, one set in terms of the longitudinal field components; the other in terms of the transverse field components.

A. Interaction equations expressed in terms of the longitudinal fields:

From (21), we get the following beam-wave interaction equations:

$$\nabla_\perp^2 E_z + aE_z = bH_z + j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0\varepsilon_1}\rho \tag{22}$$

$$\nabla_\perp^2 H_z + cH_z = dE_z - \left(\nabla \times \bar{J}\right)_z - \frac{\omega\varepsilon_2}{\varepsilon_1}\rho \tag{23}$$

where:

$$\left.\begin{aligned} &a = \left(-k_z^2 + k^2\varepsilon_1\right)\varepsilon_3 / \varepsilon_1 \\ &b = jk_z\omega\mu_0\varepsilon_2 / \varepsilon_1 \\ &c = -k_z^2 + k^2\left(\varepsilon_1^2 - \varepsilon_2^2\right) / \varepsilon_1 \\ &d = -jk_z\omega\varepsilon_0\varepsilon_2\varepsilon_3 / \varepsilon_1 \end{aligned}\right\} \tag{24}$$

The transverse field components may be expressed in terms of E_z and H_z as follows:

$$\begin{aligned} \bar{E}_\perp = \frac{1}{D}\Big[&-jk_z K^2 \nabla_\perp E_z + \omega\mu_0 k_g^2 \nabla_\perp H_z - k_z k_g^2 \nabla_\perp E_z \times \bar{e}_z \\ &-j\omega\mu_0 K^2 \nabla_\perp H_z \times \bar{e}_z + j\omega\mu_0 K^2 \bar{J}_\perp + \omega\mu_0 k_g^2 \bar{J}_\perp \times \bar{e}_z\Big] \end{aligned} \tag{25}$$

$$\bar{H}_\perp = \frac{1}{D}\Big[-\omega\varepsilon_0\varepsilon_2 k_z^2 \nabla_\perp E_z - jk_z K^2 \nabla_\perp H_z - j\omega\varepsilon_0\left(k_g^2\varepsilon_2 - K^2\varepsilon_1\right)\nabla_\perp E_z \times \bar{e}_z$$
$$-k_z k_g^2 \nabla_\perp H_z \times \bar{e}_z + k_z k_g^2 \bar{J}_\perp - jk_z K^2 \bar{J}_\perp \times \bar{e}_z\Big] \qquad (26)$$

where:

$$K^2 = k^2\varepsilon_1 - k_z^2; \quad k_g^2 = k^2\varepsilon_2; \quad D = K^4 - k_g^4; \quad k^2 = \omega^2\varepsilon_0\mu_0 \qquad (27)$$

Eq. (22) can be used to study longitudinal interactions (e.g., plasma Cherenkov radiation devices), while (23) can be used to study transverse interaction (e.g., an ECRM with plasma fill).

When the plasma fill is absent, ω_p=0 and we have ε_2=0, $\varepsilon_1=\varepsilon_3$=1. Equations (22) and (23) then reduce to:

$$\nabla_\perp^2 E_z + \left(k^2 - k_z^2\right)E_z = j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0}\rho \qquad (28)$$

$$\nabla_\perp^2 H_z + \left(k^2 - k_z^2\right)H_z = -\left(\nabla \times \bar{J}\right)_z \qquad (29)$$

Equations (25) and (26) for this case become:

$$\left.\begin{aligned}\bar{E}_\perp &= \frac{j}{k^2 - k_z^2}\left[-k_z\nabla_\perp E_z - \omega\mu_0\nabla_\perp H_z \times \bar{e}_z + \omega\mu_0\bar{J}_\perp\right] \\ \bar{H}_\perp &= \frac{j}{k^2 - k_z^2}\left[-k_z\nabla_\perp H_z + \omega\varepsilon_0\nabla_\perp E_z \times \bar{e}_z - k_z\bar{J}_\perp \times \bar{e}_z\right]\end{aligned}\right\} \qquad (30)$$

where:

$$K^2 = k^2 - k_z^2; \quad k_g^2 = 0; \quad D = K^4; \quad k^2 = \omega^2\varepsilon_0\mu_0 \qquad (31)$$

All the equations thus reduce to the well known vacuum case.

However, in the vacuum case, the TE and TM modes can be independent, so we get:

$$\left.\begin{aligned}\bar{E}_\perp &= \frac{j}{k^2 - k_z^2}\left[-k_z\nabla_\perp E_z + \omega\mu_0\bar{J}_\perp\right] \\ \bar{H}_\perp &= \frac{j}{k^2 - k_z^2}\left[\omega\varepsilon_0\nabla_\perp E_z \times \bar{e}_z - k_z\bar{J}_\perp \times \bar{e}_z\right]\end{aligned}\right\} \qquad (32)$$

for the TM modes, and

$$\left.\begin{aligned}\bar{E}_\perp &= \frac{j\omega\mu_0}{k^2-k_z^2}\left[-\nabla_\perp H_z \times \bar{e}_z + \bar{J}_\perp\right]\\ \bar{H}_\perp &= \frac{-jk_z}{k^2-k_z^2}\left[\nabla_\perp H_z + \bar{J}_\perp \times \bar{e}_z\right]\end{aligned}\right\} \tag{33}$$

for the TE modes.

For the case of a plasma fill, when $B_0 \to \infty$, (22) becomes:

$$\nabla_\perp^2 E_z - \left(k^2 - k_z^2\right)\left(1 - \omega_p^2/\omega^2\right)E_z = j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0}\rho \tag{34}$$

while when $B_0 \to 0$, we have:

$$\nabla_\perp^2 E_z + \left[k^2\left(1-\omega_p^2/\omega^2\right) - k_z^2\right]E_z = j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0\left(1-\omega_p^2/\omega^2\right)}\rho \tag{35}$$

B. Interaction equations expressed in terms of the transverse fields:

The beam-wave interaction equations (22) and (23) can also be expressed in terms of the transverse field components as follows:

$$\begin{aligned}&\nabla_\perp^2 \bar{E}_\perp + \frac{k^2\left(\varepsilon_1^2 - \varepsilon_2^2\right) - k_z^2\varepsilon_3}{\varepsilon_1}\bar{E}_\perp = \\ &\frac{jk_z\omega\mu_0\varepsilon_2}{\varepsilon_1}\bar{H}_\perp + k_z\omega\mu_0\frac{\left(\varepsilon_3-\varepsilon_1\right)}{\varepsilon_1}\bar{e}_z \times \bar{H}_\perp + j\omega\mu_0\bar{J}_\perp - \frac{\omega\mu_0\varepsilon_2}{\varepsilon_1}\bar{e}_z \times \bar{J}_\perp + \frac{\nabla_\perp\rho}{\varepsilon_0\varepsilon_1}\end{aligned} \tag{36}$$

and:

$$\nabla_\perp^2 \bar{H}_\perp + \left(k^2\varepsilon_3 - k_z^2\right)\bar{H}_\perp = -jk_z\omega\varepsilon_0\varepsilon_2\bar{E}_\perp + k_z\omega\varepsilon_0\left(\varepsilon_3-\varepsilon_1\right)\bar{e}_z \times \bar{E}_\perp - \left(\nabla \times \bar{J}\right)_\perp \tag{37}$$

Then E_z and H_z can be written as:

$$E_z = -\frac{j}{\omega\varepsilon_0\varepsilon_3}\left[\left(\nabla \times \bar{H}_\perp\right)\cdot \bar{e}_z - J_z\right] \tag{38}$$

$$H_z = \frac{j}{\omega\mu_0}\left(\nabla \times \bar{E}_\perp\right)\cdot \bar{e}_z \tag{39}$$

When ω_p=0, (36) and (37) reduce to:

$$\nabla_\perp^2 \bar{E}_\perp + \left(k^2 - k_z^2\right)\bar{E}_\perp = j\omega\mu_0 \bar{J}_\perp + \frac{\nabla_\perp \rho}{\varepsilon_0} \tag{40}$$

and:

$$\nabla_\perp^2 \bar{H}_\perp + \left(k^2 - k_z^2\right)\bar{H}_\perp = -\left(\nabla \times \bar{J}\right)_\perp \tag{41}$$

Equation (40) has been used for the kinetic theory of the ECRM with space charge taken into consideration [25]. Equations (38) and (39) remain unchanged.

For the case of a plasma fill, when $B_0 \to \infty$, $\varepsilon_1=1$, $\varepsilon_2=0$, $\varepsilon_3 = 1 - \omega_p^2/\omega^2$, (36) and (37) become:

$$\nabla_\perp^2 \bar{E}_\perp + \left[k^2 - k_z^2\left(1 - \omega_p^2/\omega^2\right)\right]\bar{E}_\perp = -\frac{k_z \mu_0 \omega_p^2}{\omega}\bar{e}_z \times \bar{H}_\perp + j\omega\mu_0 \bar{J}_\perp + \frac{\nabla_\perp \rho}{\varepsilon_0} \tag{42}$$

and:

$$\nabla_\perp^2 \bar{H}_\perp + \left[k^2\left(1 - \omega_p^2/\omega^2\right) - k_z^2\right]\bar{H}_\perp = -\frac{k_z \varepsilon_0 \omega_p^2}{\omega}\bar{e}_z \times \bar{E}_\perp - \left(\nabla \times \bar{J}\right)_\perp \tag{43}$$

and when $B_0 \to 0$, we have:

$$\nabla_\perp^2 \bar{E}_\perp + \left[k^2\left(1 - \frac{\omega_p^2}{\omega^2}\right) - k_z^2\right]\bar{E}_\perp = j\omega\mu_0 \bar{J}_\perp + \frac{\nabla_\perp \rho}{\varepsilon_0\left(1 - \omega_p^2/\omega^2\right)} \tag{44}$$

and:

$$\nabla_\perp^2 \bar{H}_\perp + \left[k^2\left(1 - \frac{\omega_p^2}{\omega^2}\right) - k_z^2\right]\bar{H}_\perp = -\left(\nabla \times \bar{J}\right)_\perp \tag{45}$$

Equations (38) and (39) again remain unchanged.

The general interaction equations obtained above cover almost all kinds of interactions in a smooth waveguide with or without a plasma fill. Which interaction equation to use depends on the specific case and on the preferences of the author.

It should be pointed out here that since the TE and TM modes are always coupled to each other, the longitudinal and transverse interactions are also both present if transverse electron motion exists.

IV. Beam-Wave Interactions in a Corrugated MPW

Wave propagation through a corrugated MPW has been previously studied [20]. According to Floquet's theorem, the fields should be expanded as follows:

$$\left.\begin{aligned}\bar{E} &= \sum_s \bar{E}_s e^{-jk_{z,s}z}\\ \bar{H} &= \sum_s \bar{H}_s e^{-jk_{z,s}z}\end{aligned}\right\} \tag{46}$$

where:

$$k_{z,s} = k_{z0} + \frac{2\pi s}{L} \tag{47}$$

L is the spatial period, and s is the spatial harmonic number. Once again, we can find the interaction equations expressed in terms of both the longitudinal as well as the transverse field components.

A. Longitudinal Field Expressions:

Substituting (46) and (47) into (22) and (23) yields the following for the longitudinal field components:

$$\sum_s \left(\nabla_\perp^2 E_{z,s} + aE_{z,s}\right) = \sum_s \left(b_s H_{z,s} + j\omega\mu_0 J_{z,s} - \frac{jk_{z,s}}{\varepsilon_0\varepsilon_1}\rho_s\right) \tag{48}$$

$$\sum_s \left(\nabla_\perp^2 H_{z,s} + c_s H_{z,s}\right) = \sum_s \left(d_s E_{z,s} - \left(\nabla \times \bar{J}_s\right)_z - \frac{\omega\varepsilon_2}{\varepsilon_1}\rho_s\right) \tag{49}$$

where:

$$\left.\begin{aligned}a_s &= \left(-k_{z,s}^2 + k^2\varepsilon_1\right)\varepsilon_3/\varepsilon_1\\ b_s &= jk_{z,s}\omega\mu_0\varepsilon_2/\varepsilon_1\\ c_s &= -k_{z,s}^2 + k^2\left(\varepsilon_1^2 - \varepsilon_2^2\right)/\varepsilon_1\\ d_s &= -jk_{z,s}\omega\varepsilon_0\varepsilon_2\varepsilon_3/\varepsilon_1\end{aligned}\right\} \tag{50}$$

The transverse field components $\bar{E}_{\perp,s}$ and $\bar{H}_{\perp,s}$ may be found from (25) and (26) by simply replacing (k_z, K^2, D and $J_\perp$) by ($k_{z,s}$, K_s^2, D_s and $J_{\perp,s}$) respectively, where:

$$K_{z,s}^2 = k^2\varepsilon_1 - k_{z,s}^2; \qquad D_{z,s} = K_{z,s}^4 - k_g^4 \tag{51}$$

Equation (48) can be used for plasma-filled devices like the TWT and BWO, while (49) can be used for a plasma-filled ECRM in a periodic system.

In the plasma-free case, $\omega_p=0$, $\varepsilon_2=0$, $\varepsilon_1=\varepsilon_3=1$, (48) and (49) reduce to:

$$\sum_s\left[\nabla_\perp^2 E_{z,s}+\left(k^2-k_z^2\right)E_{z,s}\right]=\sum_s\left(j\omega\mu_0 J_{z,s}-\frac{jk_{z,s}}{\varepsilon_0}\rho_s\right) \tag{52}$$

$$\sum_s\left[\nabla_\perp^2 H_{z,s}+\left(k^2-k_z^2\right)H_{z,s}\right]=-\sum_s\left(\nabla\times\bar{J}_s\right)_z \tag{53}$$

When $\omega_p\neq0$, $B_0\rightarrow\infty$, we have:

$$\sum_s\left[\nabla_\perp^2 E_{z,s}+\left(k^2-k_{z,s}^2\right)\left(1-\omega_p^2/\omega^2\right)E_{z,s}\right]=\sum_s\left(j\omega\mu_0 J_{z,s}-\frac{jk_{z,s}}{\varepsilon_0}\rho\right) \tag{54}$$

$$\sum_s\left[\nabla_\perp^2 H_{z,s}+\left(k^2-k_z^2\right)H_{z,s}\right]=-\sum_s\left(\nabla\times\bar{J}_s\right)_z \tag{55}$$

and when $B_0\rightarrow0$, we have:

$$\sum_s\left\{\nabla_\perp^2 E_{z,s}+\left[k^2\left(1-\omega_p^2/\omega^2\right)-k_{z,s}^2\right]E_{z,s}\right\}=\sum_s\left[j\omega\mu_0 J_{z,s}-\frac{jk_{z,s}}{\varepsilon_0\left(1-\omega_p^2/\omega^2\right)}\rho_s\right] \tag{56}$$

$$\sum_s\left\{\nabla_\perp^2 H_{z,s}+\left[k^2\left(1-\omega_p^2/\omega^2\right)-k_{z,s}^2\right]H_{z,s}\right\}=-\sum_s\left(\nabla\times\bar{J}_s\right)_z \tag{57}$$

Equations (54) and (55) are commonly used for the plasma-filled BWO and TWT, when the coupling between the TE and TM modes due to the magnetized background plasma is neglected.

B. Transverse Field Expressions:

Similarly, substituting (46) and (47) into (36) and (37) yields the following for the transverse field components:

$$\sum_s\left[\nabla_\perp^2\bar{E}_{\perp,s}+\frac{k^2\left(\varepsilon_1^2-\varepsilon_2^2\right)-k_{z,s}^2\varepsilon_3}{\varepsilon_1}\bar{E}_{\perp,s}\right]=\sum_s\left[\frac{jk_z\omega\mu_0\varepsilon_2}{\varepsilon_1}\bar{H}_{\perp,s}+j\omega\mu_0\bar{J}_{\perp,s}+\right.$$
$$\left.k_{z,s}\omega\mu_0\frac{\left(\varepsilon_3-\varepsilon_1\right)}{\varepsilon_1}\bar{e}_z\times\bar{H}_{\perp,s}-\frac{\omega\mu_0\varepsilon_2}{\varepsilon_1}\bar{e}_z\times\bar{J}_{\perp,s}+\frac{\nabla_\perp\rho_s}{\varepsilon_0\varepsilon_1}\right] \tag{58}$$

and:

$$\sum_s\left[\nabla_\perp^2\bar{H}_{\perp,s}+\left(k^2\varepsilon_3-k_{z,s}^2\right)\bar{H}_{\perp,s}\right]=\sum_s\left[-jk_{z,s}\omega\varepsilon_0\varepsilon_2\bar{E}_{\perp,s}+k_z\omega\varepsilon_0\left(\varepsilon_3-\varepsilon_1\right)\bar{e}_z\times\bar{E}_{\perp,s}-\left(\nabla\times\bar{J}_s\right)_\perp\right] \tag{59}$$

The longitudinal field components $E_{z,s}$ and $H_{z,s}$ can be found by using the same substitutions mentioned before.

In the plasma-free case, (58) and (59) reduce to:

$$\sum_s\left[\nabla_\perp^2\bar{E}_{\perp,s}+\left(k^2-k_{z,s}^2\right)\bar{E}_{\perp,s}\right]=\sum_s\left(j\omega\mu_0\bar{J}_{\perp,s}+\frac{\nabla_\perp\rho_s}{\varepsilon_0\varepsilon_1}\right) \tag{60}$$

$$\sum_s\left[\nabla_\perp^2\bar{H}_{\perp,s}+\left(k^2-k_{z,s}^2\right)\bar{H}_{\perp,s}\right]=-\sum_s\left(\nabla\times\bar{J}_s\right)_\perp \tag{61}$$

and when $\omega_p\neq0$, $B_0\rightarrow\infty$, they reduce to:

$$\sum_s\left\{\nabla_\perp^2\bar{E}_{\perp,s}+\left[k^2-k_{z,s}^2\left(1-\omega_p^2/\omega^2\right)\right]\bar{E}_{\perp,s}\right\}=\sum_s\left[j\omega\mu_0\bar{J}_{\perp,s}-\frac{k_{z,s}\mu_0\omega_p^2}{\omega}\bar{e}_z\times\bar{H}_{\perp,s}+\frac{\nabla_\perp\rho_s}{\varepsilon_0}\right] \tag{62}$$

$$\sum_s\left\{\nabla_\perp^2\bar{H}_{\perp,s}+\left[k^2\left(1-\omega_p^2/\omega^2\right)-k_{z,s}^2\right]\bar{H}_{\perp,s}\right\}=\sum_s\left[-\frac{\omega_p^2}{\omega}k_z\varepsilon_0\bar{e}_z\times\bar{E}_{\perp,s}-\left(\nabla\times\bar{J}_s\right)_\perp\right] \tag{63}$$

and when $B_0\rightarrow0$, we have:

$$\sum_s\left\{\nabla_\perp^2\bar{E}_{\perp,s}+\left[k^2\left(1-\omega_p^2/\omega^2\right)-k_{z,s}^2\right]\bar{E}_{\perp,s}\right\}=\sum_s\left(j\omega\mu_0\bar{J}_{\perp,s}+\frac{\nabla_\perp\rho_s}{\varepsilon_0\varepsilon_1}\right) \tag{64}$$

$$\sum_s\left\{\nabla_\perp^2\bar{H}_{\perp,s}+\left[k^2\left(1-\omega_p^2/\omega^2\right)-k_{z,s}^2\right]\bar{H}_{\perp,s}\right\}=-\sum_s\left(\nabla\times\bar{J}_s\right)_\perp \tag{65}$$

The interaction equations obtained above can be used for both linear and non-linear beam-wave interactions in general cases for either a vacuum or a plasma fill. It should also be pointed out that longitudinal and transverse interactions are also both present in a periodic structure provided that transverse electron motion is present.

V. Dispersion Equations of Electron Beam-Wave Interactions in an MPW Using a Fluid Model

Based on the interaction equations given in the last section, the dispersion equations for different kinds of beam-wave interactions in an MPW can be obtained. Those for longitudinal interactions, for the case of a TWT/BWO, and for transverse interactions are discussed individually in the three subsections below.

A. Dispersion Equations for Longitudinal Interactions

Starting with (22), Appendix B derives the following dispersion equation (B-5):

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = -j\omega\mu_0\varepsilon_1 \iint J_z E_{z0}^* ds + \frac{jk_z}{\varepsilon_0}\iint \rho E_{z0}^* ds \tag{66}$$

Plasma Cherenkov radiation devices typically rely on longitudinal interactions. The dispersion character of wave propagation along an MPW without an electron beam (see Appendix A) can be described by this dispersion equation. From this equation we can also get k_{z0}. One sees that without dielectric loading, all the waves, except the plasma wave modes, are fast waves. In plasma Cherenkov devices, therefore, it is necessary to insert dielectric loading or to use a corrugated waveguide.

From (22), the continuity equation yields:

$$\nabla \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0 \tag{67}$$

and by using a fluid model for the electron beam, we obtain

$$J_z = j\frac{\omega\varepsilon_0\omega_b^2}{(\omega - k_z v_0)^2}E_z \tag{68}$$

We can then rewrite the dispersion equation (66) as:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2(k_z - k_{z0})P_{HE} = \frac{\omega_b^2\left(\varepsilon_1 k^2 - k_z^2\right)}{(\omega - k_z v_0)^2}P_E \tag{69}$$

where:

$$\left.\begin{aligned} P_E &= \iint E_z \cdot E_{z0}^{*} ds \\ P_{HE} &= \iint H_z \cdot E_{z0}^{*} ds \end{aligned}\right\} \tag{70}$$

Here k_{z0} is determined by the dispersion equation for the plasma filled waveguide without an electron beam and:

$$\omega_b^2 = \frac{\rho_0 e}{m_0\gamma_0^3\varepsilon_0} \quad ; \quad \gamma_0 = \left(1-\beta_0^2\right)^{-1/2} \quad ; \quad \beta_0 = v_0\sqrt{\varepsilon_0\mu_0} \tag{71}$$

where ρ_0 is the equilibrium charge density of the electron beam, and v_0 is the equilibrium velocity of the beam. The calculation of beam current using kinetic theory is given in Part II of this paper.

The solutions of (69) can be obtained easily as follows:

$$\begin{aligned} k_z = \frac{1}{2\left[\varepsilon_3 + \dfrac{\omega_b^2}{(\omega - k_z v_0)^2}\right]P_E}\Bigg\{ & -j\omega\mu_0\varepsilon_2 P_{HE} \pm \Bigg\{-\omega^2\mu_0^2\varepsilon_2^2 P_{HE}^2 + \\ & 4\left[\varepsilon_3 + \frac{\omega_b^2}{(\omega - k_z v_0)^2}\right]P_E\left[k_{z0}^2\varepsilon_3 P_E + j\omega\mu_0\varepsilon_2 P_{HE}k_{z0} + \frac{\omega_b^2\varepsilon_1 k^2}{(\omega - k_z v_0)^2}P_E\right]\Bigg\}^{1/2}\Bigg\} \end{aligned} \tag{72}$$

From (72), we derive the instability criteria for plasma Cherenkov radiation to be:

$$4\left[\varepsilon_3 + \frac{\omega_b^2}{(\omega - k_z v_0)^2}\right]P_E^2\left[k_{z0}^2\varepsilon_3 + j\omega\mu_0\varepsilon_2\frac{P_{HE}}{P_E}k_{z0} + \frac{\omega_b^2\varepsilon_1 k^2}{(\omega - k_z v_0)^2}\right] < \omega^2\mu_0^2\varepsilon_2^2 P_{HE}^2 \tag{73}$$

When ω_p=0 and P_{HE}=0, (72) yields:

$$k_z^2 - k_{z0}^2 = \frac{\omega_b^2\left(\varepsilon_1 k^2 - k_z^2\right)}{\left(\omega - k_z v_{z0}\right)^2} \tag{74}$$

For the plasma filled case, when $B_0 \to \infty$ or $B_0 \to 0$, (74) becomes:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right) = \frac{\omega_b^2}{\left(\omega - k_z v_{z0}\right)^2}\left(\varepsilon_1 k^2 - k_z^2\right) \tag{75}$$

Equation (75) is the same as that given in [22]. The instability criteria, (73), then reduces to:

$$\left[\varepsilon_3 + \frac{\omega_b^2}{\left(\omega - k_z v_0\right)^2}\right]\left[k_{z0}^2\varepsilon_3 + \frac{\omega_b^2 \varepsilon_1 k^2}{\left(\omega - k_z v_0\right)^2}\right] < 0 \tag{76}$$

B. The Dispersion Equations for a TWT/BWO

In a BWO or TWT, a corrugated waveguide is often used. From (48), we can follow the same procedures as those of Section 1 of Appendix B to obtain:

$$\begin{aligned}&\sum_s\left[\varepsilon_3\left(k_{z.s}^2 - k_{z0.s}^2\right)P_{E.s} + j\omega\mu_0\varepsilon_2\left(k_{z.s} - k_{z0.s}\right)P_{HE.s}\right] = \\ &\sum_s\left[-j\omega\mu_0\varepsilon_1 \iint J_z E_{z0}^* ds + \frac{jk_z}{\varepsilon_0}\iint \rho E_{z0}^* ds\right]\end{aligned} \tag{77}$$

that can be rewritten as

$$\sum_s\left[\varepsilon_3\left(k_{z.s}^2 - k_{z0.s}^2\right)P_{E.s} + j\omega\mu_0\varepsilon_2\left(k_{z.s} - k_{z0.s}\right)P_{HE.s}\right] = \sum_s\left[\frac{\omega_b^2\left(\varepsilon_1 k^2 - k_{z.s}^2\right)}{\left(\omega - k_{z.s}v_0\right)^2}P_{E.s}\right] \tag{78}$$

If only the synchronous term is considered, we get:

$$\varepsilon_3\left(k_{z.q}^2 - k_{z0.q}^2\right)P_{E.q} + j\omega\mu_0\varepsilon_2\left(k_{z.q} - k_{z0.q}\right)P_{HE.q} = \frac{\omega_b^2\left(\varepsilon_1 k^2 - k_{z.q}^2\right)}{\left(\omega - k_{z.q}v_0\right)^2}P_{E.q} \tag{79}$$

In the case of no plasma fill, (78) reduces to:

$$\sum_s \left(k_{z,s}^2 - k_{z0,s}^2\right) P_{E,s} = \sum_s \left[\frac{\omega_b^2 \left(k^2 - k_{z,s}^2\right)}{\left(\omega - k_{z,s} v_0\right)^2} P_{E,s} \right] \tag{80}$$

If we only take one synchronous term s=q in (80), we get:

$$\left(k_{z,q}^2 - k_{z0,q}^2\right) P_{E,q} = \frac{\omega_b^2 \left(k^2 - k_{z,q}^2\right)}{\left(\omega - k_{z,q} v_0\right)^2} P_{E,q} \tag{81}$$

Eq. (81) is the same as that given in [25].

C. The Dispersion Equations for Transverse Interactions:

A typical and very important transverse interaction is that found in the ECRM. We now consider this interaction with a plasma fill. From (36), the procedures of Section 1 of Appendix B yields the following dispersion equation:

$$\begin{aligned} &\varepsilon_3 \left(k_z^2 - k_{z0}^2\right) P_{E_\perp} + j\omega\mu_0 \varepsilon_2 \left(k_z - k_{z0}\right) P_{HE_\perp} + \omega\mu_0 \left(\varepsilon_3 - \varepsilon_1\right)\left(k_z - k_{z0}\right) P_{EH_\perp} = \\ &-j\omega\mu_0 \varepsilon_1 \iint \bar{J}_\perp \cdot \bar{E}_{\perp 0}^* ds - \frac{1}{\varepsilon_0} \iint \left(\nabla_\perp \rho\right)_\perp \cdot \bar{E}_{\perp 0}^* ds \end{aligned} \tag{82}$$

When plasma is absent and the space charge term is neglected, (82) reduces to:

$$\left(k_z^2 - k_{z0}^2\right) P_{E_\perp} = -j\omega\mu_0 \iint \bar{J}_\varphi \cdot \bar{E}_{\varphi 0}^* ds \tag{83}$$

which can be rewritten as:

$$\left(\frac{\omega^2}{c^2} - k_c^2 - k_z^2\right) = j \frac{\omega\mu_0}{P_{E_\perp}} \iint \bar{J}_\varphi \cdot \bar{E}_{\varphi 0}^* ds \tag{84}$$

Equation (84) is simply the one used for vacuum gyrotron devices [27].

For a corrugated waveguide, (82) becomes:

$$\begin{aligned} &\sum_s \left[\varepsilon_3 \left(k_{z,s}^2 - k_{z0}^2\right) P_{E_\perp,s} + j\omega\mu_0 \varepsilon_2 \left(k_{z,s} - k_{z0}\right) P_{HE_\perp,s} + \omega\mu_0 \left(\varepsilon_3 - \varepsilon_1\right)\left(k_{z,s} - k_{z0}\right) P_{EH_\perp,s} \right] \\ &= -\sum_s \left[j\omega\mu_0 \varepsilon_1 \iint \bar{J}_{\perp,s} \cdot \bar{E}_{\perp 0,s}^* ds + \frac{1}{\varepsilon_0} \iint \left(\nabla_\perp \rho\right)_\perp \cdot \bar{E}_{\perp 0,s}^* ds \right] \end{aligned} \tag{85}$$

For the vacuum case, the space charge term is neglected and we obtain:

$$\sum_s \left(k_{z,s}^2 - k_{z0}^2\right) P_{E_\perp,s} = -\sum_s j\omega\mu_0 \iint \bar{J}_{\varphi,s} \cdot \bar{E}_{\varphi 0,s}^* ds \tag{86}$$

At this point, it is important to again remember that, for the case of a magnetized plasma fill, transverse interactions are accompanied by longitudinal interactions.

Several important points must be highlighted here. First, according to (9)-(14), each field component is split into four parts, and correspondingly, J_z and $\bar{J}_\perp$ are also divided into four parts. Second, it therefore follows that each of the integrations (P_E, $P_{E\perp}$, P_{HE}, $P_{HE\perp}$, $\iint J_z \cdot E_{z0}^* ds$ and $\iint \bar{J}_\varphi \cdot \bar{E}_{\varphi 0}^* ds$) has 16 terms. This makes the interaction and dispersion equations very complicated. Third, because of the coupling of the TE and TM modes, E_z and $E_\perp$ always exist simultaneously. Therefore, we always have transverse and longitudinal interactions together. This is the most important feature of beam-wave interactions in a magnetized plasma waveguide.

VI. Theoretical Analysis of Electron Beam-Wave Interactions in an MPW with an Ion-Channel Taken into Account

As mentioned in the introduction, when an electron beam is injected into a plasma, an ion-channel may be formed if the beam density is relatively high. In most published papers dealing with beam-wave interactions in plasma waveguides [1], [3]-[5], [8], [9] the ion-channel effect is neglected. This is valid when the beam density is so low that the effects of the electron beam on the plasma background are not significant. However, when the beam density is high, say $n_b >> n_p$ (or even $n_b \geq n_p$), where n_b is the beam density and the n_p is the background plasma density, the ion-channel may play an essential role in beam-wave interactions and cannot be neglected.

A theoretical analysis of electron beam-wave interactions in an MPW with the ion-channel taken into consideration is given in this section using a fluid model. The theory for a plasma filled ECRM with an ion-channel will be given in Part II of this paper. First, a general review of the formation of an ion-channel is presented in Subsection A below. Subsection B then deals with the dispersion equations for longitudinal beam-wave interactions in an MPW with the ion-channel taken into account. The corresponding transverse interactions are analyzed in Part II of this paper using kinetic theory. Beam-wave interactions in an unmagnetized plasma waveguide are also included there. Some detailed derivations are provided in Appendix C.

A. The Formation of an Ion Channel in an MPW

Electron beam propagation through plasma has been an important topic of study in both physics and electrical engineering for a long time [16]-[18]. In this section, we first give a general review and then present some assumptions to be used in the detailed analysis.

As an electron beam propagates through a plasma, it continuously expels plasma electrons away from the beam volume. This expulsion is partial or complete depending on the ratio of the beam density to the background plasma density (n_b/n_p), leaving the heavy ions of the plasma to provide focusing and neutralization for the beam. Therefore, for the case where the beam density is relatively high, an ion-channel can be formed. A rough estimate can be made that the radius of the ion-channel, R_i, as $R_i \cong b(n_b / n_p)^{1/2}$, where b is the radius of electron beam. This implies that when $n_b > n_p$, the ion-channel radius is even larger than that of the electron beam.

In the theoretical analysis that follows, we will deal with both a solid and a hollow electron beam. The solid beam case is shown in Fig.1(a) and that for a hollow beam in Fig.1(b).

Fig. 1 shows that, for a solid beam, the ion-channel is formed both inside and outside the beam radius if $n_b > n_p$. For a hollow beam, an ion-channel can be formed in the beam volume itself as well as outside that volume, but inside the beam radius, the plasma still remains.

Based on the above discussion of ion-channel formation, the following concepts of the model for further analysis can be deduced. Since we are using the "applied field" approach, the following models for studying the longitudinal and transverse interactions are very useful, as shown in Fig. 2. Under the "applied field" approach, it is understood that both the electron motion and also the perturbed electron current density are calculated by assuming that the beam electrons are moving in the field of the plasma waveguide without the electron beam present. Therefore, now with the ion-channel, we have two "plasma waveguide models", as shown in Fig. 2(a) and Fig. 2(b). Introducing the concept of the "plasma waveguide model" greatly simplifies the theoretical analysis that follows. Our first step in studying the beam-wave interactions is simply to calculate the field in the "simplified plasma waveguide model."

Using this approach, the interaction equations and the dispersion equations given in Sections III, IV, and V can also be used. The only necessary modification is that now the beam-wave interactions take place in the ion-channel region.

B. Dispersion Equations for the Longitudinal Interactions

As mentioned above, we now calculate the field in the "simplified MPW Model" for longitudinal interactions involving a solid electron beam. This model is shown in Fig. 2(a), and the detailed derivations can be found in Appendix C. Since the beam-wave interactions take place in the ion-channel region, (66) from Section V above can be used:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E = -j\omega\mu_0\varepsilon_1 \iint J_z E_{z0}^* ds + \frac{jk_z}{\varepsilon_0} \iint \rho E_{z0}^* ds \tag{87}$$

In this equation, we consider that in the ion-channel, $\varepsilon_1 = 1 - \omega_p^2/\omega^2$ and $\varepsilon_2 = 0$, and that for the heavy ions, $\omega_{ci}/\omega = 0$. Then, a similar dispersion equation can be obtained:

$$k_z^2 - k_{z0}^2 = \frac{\omega_b^2\left(k^2 - k_z^2\right)}{\left(\omega - k_z \upsilon_0\right)^2} \tag{88}$$

It should be noted that in (87) and (88), k_{z0} denotes the phase constant for wave propagation in the "simplified MPW model" shown in Fig. 2(a).

It is not difficult to derive the dispersion relation for longitudinal interactions in a corrugated MPW with an ion-channel as follows:

$$\sum_s \left(k_{z,s}^2 - k_{z0,s}^2\right) P_{E,s} = \sum_s \frac{\omega_b^2\left(k^2 - k_{z,s}^2\right)}{\left(\omega - k_{z,s} v_0\right)^2} P_{E,s} \tag{89}$$

If one only considers the synchronous term, this reduces to:

$$k_{z,s}^2 - k_{z0,s}^2 = \frac{\omega_b^2\left(k^2 - k_{z,s}^2\right)}{\left(\omega - k_{z,s} v_0\right)^2} \tag{90}$$

where "s" indicates the s-th spatial harmonic.

C. Interactions in Devices Such as the PASOTRON

Beam-wave interactions may also take place in plasma waveguides without a magnetic field. The PASOTRON device [14], [26] is a good example of this case. Here we consider such interactions with an ion-channel taken into account. We begin with the smooth-walled configuration. For that case, the field expressions are as follows (only considering the TM mode) (see Fig. 2a.):

For Region I (~~ion-channel~~ plasma region, $R_i \le R \le a$ ~~$0 \le R \le R_b$~~):
plasma

$$E_z = A_1 J_m(pR) + A_2 N_m(pR) \tag{91}$$

√

$$E_R = -\frac{jk_z}{p}\left[A_1 J_m'(pR) + A_2 N_m'(pR)\right] \tag{92}$$

$$E_\varphi = \frac{k_z m}{p^2 R}[A_1 J_m(pR) + A_2 N_m(pR)] \tag{93}$$

$$H_R = \frac{-\omega\varepsilon_0\varepsilon_1 m}{p^2 R}[A_1 J_m(pR) + A_2 N_m(pR)] \tag{94}$$

$$H_\varphi = -\frac{j\omega\varepsilon_0\varepsilon_1}{p}\left[A_1 J_m'(pR) + A_2 N_m'(pR)\right] \tag{95}$$

where

$$\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2} \tag{96}$$

Ion-channel $0 \le R \le R_i$

For Region II (~~plasma~~ Ion-channel region, ~~$Ri \ge R \le 0$~~):

$$E_z = A_3 J_m(k_c R) \tag{97}$$

$$E_R = -\frac{jk_z}{k_c}[A_3 J_m'(k_c R)] \tag{98}$$

$$E_\varphi = \frac{k_z m}{k_c^2 R}[A_3 J_m(k_c R)] \tag{99}$$

$$H_R = -\frac{\omega\varepsilon_0 m}{k_c^2 R}[A_3 J_m(k_c R)] \tag{100}$$

$$H_\varphi = -\frac{j\omega\varepsilon_0}{k_c}[A_3 J_m'(k_c R)] \tag{101}$$

where

$$p^2 = k^2 - k_z^2 \varepsilon_1 \tag{102}$$

$$k_c^2 = k^2 - k_z^2 \tag{103}$$

The boundary conditions are:

At R=a: $E_{zI} = 0$ (104)

At R=Ri: $E_{zI} = E_{zII}$, $E_{\varphi I} = E_{\varphi II}$ $H_{\varphi I} = H_{\varphi II}$ (105)

Simple mathematics yields the coefficients:

$$A_2 = -\frac{J_m(pa)}{N_m(pa)} A_1, \tag{106}$$

$$A_3 = \frac{J_m(pR_i)N_m(pa) - J_m(pa)N_m(pR_i)}{J_m(k_cR_i)N_m(pa)} A_1 \tag{107}$$

and dispersion equations for the simplified plasma waveguide model:

$$\Delta = \begin{vmatrix} J_m(pa) & N_m(pa) & 0 \\ J_m(pR_i) & N_m(pR_i) & -J_m(k_cR_i) \\ \frac{\varepsilon_1}{p^2} J_m(pR_i) & \frac{\varepsilon_1}{p^2} N_m(pR_i) & -\frac{1}{k_c^2} J_m(k_cR_i) \end{vmatrix} = 0 \tag{108}$$

$$\frac{\varepsilon_1}{p} \qquad \frac{\varepsilon_1}{1} \qquad -\frac{\varepsilon_3}{k_c}$$

The dispersion relation for beam-wave interactions can be obtained as :

$$k_z^2 - k_{z0}^2 = \frac{\omega_b^2 (k^2 - k_z^2)}{(\omega - k_z \upsilon_{z0})^2} \tag{109}$$

where k_{z0} may be derived from (108).

The dispersion relation for interactions in the full corrugated-waveguide PASOTRON device is:

$$\sum_s \left(k_{z,s}^2 - k_{z0,s}^2\right) \overset{P_{\varepsilon,s}}{\cancel{P_{\varepsilon,s}}} = \sum_s \frac{\omega_b^2 \left(k^2 - k_{z,s}^2\right)}{\left(\omega - k_{z,s}\upsilon_{z0}\right)^2} \overset{P_{\varepsilon,s}}{\cancel{P_{\varepsilon,s}}} \tag{110}$$

Equation (110) can be used for the cases of a BWO/TWT, a plasma waveguide without a magnetic field (unmagnetized plasma), and for the cases where an ion-channel is present.

VII. Sample Numerical Calculations for Longitudinal Interactions

In the preceding sections of this paper, complete theoretical formulations for beam-wave interactions in an MPW have been derived. In order to illustrate the theory, computer calculations have been carried out for longitudinal interactions for some sample cases. The calculations clearly show that there are many unique and significant features of beam-wave interactions when a magnetized plasma fill is present.

Computer calculations for BWO and Cherenkov radiation have appeared in previous papers [9], [10], [22]. Therefore we concentrate on a different configuration, namely that of a circular cylindrical waveguide filled with a thin annular plasma and a thin annular electron beam propagating and interacting with the wave as shown in Fig. 3. Since the corresponding experimental work has already been published [28], we are able to compare our calculations directly to those results. Since there is only a thin annular plasma background, the ion-channel effect can be neglected and the interaction equations (66) and (69) from above can be used:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2(k_z - k_{z0})P_{HE} = -j\omega\mu_0\varepsilon_1 \iint J_z E_{z0}^* ds + \frac{jk_z}{\varepsilon_0}\iint \rho E_{z0}^* ds \tag{111}$$

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2(k_z - k_{z0})P_{HE} = \frac{\omega_b^2(\varepsilon_1 k^2 - k_z^2)}{(\omega - k_z v_z)^2}P_E \tag{112}$$

In order to find k_{z0}, P_E and P_{HE}, the simplified plasma-filled waveguide model, that is the waveguide filled with an annular plasma but without an electron beam, will be studied first. The detailed solution is as follows.

Referring to Fig. 3, a hollow waveguide is filled with an annular plasma (Region II) of outer and inner radius, r_p^+ and r_p^-, but without an electron beam. Then the field component expressions for the different regions can be written as follows:

For Region I: $r_p^+ \le R \le R_c$ between the annular plasma and the waveguide wall

$$\left.\begin{aligned} E_z^I &= A_1 J_m(k_c R) + A_2 N_m(k_c R) \\ H_z^I &= B_1 J_m(k_c R) + B_2 N_m(k_c R) \end{aligned}\right\} \tag{113}$$

For Region II: $r_p^- \le R \le r_p^+$ inside the annular plasma volume itself

$$\left.\begin{aligned} E_z^{II} &= A_3 J_m(p_1 R) + A_4 N_m(p_1 R) + A_5 J_m(p_2 R) + A_6 N_m(p_2 R) \\ H_z^{II} &= A_3 h_1 J_m(p_1 R) + A_4 h_1 N_m(p_1 R) + A_5 h_2 J_m(p_2 R) + A_6 h_2 N_m(p_2 R) \end{aligned}\right\} \tag{114}$$

For Region III: $0 \le R \le r_p^-$ inside the volume enclosed by the annular plasma

$$\left.\begin{aligned} E_z^{III} &= A_7 J_m(k_c R) \\ H_z^{III} &= A_8 J_m(k_c R) \end{aligned}\right\} \tag{115}$$

with the boundary conditions:

$$\text{at } R = R_c: \qquad E_z^I = 0, \qquad E_\varphi^I = 0 \tag{116}$$

$$\text{at } R = r_p^+: \qquad E_z^I = E_z^{II}, \qquad E_\varphi^I = E_\varphi^{II}$$

$$H_z^I = H_z^{II}, \qquad H_\varphi^I = H_\varphi^{II} \tag{117}$$

$$\text{at } R = r_p^-: \qquad E_z^{II} = E_z^{III}, \qquad E_\varphi^{II} = E_\varphi^{III}$$

$$H_z^{II} = H_z^{III}, \qquad H_\varphi^{II} = H_\varphi^{III} \tag{118}$$

The components E_R, E_φ, H_R, H_φ can be found by using (A-3)-(A-6) in Appendix A. Then, substituting the field component expressions, (113)-(115), into the boundary conditions, (116)-(118), yields the individual elements of the dispersion relation:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1,10} \\ a_{21} & a_{22} & \cdots & a_{2,10} \\ \cdots & \cdots & \cdots & \cdots \\ a_{10,1} & a_{10,2} & \cdots & a_{10,10} \end{vmatrix} = 0 \tag{119}$$

Using (119), we can calculate the dispersion relation without an electron beam present. Thus, the fields E_{z0} and H_{z0} can also be obtained. Then, we can get P_E and P_{HE} by using the equations,

$$P_E = \iint E_{z0} \cdot E_{z0}^* ds \tag{120}$$

$$P_{HE} = \iint H_{z0} \cdot E_{z0}^* ds \tag{121}$$

So, according to (66) and (69) in Section V or (111) and (112), the longitudinal interactions of the plasma waves and the electron beam have been calculated.

We find that there are at least two differences between our work and the experiment cited in [28]. They are as follows:

1) The authors of [28] stated specifically that only "the E-mode" (i.e. – the TM mode) were considered there. However, theory [19], [20] demands that in an MPW the "E-mode" cannot exist independently. Only the hybrid modes, HE or EH, are allowed.

2) It was also indicated in [28] that the modes for which the cut-off frequency is zero and the phase velocity is less than the speed of the light were used. Therefore, the mode used in [28] belongs to the "plasma waves" or "T-G modes". With an annular plasma fill, our calculations show that only a symmetrical mode with m=0, (HE_{0n} or EH_{0n} with no azimuthal variation), may have zero cut-off frequencies. The cut-off frequencies of all asymmetrical modes are not zero. That is the essential difference between a solid plasma fill and the annular plasma fill. Therefore, our calculations are mainly focused on the m=0 mode.

The results of the numerical calculations are shown in Figs. 4 - 9. Fig.4 shows the dispersion curve of the HE_{01} mode for different values of n_p ($2\times10^{13}/cm^3$, $4\times10^{13}/cm^3$, $6\times10^{13}/cm^3$, $8\times10^{13}/cm^3$ and $10\times10^{13}/cm^3$). The cut-off frequency for all the curves is zero. Fig.5 shows the dispersion curves for the simplified plasma waveguide model (plasma-filled waveguide without an electron beam) for the asymmetric mode, HE_{11}. It shows the intersections of the dispersion curves with the light line, c, and the electron beam line, 500keV. The intersection is at about 15 GHz, higher than that for the symmetrical mode. Fig. 5 clearly shows that the cut-off frequencies for the asymmetrical modes are not zero. Fig. 6 shows the real part, $Re(k_z)$, for the beam-wave interactions of the HE_{01} mode. The imaginary part, $Im(k_z)$, is shown in Fig. 7. These plots show that the instability may exist in quite a

It should be noticed also that in [28] there is no specification of mode that was used in their experimental work.

broad frequency band. The corresponding plots for the interactions with the HE_{11} mode are shown in Fig. 8 and Fig. 9.

Direct comparison between our numerical calculations and the experimental results given in [28] (Fig. 2(d) in [28] for example) shows good basic agreement. However, our calculations do not agree with the theoretical calculations given in Fig. 4 of [28]. This is understandable since we calculated the hybrid mode while in [28] the "E-mode" was used.

6

A very interesting new result of our calculations is that the frequency for the peak growth rate of the interaction instability is far from the frequency at which the beam line is tangential to the dispersion curve for the plasma waveguide without a beam. This is perhaps one of the main differences between the MPW and the vacuum case. This is worthy of further study.

VIII. Some Possible New Interactions in an MPW

$\boxed{1.2}$

There are varieties of propagating waves in an MPW. When a driving electron beam is present, there must naturally be some coupling between/among waves. Such coupling, of course, may lead to some instabilities. In reference [10], for example, a parametric coupling excitation was presented. It is suggested that the T-G mode parametrically couples with a TM mode to excite a negative energy beam mode. The beam mode feeds energy into the positive energy T-G and TM modes giving rise to an explosive instability.

In an MPW, the mechanism is actually somewhat different from that given in [10]. Here the T-G mode parametrically couples with either the EH_{mn} or HE_{mn} mode, or even with one of the cyclotron modes, and then excites the electron beam. Now there are two transverse wave numbers, p_1 and p_2, so the excitation condition should be modified also. Considering the varieties of waves that can propagate in an MPW, the formulations become much more complicated, some results will be given in a future paper by the authors.

There exists a special kind of wave family that can propagate in an MPW in the frequency range: ($\omega_p < \omega < \omega_c$ or $\omega_c < \omega < \omega_p$), called the cyclotron modes. In particular, some of these waves are inherently backward waves (negative dispersion). This may therefore constitute a new interaction mechanism with the cyclotron waves. Similarly for backward waves, a new type of BWO might be constructed without the need for a periodic structure. The formulations of beam-wave interactions with cyclotron waves parallel those given in Sections III and IV above. In principle, the slow cyclotron waves may also be used as pump waves for parametric excitation.

The ion-channel produced by an electron beam propagating in a plasma complicates the beam-wave interactions. When there is no external guide magnetic field, an ion-channel laser may result. When the external guide magnetic field is not zero, there will exist a new kind of hybrid interaction. The authors are preparing another paper to analyze and discuss this new hybrid instability [29].

IX. Discussion ~~and~~ Analysis and ~~Discussion~~ Conclusion

The general formulations, including the general interaction equations and dispersion equations, given in this paper cover almost all kinds of beam-wave interactions in a waveguide with and without a plasma fill. Now based on these formulations, the following observations can be made concerning the specific influences and roles of the magnetized plasma fill itself.

1. Physically, the cyclotron motion of the background plasma electron plays a very important role. Because of it, we have $\varepsilon_2 = -\frac{\xi^2\tau}{1-\tau^2} \neq 0$ {see (7)}. This gives rise to the coupling between the TE and TM modes ~~because $b\neq0$ and $d\neq0$.~~ It also produces the additional parts of the wave field

components associated with $k_g^2 = k^2\varepsilon_2$. These parts of the wave field are directly involved in the beam-wave interactions.

The field patterns in an MPW are much more complicated than those in a vacuum waveguide. The field structure is completely changed. There are two eigenvalues and two corresponding eigenfunctions. The plasma background produces additional parts of the wave. Therefore, all field components, E_φ for example, may be divided into four parts: $E_\varphi = E_{\varphi 1} + E_{\varphi 2} + E_{\varphi 1p} + E_{\varphi 2p}$ where $E_{\varphi 1}$ is produced by $H_z = A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R)$ and is called the TE-like part. $E_{\varphi 2}$ is produced by $E_z = A_1 J_m(p_1 R) + A_2 J_m(p_2 R)$ and is called the TM-like part, and $E_{\varphi 1p}$ and $E_{\varphi 1p}$ are additional parts due to the plasma background. All of these parts are involved in the beam-wave interactions, vastly complicating them.

Similarly, the RF current density components gain two plasma parts as follows: $J_\varphi = J_{\varphi 1} + J_{\varphi 2} + J_{\varphi 1p} + J_{\varphi 2p}$, and $J_z = J_{z1} + J_{z2} + J_{z1p} + J_{z2p}$ (See Part II of this paper).

Beam-wave interactions mainly depend on the term $\iint \bar{J} \cdot \bar{E}^* ds$. Therefore, the magnetized plasma fill greatly complicates the interactions in the following manner:

$$\iint \bar{J}_\perp \cdot \bar{E}_\perp^* ds = \iint J_\varphi \cdot E_\varphi^* ds = \iint \big(J_{\varphi 1}E_{\varphi 1}^* + J_{\varphi 1}E_{\varphi 2}^* + J_{\varphi 1}E_{\varphi 1p}^* + J_{\varphi 1}E_{\varphi 2p}^* + J_{\varphi 1p}E_{\varphi 1}^* + J_{\varphi 1p}E_{\varphi 2}^* + J_{\varphi 1p}E_{\varphi 1p}^* + J_{\varphi 1p}E_{\varphi 2p}^* + J_{\varphi 2}E_{\varphi 1}^* + J_{\varphi 2}E_{\varphi 2}^* + J_{\varphi 2}E_{\varphi 1p}^* + J_{\varphi 2}E_{\varphi 2p}^* + J_{\varphi 2p}E_{\varphi 1}^* + J_{\varphi 2p}E_{\varphi 2}^* + J_{\varphi 2p}E_{\varphi 1p}^* + J_{\varphi 2p}E_{\varphi 2p}^* \big) ds.$$

The same is true for $\iint J_z \cdot E_z^* ds$. Since $\iint \bar{J}_\perp \cdot \bar{E}_\perp^* ds$ represents the transverse beam-wave interaction, and since $\iint J_z \cdot E_z^* ds$ represents the longitudinal beam-wave interaction, ~~and~~ since both $\bar{E}_\perp$ and E_z always exist simultaneously, we can see clearly that transverse interactions are always accompanied by longitudinal interactions. So beam-wave interactions in an MPW are much more complicated and richer than those for the vacuum case.

3. An instability of the longitudinal interactions and that of the transverse interactions may or may not occur at the same frequency. The instability will of course be enhanced when they do occur at same frequency. When they occur at different frequencies, a spurious spectrum will occur.

4. When an electron beam is injected into a plasma, an ion-channel may be formed if the beam density and energy are sufficiently high. Then the analysis of beam-wave interactions must take the ion-channel into account.

~~X. Conclusions~~

Then, ~~T~~he basic theory of electron beam-wave interactions in a waveguide filled with plasma immersed in a finite magnetic field has been presented in this paper. The interaction equations and dispersion relations for both longitudinal and transverse interactions in a magnetized plasma have been formulated. These equations cover almost all kinds of beam-wave interactions. They can also be used for any parametric excitations that may exist. The interaction equations can be used for both linear and non-linear waves, but the dispersion relations can only be used for the linear case. The theory given in this paper is only valid as long as the plasma background is not distorted; that is, as long as the background plasma can be described by the permittivity tensor given in (6)-(8).

From the formulations given in this paper, eight major results were obtained. First, the importance of the background plasma is that: 1) The electron gyrating motion of the background plasma couples the TE modes and TM modes, 2) This coupling generates the hybrid HE mode and EH mode; also, because of the magnetized plasma, there are varieties of modes propagating along the waveguide, and 3) The background plasma itself is involved in the electron beam-wave interactions by producing additional parts of the wave that depend on the gyrating motion. Thus, the magnetized background plasma makes the electron beam-wave interactions much more complicated and rich.

Second, since the TE and TM modes are always coupled, in an MPW, E_z and $\bar{E}_\perp$ always exist simultaneously. In plasma-filled microwave devices, therefore, there are no pure transverse interactions. They are always accompanied by longitudinal interactions with corresponding slow waves that can exist in an MPW both for smooth and corrugated walls. Likewise there is no pure longitudinal interaction. If there is any, even small, transverse electron motion, there must be some transverse interaction. It is inherent in an MPW that the transverse and longitudinal interactions are coupled.

Third, since there are varieties of waves in an MPW, when the electron beam is present, coupling between/among waves may happen. The low frequency plasma modes (T-G modes or even cyclotron waves) may serve as the pump wave and parametric excitations may be obtained.

Fourth, there is a special kind of wave family in the frequency range: ($\omega_p < \omega < \omega_c$ or $\omega_c < \omega < \omega_p$), called cyclotron modes. The waves in this family are all electromagnetic waves. They can interact directly with the electron beam, since their phase velocities may be less than the speed of light. In particular, some of the cyclotron waves are inherently negative; they are natural backward-waves. These inherent backward-waves may even be used for building backward-wave oscillators without periodic structures.

Fifth, the instabilities caused by longitudinal and transverse interactions may lead to two cases: (a). If two instability mechanisms occur at the same frequency or in the same frequency band, if properly adjusted (i.e.- carefully tuned), the instability will be dramatically enhanced. (b). If two instability mechanisms occur at different frequencies or frequency bands, then, spurious oscillations may occur.

Sixth, the coupling between TE and TM modes in the waveguide and the intensity of the interactions due to the participation of the plasma depend on the plasma electron gyrating motion and the plasma background density. It is proportional to the parameter $k_g^2 = k^2\varepsilon_2 = -k^2 \frac{\xi^2\tau}{1-\tau^2}$, $\xi = \frac{\omega_p}{\omega}$, $\tau = \frac{\omega_c}{\omega}$. Thus, adjusting the magnetic field and the density of the background plasma is important for the design and operation of plasma-filled devices. Those parameters thereby permit frequency-tuning of a given device [30].

Seventh, when an ion-channel is formed, any theoretical analysis of the system must take it into consideration. One of the most convenient approaches is to use our simplified plasma waveguide model for a solid beam or for a hollow beam. The electron beam-wave interactions take place in the ion-channel region itself.

~~Finally.~~ Our theoretical predictions confirm that, in general, the frequency spectrum and the spurious output of plasma filled devices cannot be of as high a quality as that of vacuum devices. That appears to be the main price we must pay for enhancing the output power and efficiency of microwave devices by means of a plasma fill.

Appendix A: Basic Equations / Properties of Wave Propagation in an MPW

In this appendix, a detailed derivation of the basic equations and illustrations of the character of wave propagation in an MPW are given. These equations and characteristics are necessary for a full appreciation of the results presented in this paper.

1. Basic Equations:

The longitudinal field components in a smooth-walled MPW are given in (1) and (2) as:

$$E_z = A_1 J_m(p_1 R) + A_2 J_m(p_2 R) \tag{A-1}$$

$$H_z = A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R) \tag{A-2}$$

The transverse field components can be derived from (11) by assuming a form like (A-1) and (A-2) for each field subcomponent and making the substitution from (3) and (4):

$$\begin{aligned} E_R = \frac{1}{D}\Big\{ & -jk_z K^2\left[A_1 p_1 J_m'(p_1 R) + A_2 p_2 J_m'(p_2 R)\right] - jk_z k_g^2 \frac{m}{R}\left[A_1 J_m(p_1 R) + A_2 J_m(p_2 R)\right] + \\ & \omega\mu_0 k_g^2\left[A_1 h_1 p_1 J_m'(p_1 R) + A_2 h_2 p_2 J_m'(p_2 R)\right] + \omega\mu_0 K^2 \frac{m}{R}\left[A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R)\right]\Big\} \end{aligned} \tag{A-3}$$

$$\begin{aligned} E_\phi = \frac{1}{D}\Big\{ & k_z k_g^2\left[A_1 p_1 J_m'(p_1 R) + A_2 p_2 J_m'(p_2 R)\right] + k_z K^2 \frac{m}{R}\left[A_1 J_m(p_1 R) + \right. \\ & \left. A_2 J_m(p_2 R)\right] + j\omega\mu_0 K^2\left[A_1 h_1 p_1 J_m'(p_1 R) + A_2 h_2 p_2 J_m'(p_2 R)\right] + \\ & j\omega\mu_0 k_g^2 \frac{m}{R}\left[A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R)\right]\Big\} \end{aligned} \tag{A-4}$$

$$\begin{aligned} H_R = \frac{1}{D}\Big\{ & -\omega\varepsilon_0\varepsilon_2 k_z^2\left[A_1 p_1 J_m'(p_1 R) + A_2 p_2 J_m'(p_2 R)\right] - \omega\varepsilon_0\left(\varepsilon_1 K^2 - \varepsilon_2 k_g^2\right)\frac{m}{R}\cdot \\ & \left[A_1 J_m(p_1 R) + A_2 J_m(p_2 R)\right] - jk_z K^2\left[A_1 h_1 p_1 J_m'(p_1 R) + A_2 h_2 p_2 J_m'(p_2 R)\right] \\ & -jk_z k_g^2 \frac{m}{R}\left[A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R)\right]\Big\} \end{aligned} \tag{A-5}$$

Eq.(A-3) – (A-6) should be replaced by Eq.(12)–(13) in page 9 10.

$$H_\varphi = \frac{1}{D}\Big\{-j\omega\varepsilon_0\left(\varepsilon_1 K^2 - \varepsilon_2 k_g^2\right)\left[A_1 p_1 J_m'(p_1 R) + A_2 p_2 J_m'(p_2 R)\right] - j\omega\varepsilon_0\varepsilon_2 k_z^2 \frac{m}{R}\cdot$$
$$\left[A_1 J_m(p_1 R) + A_2 J_m(p_2 R)\right] + k_z k_g^2\left[A_1 h_1 p_1 J_m'(p_1 R) + A_2 h_2 p_2 J_m'(p_2 R)\right] \tag{A-6}$$
$$+k_z K^2 \frac{m}{R}\left[A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R)\right]\Big\}$$

where: $D = K^4 - k_g^4 \quad K^2 = -k_z^2 + k^2\varepsilon_1 \quad k_g^2 = \omega^2\mu_0\varepsilon_0\varepsilon_2 = k^2\varepsilon_2 \quad \varepsilon_g = j\varepsilon_2$ (A-7)

By using $R = a$ $E_z = 0$, $E_\varphi = 0$,

The dispersion relation can be obtained from (22), (A-1), and (A-2) as follows:

$$p_1 J_m'(p_1 R_c) J_m(p_2 R_c)\left[jk_z k_g^2 + j\frac{K^2}{k_z\varepsilon_2}\left(\varepsilon_3 K^2 - \varepsilon_1 p_1^2\right)\right] - p_2 J_m'(p_2 R_c) J_m(p_1 R_c)\cdot$$
$$\left[jk_z k_g^2 + j\frac{K^2}{k_z\varepsilon_2}\left(\varepsilon_3 K^2 - \varepsilon_1 p_2^2\right)\right] - j\left(\frac{m}{R_c}\right)\frac{\varepsilon_1 k_g^2}{k_z\varepsilon_g}\left(p_1^2 - p_2^2\right) J_m(p_1 R_c) J_m(p_2 R_c) = 0 \tag{A-8}$$

For a corrugated waveguide, the field components should be expanded using Floquet's theorem [31].

$$E_z = \sum_n\left[A_{1,n} J_m(p_{1,n} R) + A_{2,n} J_m(p_{2,n} R)\right] e^{-jk_{z,n}z} \tag{A-9}$$

$$H_z = \sum_n\left[A_{1,n} h_{1,n} J_m(p_{1,n} R) + A_{2,n} h_{2,n} J_m(p_{2,n} R)\right] e^{-jk_{z,n}z} \tag{A-10}$$

Where the wave factor $exp(j\omega t + jm\varphi - jk_{//}z)$ is implied in the above but not shown, and

$$k_{//,n} = k_{//} + \frac{2\pi n}{L} \tag{A-11}$$

and L is the spatial period.

The expressions for the transverse field components $\left(E_R, E_\varphi, H_R, H_\varphi\right)$ may be obtained by using Floquet's expansion of the respective equations. We also have the eigenvalues for the corrugated waveguide:

$$(p_{1,2})_s^2 = \frac{1}{2\varepsilon_1}\left[-k_{z,s}^2(\varepsilon_1 + \varepsilon_2) + k^2\left(\varepsilon_1\varepsilon_3 + \varepsilon_1^2 - \varepsilon_2^2\right)\right] \pm \frac{1}{2\varepsilon_1}\cdot$$
$$\left\{\left[-k_{z,s}^2(\varepsilon_3 - \varepsilon_1) + k^2\left(\varepsilon_1\varepsilon_3 - \varepsilon_1^2 + \varepsilon_2^2\right)\right]^2 + 4k^2 k_{z,s}^2\varepsilon_2^2\varepsilon_3\right\}^{\frac{1}{2}} \tag{A-12}$$

$$\left(h_{1,2}\right)_s = \frac{\left(-k_{z,s}^2 + k^2\varepsilon_1\right)\varepsilon_3 - \varepsilon_1\left(p_{1,2}\right)_s^2}{j\omega\mu_0 k_{z,s}\varepsilon_2} \tag{A-13}$$

$$\mathrm{D}_s = \mathrm{K}_s^4 - \mathrm{k}_g^4 \tag{A-14}$$

$$\mathrm{K}_s^2 = -\mathrm{k}_{z,s}^2 + \mathrm{k}^2\varepsilon_1 \tag{A-15}$$

The boundary conditions along the corrugated waveguide surfaces may be written as:

$$\mathrm{R} = \mathrm{a} + \mathrm{h}\cos\mathrm{k}_0\mathrm{z} \tag{A-16}$$

$$\mathrm{E}_t = 0, \qquad \mathrm{H}_n = 0 \tag{A-17}$$

where we assume that the waveguide has a sinusoidal boundary wall, h is the depth of the ripple, and $\mathrm{k}_0 = \frac{2\pi}{\mathrm{L}}$. So (A-16) and (A-17) reduce to:

$$\left(E_z - E_R k_0 h \sin k_0 z\right)_{R=a+h\cos k_0 z} = 0 \tag{A-18}$$

$$\left(H_z k_0 h \sin k_0 z - H_r\right)_{R=a+h\cos k_0 z} = 0 \tag{A-19}$$

Substituting the field component expressions into the boundary conditions, and taking the average value in one spatial period, we obtain:

$$\begin{bmatrix} \left(F_{m,1}\right)_{n,s} & \left(F_{m,2}\right)_{n,s} \\ \left(G_{m,1}\right)_{n,s} & \left(G_{m,2}\right)_{n,s} \end{bmatrix} \begin{bmatrix} A_{1,n} \\ A_{2,n} \end{bmatrix} = 0 \tag{A-20}$$

Where:

$$\begin{aligned}\left(F_{m,1}\right)_{n,s} = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} dz \Big\{ J_m\left(p_{1,n}R\right) + \frac{1}{D_n}\Big[jk_{//,n}K_n^2 p_{1,n} J_m'\left(p_{1,n}R\right) + j\frac{m}{R}k_g^2 k_{//,n} J_m\left(p_{1,n}R\right) - \\ \omega\mu_0 k_g^2 h_{1,n} p_{1,n} J_m'\left(p_{1,n}R\right) - \frac{m}{R}\omega\mu_0 K_n^2 h_{1,n} J_m\left(p_{1,n}R\right)\Big] k_0 h \sin k_0 z \Big\} e^{-j(n-s)k_0 z}\end{aligned} \tag{A-21}$$

$$\begin{aligned}\left(F_{m,2}\right)_{n,s} = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} dz \Big\{ J_m\left(p_{2,n}R\right) + \frac{1}{D_n}\Big[jk_{//,n}K_n^2 p_{2,n} J_m'\left(p_{2,n}R\right) + j\frac{m}{R}k_g^2 k_{//,n} J_m\left(p_{2,n}R\right) - \\ \omega\mu_0 k_g^2 h_{2,n} p_{2,n} J_m'\left(p_{2,n}R\right) - \frac{m}{R}\omega\mu_0 K_n^2 h_{2,n} J_m\left(p_{2,n}R\right)\Big] k_0 h \sin k_0 z \Big\} e^{-j(n-s)k_0 z}\end{aligned} \tag{A-22}$$

$$\left(G_{m,1}\right)_{n,s} = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} dz\, e^{-j(n-s)k_0 z}\Big\{h_{1,n}J_m\left(p_{1,n}R\right)k_0 h \sin k_0 z +$$
$$\frac{1}{D_n}\Big[-\omega\varepsilon_0 k_{//,n}^2 \varepsilon_2 p_{1,n} J_m'\left(p_{1,n}R\right) - \omega\varepsilon_0 \frac{m}{R}\left(\varepsilon_1 K_n^2 - \varepsilon_2 k_g^2\right) J_m\left(p_{1,n}R\right) \qquad \text{(A-23)}$$
$$-jk_{//,n}K_n^2 h_{1,n} p_{1,n} J_m'\left(p_{1,n}R\right) - jk_{//,n}k_g^2 \frac{m}{R} h_{1,n} J_m\left(p_{1,n}R\right)\Big]\Big\}$$

$$\left(G_{m,2}\right)_{n,s} = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} dz\, e^{-j(n-s)k_0 z}\Big\{h_{2,n}J_m\left(p_{2,n}R\right)k_0 h \sin k_0 z +$$
$$\frac{1}{D_n}\Big[-\omega\varepsilon_0 k_{//,n}^2 \varepsilon_2 p_{2,n} J_m'\left(p_{2,n}R\right) - \omega\varepsilon_0 \frac{m}{R}\left(\varepsilon_1 K_n^2 - \varepsilon_2 k_g^2\right) J_m\left(p_{2,n}R\right) \qquad \text{(A-24)}$$
$$-jk_{//,n}K_n^2 h_{2,n} p_{2,n} J_m'\left(p_{2,n}R\right) - jk_{//,n}k_g^2 \frac{m}{R} h_{2,n} J_m\left(p_{2,n}R\right)\Big]\Big\}$$

The general dispersion relation for a corrugated MPW is then obtained as follows:

$$\begin{bmatrix} \left(F_{m,1}\right)_{n,s} & \left(F_{m,2}\right)_{n,s} \\ \left(G_{m,1}\right)_{n,s} & \left(G_{m,2}\right)_{n,s} \end{bmatrix} = 0 \qquad \text{(A-25)}$$

and for symmetric modes,

$$\begin{bmatrix} \left(F_{0,1}\right)_{n,s} & \left(F_{0,2}\right)_{n,s} \\ \left(G_{0,1}\right)_{n,s} & \left(G_{0,2}\right)_{n,s} \end{bmatrix} = 0 \qquad \text{(A-26)}$$

2. Characteristics of Wave Propagation.

Fig. A-1 gives the dispersion curves of waves that may propagate in an MPW. It is obtained from (A-8). The figure shows that there are at least the following three kinds of waves (modes) that can propagate in an MPW: plasma waves (T-G mode), in the frequency range ($0 < \omega < \omega_p$); waveguide waves in the frequency ($\omega_h = \sqrt{\omega_c^2 + \omega_p^2} < \omega$); and cyclotron waves in the frequency range ($\mathrm{Max}(\omega_c, \omega_p) < \omega < \omega_h$). We can see clearly that the plasma waves are slow waves (the phase velocity is less than the speed of light); the waveguide waves are fast waves (the phase velocity is higher than the speed of light); as for the cyclotron waves the phase velocity can be either less than or greater than the speed of light depending on the frequency. It is interesting to point out that for some cyclotron waves the group velocity is negative (negative dispersion), and thus a natural backward wave can propagate. Therefore, the waveguide waves cannot directly interact with the electron beam, while cyclotron waves and plasma waves can. It has been shown in [22] that by means of dielectric loading, the phase velocity of some waveguide waves can be slowed down to less than the speed of light, thus allowing beam-wave interactions.

Appendix B. Derivation of Dispersion Equations for Electron Beam-Wave Interactions in an MPW

1. For Longitudinal Interactions:

From Maxwell's equations, we obtained the beam-wave interaction expression (22) and (23) as:

$$\nabla_{\perp}^{2} E_z + aE_z = bH_z + j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0 \varepsilon_1}\rho \tag{B-1}$$

$$\nabla_{\perp}^{2} H_z + cH_z = dE_z - \left(\nabla \times \vec{J}\right)_z - \frac{\omega\varepsilon_2}{\varepsilon_1}\rho \tag{B-2}$$

where a, b, c, and d are defined in (24) and where J_z and ρ are the z-component of the RF electron beam current density and the RF space-charge density, respectively. For the case of no electron beam, we have:

$$\nabla_{\perp}^{2} E_{z0} + a_0 E_{z0} = b_0 H_{z0} \tag{B-3}$$

$$\nabla_{\perp}^{2} H_{z0} + c_0 H_{z0} = d_0 E_{z0} \tag{B-4}$$

where a_0, b_0, c_0 and d_0 denote the parameters for the case when the beam is absent, and similarly for E_{z0} and H_{z0}.

From (B-1)-(B-4), we obtain the dispersion equations respectively:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = -j\omega\mu_0\varepsilon_1\iint J_z E_{z0}^{*}ds + \frac{jk_z}{\varepsilon_0}\iint \rho E_{z0}^{*}ds \quad \text{(B-5)}$$

and

$$\varepsilon_1\left(k_z^2 - k_{z0}^2\right)P_H - j\omega\varepsilon_0\varepsilon_2\varepsilon_3\left(k_z - k_{z0}\right)P_{EH} = \iint\left(\nabla\times\bar{J}\right)_z\cdot H_{z0}^{*}ds + \omega\varepsilon_2\iint \rho H_{z0}^{*}ds \quad \text{(B-6)}$$

To obtain (B-5) and (B-6), the following assumptions have been made:

$$P_E = \iint E_z\cdot E_{z0}^{*}ds \cong \iint E_{z0}\cdot E_{z0}^{*}ds \quad \text{(B-7)}$$

$$P_H = \iint H_z\cdot H_{z0}^{*}ds \cong \iint H_{z0}\cdot H_{z0}^{*}ds \quad \text{(B-8)}$$

$$P_{HE} = \iint H_z\cdot E_{z0}^{*}ds \cong \iint H_{z0}\cdot E_{z0}^{*}ds \quad \text{(B-9)}$$

$$P_{EH} = \iint E_z\cdot H_{z0}^{*}ds \cong \iint E_{z0}\cdot H_{z0}^{*}ds \quad \text{(B-10)}$$

$$\begin{aligned}\iint\left(\nabla_\perp^2 E_z\right)\cdot E_{z0}^{*}ds &= \iint\left(\nabla_\perp^2 E_{z0}\right)\cdot E_{z0}^{*}ds\\ \iint\left(\nabla_\perp^2 H_z\right)\cdot H_{z0}^{*}ds &= \iint\left(\nabla_\perp^2 H_{z0}\right)\cdot H_{z0}^{*}ds\end{aligned} \quad \text{(B-11)}$$

These assumptions are commonly used for linear theory when the influence of the electron beam on the field profile is negligible.

2. For Transverse Interactions

The same procedures can be used to obtain the dispersion equations for transverse interactions:

$$\begin{aligned}&\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_{E_\perp} + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE_\perp} + \omega\mu_0\left(\varepsilon_3 - \varepsilon_1\right)\left(k_z - k_{z0}\right)P_{EH_\perp} =\\ &-j\omega\mu_0\varepsilon_1\iint \bar{J}_\perp\cdot\bar{E}_{\perp 0}^{*}ds - \frac{1}{\varepsilon_0}\iint\left(\nabla_\perp\rho\right)_\perp\cdot\bar{E}_{\perp 0}^{*}ds\end{aligned} \quad \text{(B-16)}$$

and

$$\begin{aligned}\left(k_z^2 - k_{z0}^2\right)P_{H_\perp} - j\omega\varepsilon_0\varepsilon_2\left(k_z - k_{z0}\right)P_{EH_\perp} &= \iint\left(\nabla\times\bar{J}\right)_\perp\cdot\bar{H}_{\perp 0}^{*}ds -\\ &\omega\varepsilon_0\left(\varepsilon_3 - \varepsilon_1\right)\left(k_z - k_{z0}\right)\iint\left(\bar{e}_z\times\bar{E}_\perp\right)\cdot\bar{H}_{\perp 0}^{*}ds\end{aligned} \quad \text{(B-17)}$$

where:

$$P_{E_\perp} = \iint \bar{E}_\perp\cdot\bar{E}_{\perp 0}^{*}ds \cong \iint \bar{E}_{\perp 0}\cdot\bar{E}_{\perp 0}^{*}ds$$

$$P_{H_\perp} = \iint \bar{H}_\perp \cdot \bar{H}^*_{\perp 0} ds \cong \iint \bar{H}_{\perp 0} \cdot \bar{H}^*_{\perp 0} ds$$

$$P_{HE_\perp} = \iint \bar{H}_\perp \cdot \bar{E}^*_{\perp 0} ds \cong \iint \bar{H}_{\perp 0} \cdot \bar{E}^*_{\perp 0} ds$$

$$P_{EH_\perp} = \iint \bar{E}_\perp \cdot \bar{H}^*_{\perp 0} ds \cong \iint \bar{E}_{\perp 0} \cdot \bar{H}^*_{\perp 0} ds \tag{B-18}$$

Similar procedures combined with the methodology used in Section IV can be used for the formulation of electron beam-wave interactions in a plasma filled corrugated waveguide.

Appendix C. Field Expressions and Dispersion Relations for a Simplified MPW with a Solid Electron Beam

In order to derive the dispersion relations for electron beam-wave interactions for a solid beam in an MPW with the ion-channel taken into account, we need to know the unperturbed plasma field distribution. The cross section diagram is shown in Fig. 1(a). According to the approach used in this paper (in both Part I and Part II), we calculate the unperturbed phase constant k_{z0} for wave propagation in the waveguide system without a beam. In the MPW with the ion-channel taken into account, the unperturbed field E_{z0} , $E_{\perp 0}$ and k_{z0} can be found by using the configuration shown in Fig. 1(b). This is our simplified MPW model, only used for calculating the unperturbed field that interacts with the electron beam.

The field expressions for the simplified MPW model for a solid beam are as follows:

In Region II:

$$E_z = B_1 J_m(pR) \tag{C-1}$$

$$H_z = B_2 J_m(pR) \tag{C-2}$$

$$E_R = \frac{1}{p^2}\left\{-jk_z p B_1 J'_m(pR) + \omega\mu_0 \frac{m}{r} B_2 J_m(pR)\right\} \tag{C-3}$$

$$E_\phi = \frac{1}{p^2}\left\{-jk_z\frac{m}{r}B_1J_m(pR) + j\omega\mu_0 pB_2J_m'(pR)\right\} \tag{C-4}$$

$$H_\phi = \frac{1}{p^2}\left\{-j\omega\mu_0 pB_1J_m'(pR) + k_z\frac{m}{r}B_2J_m(pR)\right\} \tag{C-5}$$

$$H_r = \frac{1}{p^2}\left\{-\omega\varepsilon_0\frac{m}{r}B_1J_m(pR) - jk_z pB_2J_m'(pR)\right\} \tag{C-6}$$

In Region I:

$$E_z = A_1J_m(p_1R) + A_2N_m(p_1R) + A_3J_m(p_2R) + A_4N_m(p_2R) \tag{C-7}$$

$$H_z = A_1h_1J_m(p_1R) + A_2h_1N_m(p_1R) + A_3h_2J_m(p_2R) + A_4h_2N_m(p_2R) \tag{C-8}$$

$$\begin{aligned} E_r = \frac{1}{D}\Big\{&-jk_zK^2\left[A_1p_1J_m'(p_1R) + A_2p_1N_m'(p_1R) + A_3p_2J_m'(p_2R) + A_4p_2N_m'(p_2R)\right] \\ &-jk_zk_g^2\frac{m}{r}\left[A_1J_m(p_1R) + A_2N_m(p_1R) + A_3J_m(p_2R) + A_4N_m(p_2R)\right] + \omega\mu_0k_g^2 \\ &\left[A_1h_1p_1J_m'(p_1R) + A_2h_1N_m'(p_1R) + A_3h_2p_2J_m'(p_2R) + A_4h_2p_2N_m'(p_2R)\right] + \omega\mu_0K^2\frac{m}{r} \\ &\left[A_1h_1J_m(p_1R) + A_2h_1N_m(p_1R) + A_3h_2J_m(p_2R) + A_4h_2N_m(p_2R)\right]\Big\} \end{aligned} \tag{C-9}$$

$$\begin{aligned} E_\phi = \frac{1}{D}\Big\{&k_zK_g^2\left[A_1p_1J_m'(p_1R) + A_2p_1N_m'(p_1R) + A_3p_2J_m'(p_2R) + A_4p_2N_m'(p_2R)\right] \\ &+k_zK^2\frac{m}{r}\left[A_1J_m(p_1R) + A_2N_m(p_1R) + A_3J_m(p_2R) + A_4N_m(p_2R)\right] + j\omega\mu_0K^2 \\ &\left[A_1h_1p_1J_m'(p_1R) + A_2h_1N_m'(p_1R) + A_3h_2p_2J_m'(p_2R) + A_4h_2p_2N_m'(p_2R)\right] + j\omega\mu_0K_g^2\frac{m}{r} \\ &\left[A_1h_1J_m(p_1R) + A_2h_1N_m(p_1R) + A_3h_2J_m(p_2R) + A_4h_2N_m(p_2R)\right]\Big\} \end{aligned} \tag{C-10}$$

$$\begin{aligned} H_r = \frac{1}{D}\Big\{&-\omega\varepsilon_0\varepsilon_gk_z^2\left[A_1p_1J_m'(p_1R) + A_2p_1N_m'(p_1R) + A_3p_2J_m'(p_2R) + A_4p_2N_m'(p_2R)\right] \\ &-\omega\varepsilon_0\left(\varepsilon_1K^2 - \varepsilon_gK_g^2\right)\frac{m}{r}\left[A_1J_m(p_1R) + A_2N_m(p_1R) + A_3J_m(p_2R) + A_4N_m(p_2R)\right] + jk_zK^2 \\ &\left[A_1h_1p_1J_m'(p_1R) + A_2h_1N_m'(p_1R) + A_3h_2p_2J_m'(p_2R) + A_4h_2p_2N_m'(p_2R)\right] - jk_zk_g^2\frac{m}{r} \\ &\left[A_1h_1J_m(p_1R) + A_2h_1N_m(p_1R) + A_3h_2J_m(p_2R) + A_4h_2N_m(p_2R)\right]\Big\} \end{aligned} \tag{C-11}$$

$$H_\phi = \frac{1}{D}\{-j\omega\varepsilon_0(\varepsilon_1 K^2 - \varepsilon_g K_g^2)[A_1 p_1 J_m'(p_1 R) + A_2 p_1 N_m'(p_1 R) + A_3 p_2 J_m'(p_2 R) + A_4 p_2 N_m'(p_2 R)]$$
$$- j\omega\varepsilon_0\varepsilon_g k_z^2 \frac{m}{r}[A_1 J_m(p_1 R) + A_2 N_m(p_1 R) + A_3 J_m(p_2 R) + A_4 N_m(p_2 R)] + k_z k_g^2$$
$$[A_1 h_1 p_1 J_m'(p_1 R) + A_2 h_1 N_m'(p_1 R) + A_3 h_2 p_2 J_m'(p_2 R) + A_4 h_2 p_2 N_m'(p_2 R)] + k_z K^2 \frac{m}{r}$$
$$[A_1 h_1 J_m(p_1 R) + A_2 h_1 N_m(p_1 R) + A_3 h_2 J_m(p_2 R) + A_4 h_2 N_m(p_2 R)]\}$$

(C-12)

The boundary conditions are:

at $R = a$: $E_z^I = 0,\quad E_\varphi^I = 0$ (C-13)

at $R = Ri$: $E_z^I = E_z^{II},\quad E_\varphi^I = E_\varphi^{II}$

$H_z^I = H_z^{II},\quad H_\varphi^I = H_\varphi^{II}$ (C-14)

By using (C-7)-(C-12), the dispersion relation for Region I becomes:

$$\Delta = \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{16} \\ a_{21} & a_{22} & \cdots\cdots & a_{26} \\ \cdot & \cdot & \cdot & \cdot \\ a_{61} & a_{62} & \cdots\cdots & a_{66} \end{bmatrix} = 0 \qquad \text{(C-15)}$$

where:

$$a_{11} = J_m(p_1 a) \quad a_{12} = J_m(p_2 a) \quad a_{13} = N_m(p_1) \quad a_{14} = N_m(p_2 a)$$
$$a_{15} = 0 \quad a_{16} = 0$$

$$a_{21} = (j\omega\mu_0 K^2 h_1 + k_z K_g^2) p_1 J_m'(p_1 a) \quad a_{22} = (j\omega\mu_0 K^2 h_2 + k_z K_g^2) p_1 J_m'(p_2 a)$$
$$a_{23} = (j\omega\mu_0 K^2 h_1 + k_z K_g^2) p_1 N_m'(p_1 a) \quad a_{22} = (j\omega\mu_0 K^2 h_2 + k_z K_g^2) p_1 N_m'(p_2 a)$$
$$a_{25} = 0 \quad a_{26} = 0$$

$$a_{31} = J_m(p_1 a) \quad a_{32} = J_m(p_2 R_i) \quad a_{33} = N_m(p_1 R_i) \quad a_{34} = N_m(p_2 R_i)$$
$$a_{35} = 0 \quad a_{36} = 0$$

$$a_{41} = h_1 J_m(p_1 a) \quad a_{42} = h_2 J_m(p_2 R_i) \quad a_{43} = h_1 N_m(p_1 R_i) \quad a_{44} = h_2 N_m(p_2 R_i)$$
$$a_{45} = 0 \quad a_{46} = 0$$

$$a_{5j} = (j\omega\mu_0 K^2 h_j + k_z K_g^2) p_j J_m'(p_j R_i) - (j\omega\mu_0 K^2 h_j + k_z K_g^2) \frac{m}{R_i} J_m(p_2 R_i)$$

$$a_{5(2+j)} = (j\omega\mu_0 K^2 h_j + k_z K_g^2) p_j N_m'(p_j R_i) - (j\omega\mu_0 K^2 h_j + k_z K_g^2) p_j \frac{m}{R_i} N_m(p_j R_i)$$

$$a_{55} = \frac{1}{p^2} k_z \frac{m}{R_i} J_m(pR_i) \quad a_{56} = \frac{j\omega\mu_0}{p} J_m'(pR_i)$$

$$a_{6j} = \frac{1}{D}\left\{[-j\omega\varepsilon_0(\varepsilon_1 K^2 - \varepsilon_2 k_g^2) + k_z k_g^2 h_1] p_j J_m'(p_j R_i) + (j\omega\varepsilon_0\varepsilon_2 k_g^2 - K^2 k_z h_1) \frac{m}{R_i} J_m(p_j R_i)\right\}$$

$$a_{6(j+2)} = \frac{1}{D}\left\{[-j\omega\varepsilon_0(\varepsilon_1 K^2 - \varepsilon_2 k_g^2) + k_z k_g^2 h_1] p_j N_m'(p_j R_i) + (j\omega\varepsilon_0\varepsilon_2 k_g^2 - K^2 k_z h_1) \frac{m}{R_i} N_m(p_j R_i)\right\}$$

$$a_{65} = \frac{1}{p} j\omega\varepsilon_0 J_m'(pR_i)$$

$$a_{66} = \frac{1}{p^2} k_z \frac{m}{R_i} J_m(pR_i)$$

For longitudinal interactions we only need the k_{z0} term, but for transverse interactions, we also need to know the constants A_1, A_2, A_3, A_4 and B_1, B_2, which can be found by using the boundary conditions (C-13) and (C-14).

REFERENCES

[1] V. L. Granatstein and I. Alexeff, *High-Power Microwave Source*, Artech House Inc. 1987.

[2] O.T. Loza, A.G. Shkvarunets, and P.S. Strelkov, "Experimental Plasma Relativistic Microwave Electronics," *IEEE Trans. on Plasma Science*, Vol. 26, No. 3, pp. 615-627, June 1998.

[3] C. J. Edgcombe, *Gyrotron Oscillators: Their Principles and Practice*, Taylor & Francis, 1993.

[4] K. R. Chu, et al., "Stabilization of Absolute Instabilities in the Gyrotron Travelling Wave Amplifier," *Phys. Rev. Lett.*, Vol. 74, No. 7, pp. 1103-1106, 1995.

[5] Mikhial V. Neglin, *Physics of Intense Beams in Plasma*, Chapter1, Institute of Physics Publishing, Bristol and Philadelphia, 1993.

[6] R.J. Barker, "Advanced Research in Millimeter-Wave Vacuum Electronics," *Int. Journal of IR & MM-Waves*, Vol. 10, No. 10, pp. 1213-1222, 1989.

[7] X. Zhai, E. Garate, R. Prohaska, G. Benford, and A. Fisher, "Experimental Study of a Plasma-Filled Backward Wave Oscillator," *IEEE Trans. on Plasma Sci.*, Vol. 21, pp. 142-150, Feb. 1993.

[8] L. S. Bogdankevich, et al., "Plasma Microwave Electronics," *Soviet Physics*, Vol. 24, p.1, 1981.

[9] Y. Carmel, et al., "Relativistic Plasma Microwave Electronics: Studies of High-Power Plasma Filled Backward-wave Oscillators," *Phys. Fluids. B*, Vol. 4, No. 7, pp. 2286-2292, 1992.

[10] C. S. Liu and V. K. Tripathi, "Explosive Instability in a Backward Wave Oscillators," *IEEE Trans. on Plasma Sci.*, Vol. 21, No. 1, pp. 191-193, Feb. 1993.

[11] C. Joshi, et al., "Plasma Wave Wiggler for FEL: Plasma Effects in FEL," *IEEE Trans. on Plasma Sci.*, Vol. 18, pp. 446, 1990.

[12] V. K. Tripathi and C. S. Liu, "Plasma Effects in a Free Electron Laser," *IEEE Trans. on Plasma Sci.*, Vol. 18, pp. 466-471, 1990.

[13] P. Jha and P. Kumar, "Dispersion Relation and Growth in a Free-Electron Laser with Ion-Channel Guiding," *Phys. Rev. E*, Vol. 57, No. 2, pp. 2256, 1998.

[14] D. M. Goebel, J.M. Butler, R.W. Schumacher, J. Santoru, and R.L. Eisenhart, "High-Power Microwave Source Based on an Unmagnetized Backward-Wave Oscillator," *IEEE Trans. on Plasma Sci.*, Vol. 22, No. 5, pp. 547-553, Oct. 1994.

[15] J. Santoru, J.M. Butler, D.M. Goebel, and R.W. Schumacher, "High-Power, Single-Beam Plasma Wave Tube," *IEEE Trans. on Plasma Sci.*, Vol. 22, No. 5, pp. 593-599, Oct. 1994.

[16] David H. Whittem, Andrew M. Sessler and J.M. Dawson, "Ion-Channel Laser," *Phys. Rev. Lett.*, Vol. 64, No. 21, pp. 2511-2514, 1990.

[17] Chang J. Tang, Pu K. Lin and Shenggang Liu, "Electromagnetic Instability in an Ion-Channel Electron Cyclotron Maser," *J. Phys. D: Appl. Phys.*, Vol. 29, pp. 90-94, 1996.

[18] K. R. Chen, J. M. Dawson, A. T. Lin and T. Katsouleas, "Unified Theory and Comparative Study of Cyclotron Maser, Ion-Channel Laser, and Free Electron Laser", *Phys. Fluids B*. Vol. 3, No. 5, pp. 1270, 1991.

[19] S. T. Ivanov and E.G. Alexov, "Electromagnetic Waves in a Plasma Waveguide," *J. Plasma Physics*, Vol. 43, pp. 51, 1990.

[20] Liu Shenggang, J.K. Lee, Song Liqun, Yan Yang, and Zhu Dajun, "Theory of Wave Propagation Along Corrugated Waveguide Filled with Plasma Immersed in an Axial Magnetic Field," *IEEE Trans. on Plasma Sci.*, Vol. 24, No. 3, pp. 918-923, June 1996.

[21] G.S. Nusinovich, Y. Carmel, T.M. Antonsen, Jr., D.M. Goebel, and J. Santoru, "Recent Progress in the Development of Plasma-Filled Travelling-Wave Tubes and Backward-Wave Oscillators," *IEEE Trans. on Plasma Sci.*, Vol. 26, No. 3, pp. 628-645, June 1998.

[22] K. P. Maheshwar, et.al, "Cherenkov-Cyclotron Resonance Interaction in a High-Power Plasma-Filled Backward-Wave Oscillator," *Phys. Rev. A*, Vol. 45, No. 8, pp. 5866-5871, 1992.

[23] R. Sawhney, K.P. Maheshari, and Y. Choyal, "Effect of Plasma on Efficiency Enhancement in a High Power Relativistic Backward Wave Oscillator," *IEEE Trans. on Plasma Sci.*, Vol. 21, No. 6, pp. 609-613, Dec. 1993.

[24] V. U. Kremenzev, et al., "Gyrotron with Relativistic Electron Beam and Plasma Filling," *J. of Exp. and Theor. Physics*, Vol. 6, No. 12, pp. 2151-2154, 1978.

[25] Liu Shenggang and Y. Zhonghai, "The Kinetic Theory of the Electron Cyclotron Resonance Maser with Space Charge Effect Taken into Consideration," *Int. Journal of Electronics*, Vol. 51, p. 341, 1981.

[26] J. M. Butler, et al., "PASOTRON High-Energy Microwave Source," *IEEE MTT-S Digest*, pp. 115, 1992.

[27] Liu Shenggang, "The Kinetic Theory of Electron Cyclotron Resonance Maser," *Scientific Sinica*, Vol. 22, p. 901, 1979.

[28] A.V. Ponomarev, P.S. Strelkov, and A.G. Shkvarunets, "Relativistic Plasma-Beam Microwave Amplifier," *Plasma Phy. Rep.* Vol. 24, No. 1, pp. 48-52, 1998.

[29] Liu Shenggang, et al., "A New Hybrid Ion-Channel Instability," *Proc. 23rd Int. Conf. on IR/MM Waves*, Essex University, UK, Sept. 7-11, 1998.

[30] A.G. Shkvarunets, S. Kobayashi, Y. Carmel, J. Rodgers, T.M. Antonsen, L. Duan, and V.L. Granatstein, "Operation of a Relativistic Backward-Wave Oscillator Filled with a Preionized, High-Density, Radially Inhomogeneous Plasma," *IEEE Trans. on Plasma Sci.*, Vol. 26, No. 3, pp. 615-627, June 1998.

[31] C.C. Johnson, *Field and Wave Electrodynamics*, McGraw-Hill, New York, 1965.

[32] Liu Shenggang, et al., "Theory of Wave Propagation Along Waveguide Filled with Plasma in Magnetic Field," *Science in China*, Vol. 39, No. 5, pp. 485-503, 1996.

FIGURE CAPTIONS

Figure 1. Ion-channels produced by a solid (a) and hollow (b) high density electron beam in an MPW.

Figure 2. Simplified models for the study of beam-wave interactions in a plasma waveguide with an ion-channel (a) for a solid beam and (b) for a hollow beam.

Figure 3. Schematic structure of a hollow waveguide with an annular plasma and electron-beam.

Figure 4. Dispersion relation for the HE_{01} mode for different values of n_p. (v_0 and c are the speed of the electron beam and of light, respectively.)

Figure 5. Dispersion relation for the HE_{11} mode for different values of n_p. (Note that the cut-off frequency is not zero.)

Figure 6. The real part of $k_z R_c$ for the HE_{01} mode *vs* the operating frequency.

Figure 7. The imaginary part of $k_z R_c$ showing the spatial growth rate of the HE_{01} mode *vs* the operating frequency.

Figure 8. The real part of $k_z R_c$ of the HE_{11} mode *vs* the operating frequency.

Figure 9. The imaginary part of $k_z R_c$ showing the spatial growth rate for the HE_{11} mode *vs* the operating frequency.

Figure A-1. A typical plot of dispersion curves (B_0=0.175T, $\omega_p=1.85\times10^{10}s^{-1}$, R_c=1.5cm)

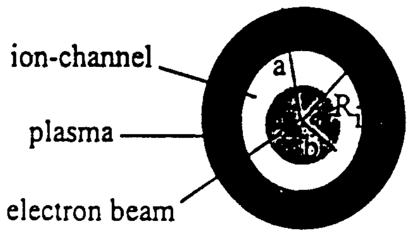

FIGURE 1 (a) - Barker, Part I

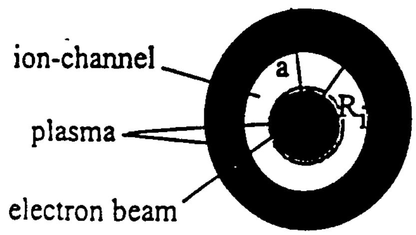

FIGURE 1 (b) - Barker, Part I

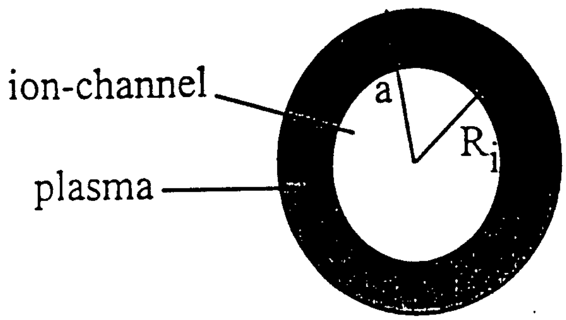

FIGURE 2 (a) - Barker, Part I

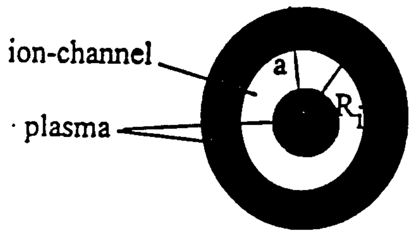

FIGURE 2 (b) - Barker, Part I

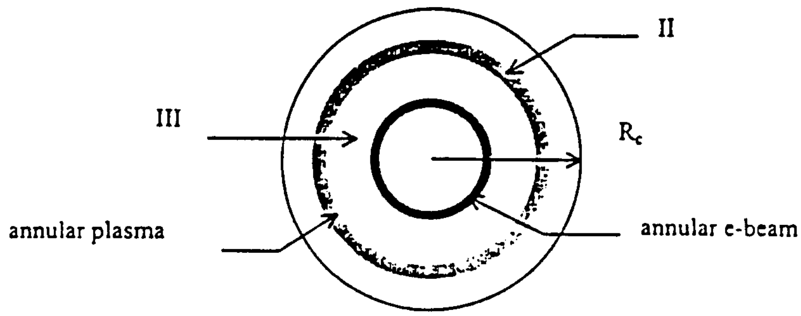

FIGURE 3 - Barker, Part I

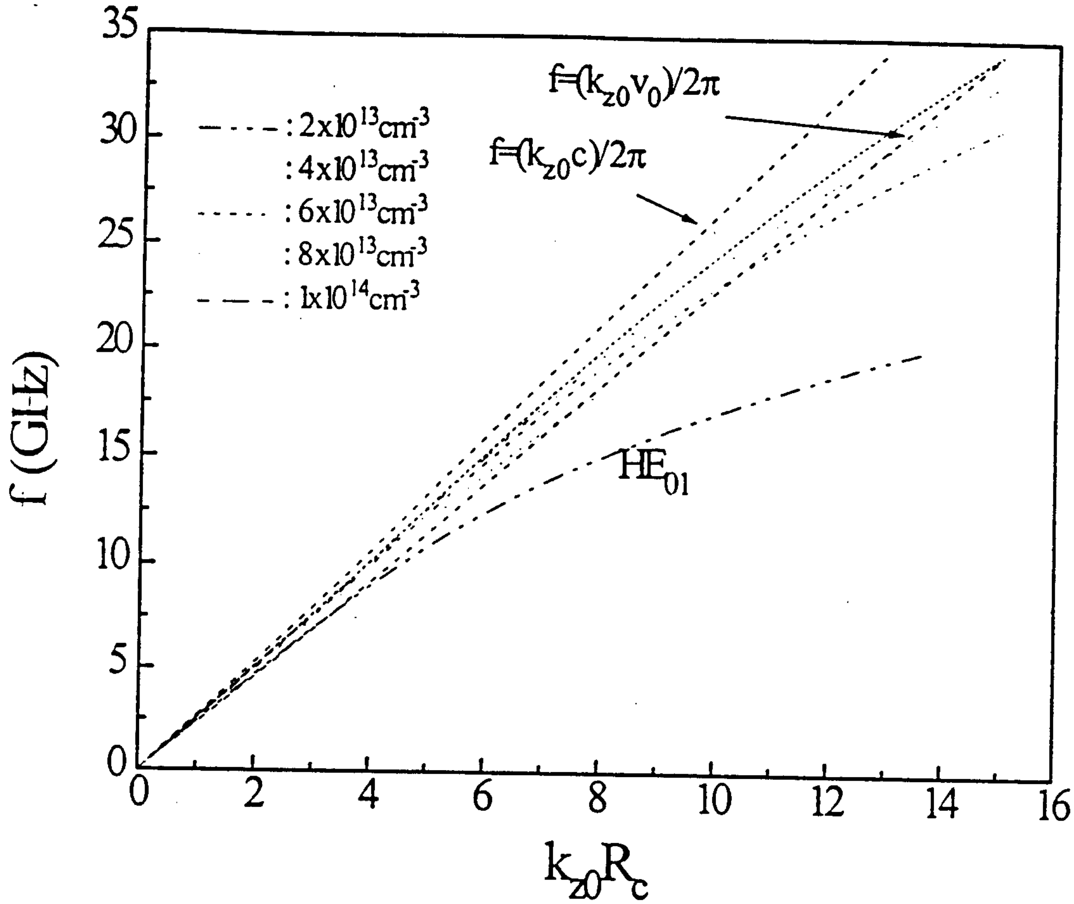

FIGURE 4 - Barker, Part I

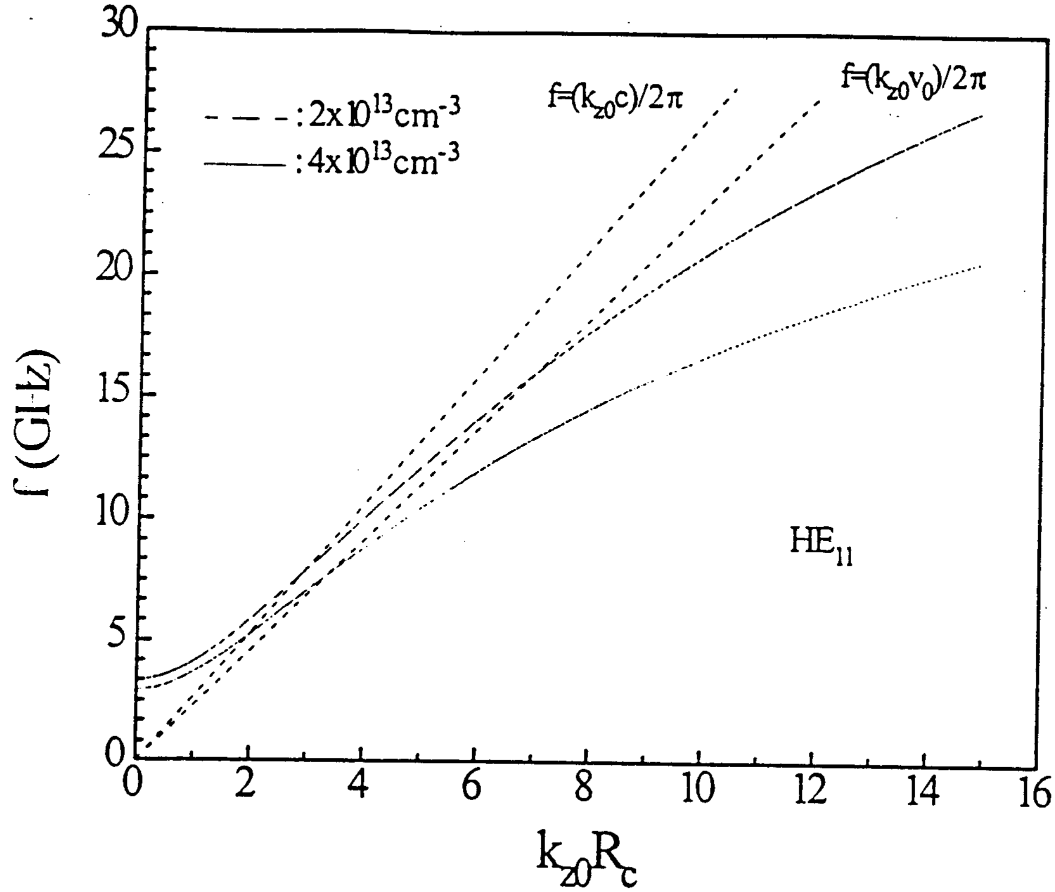

FIGURE 5 - Barker, Part I

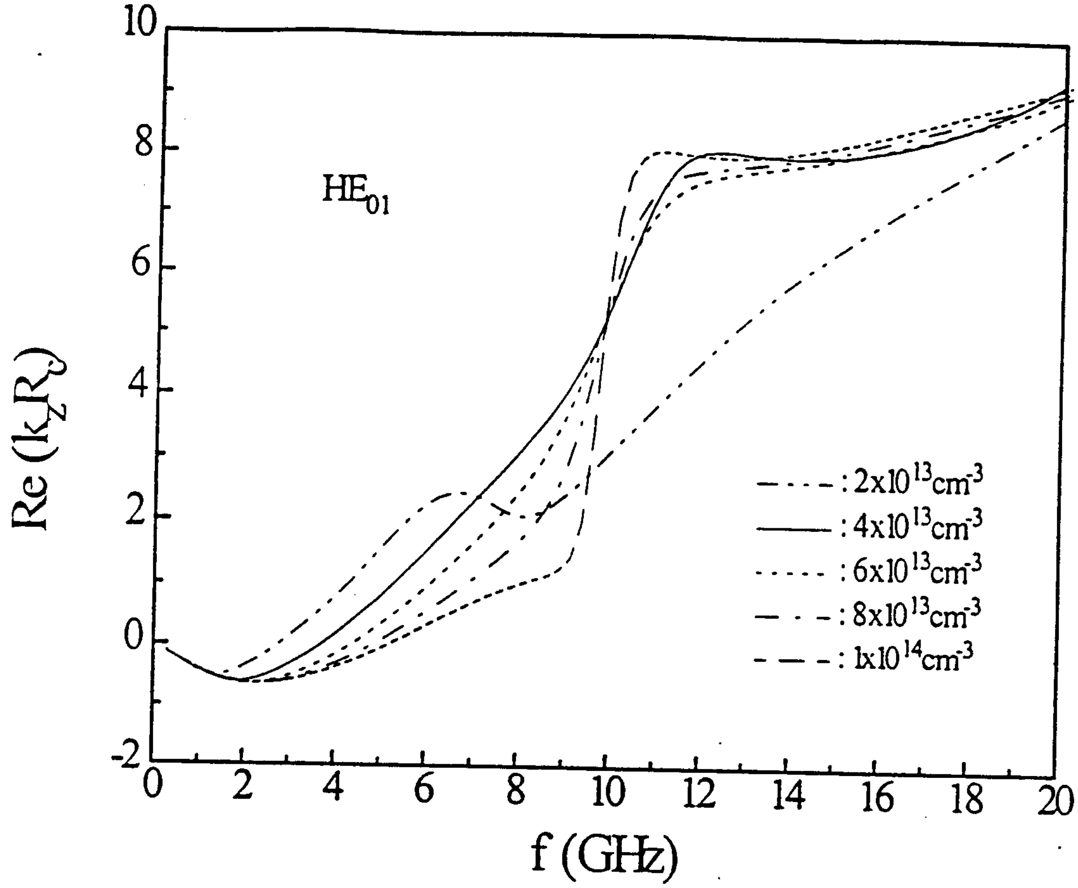

FIGURE 6 - Barker, Part I

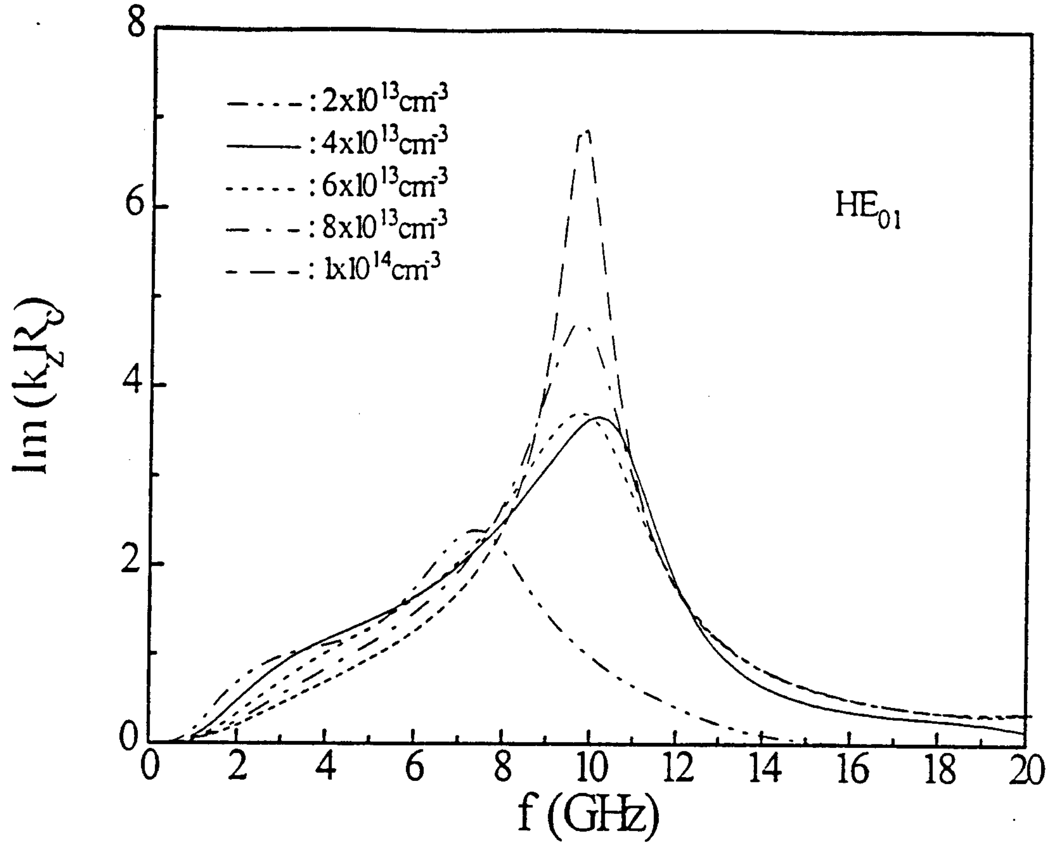

FIGURE 7 - Barker, Part I

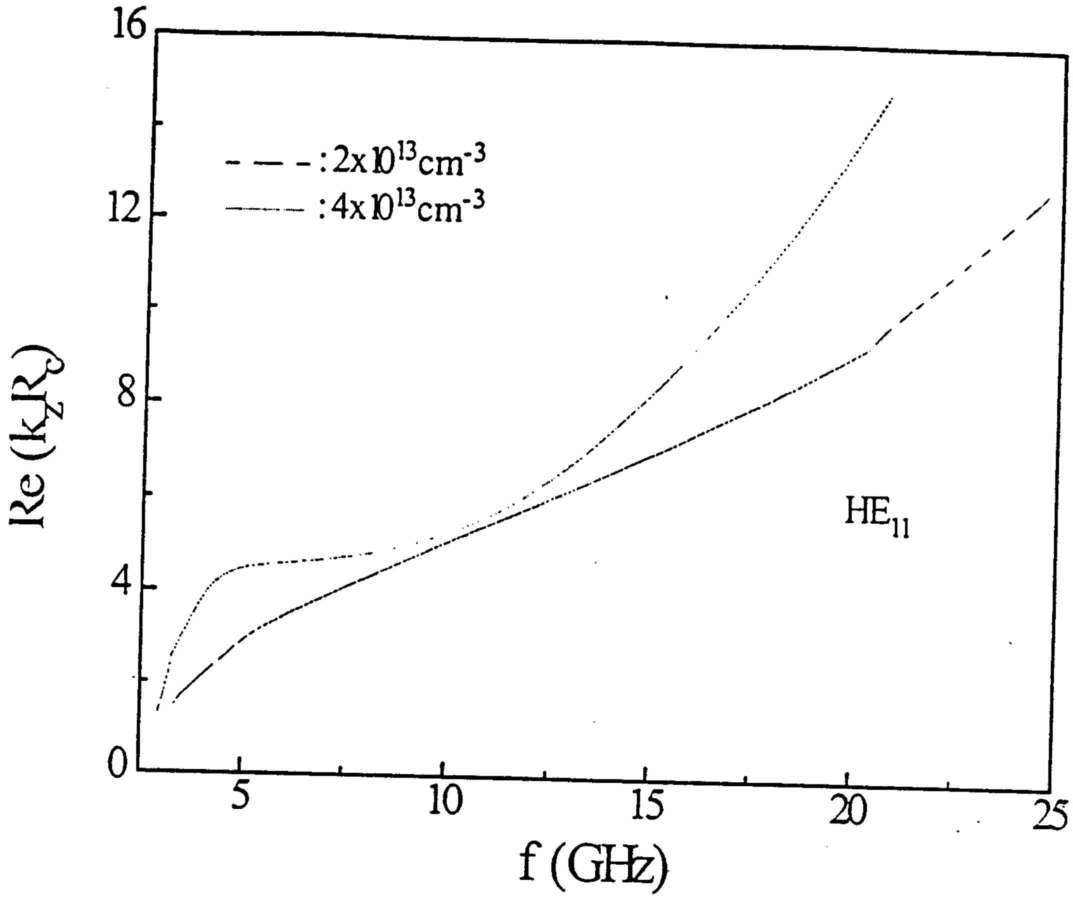

FIGURE 8 - Barker, Part I

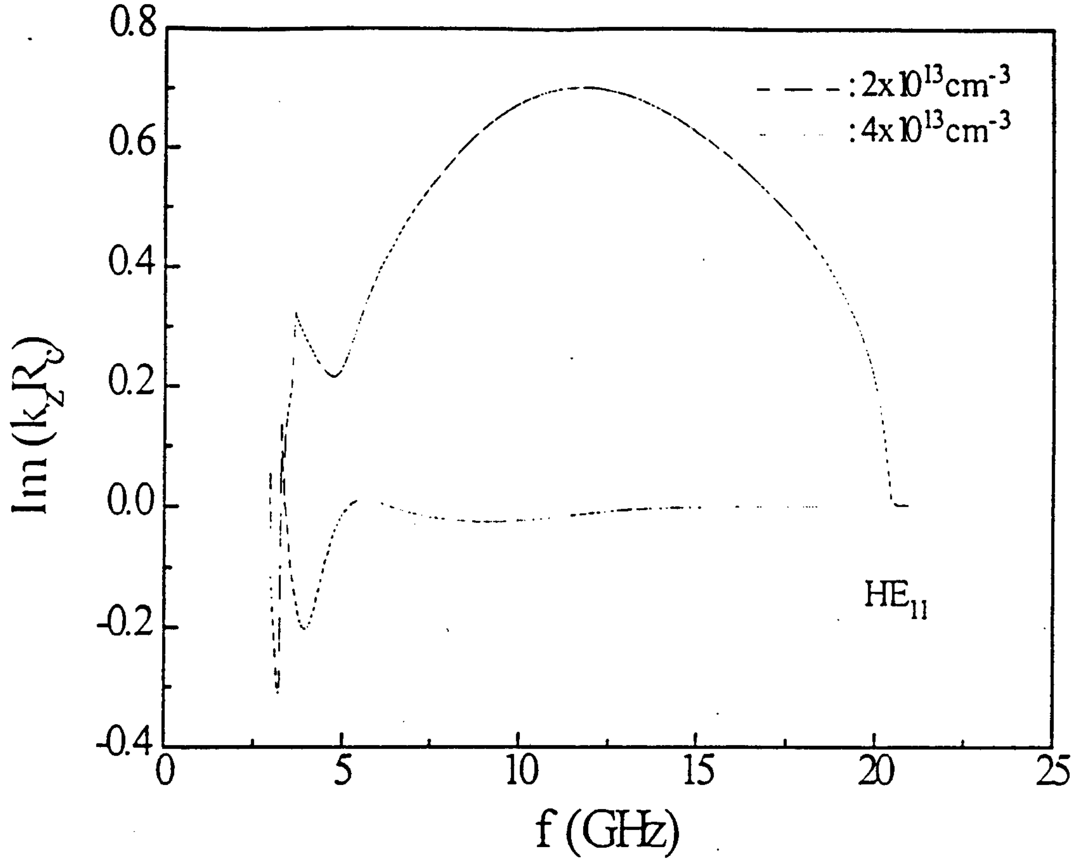

FIGURE 9 - Barker, Part I

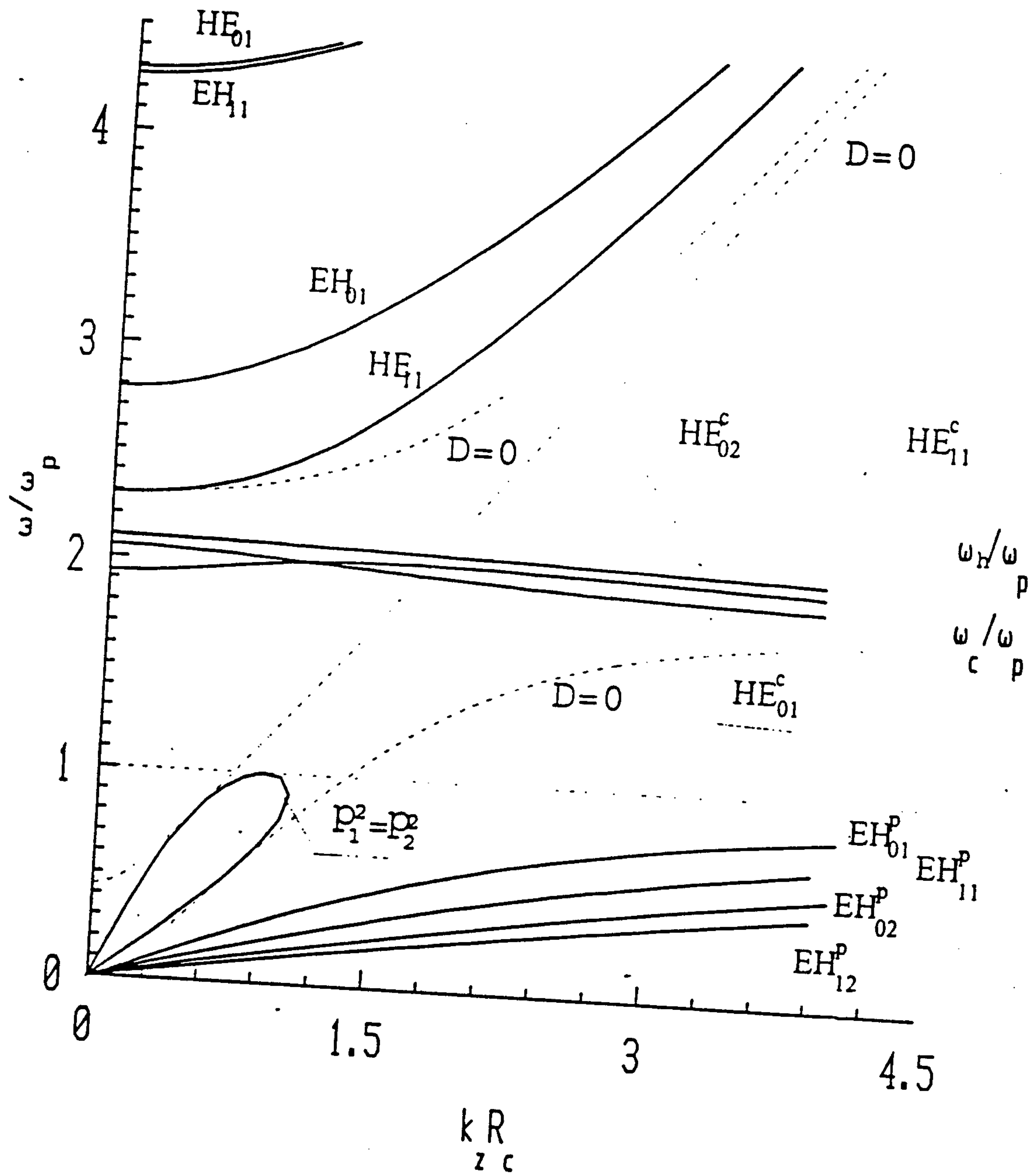

FIGURE A-1 - Barker, Part I

2. Shenggang Liu, Robert J. Barker, Yan Yang, and Zhu Dajun, "Basic Theoretical Formulation of Plasma Microwave Electronics Part II: Kinetic Theory of Electron Beam-Wave Interactions".

Basic Theoretical Formulation of Plasma Microwave Electronics *

Part II: Kinetic Theory of Electron Beam-Wave Interactions

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Yan Yang**, and Zhu Dajun**

Key Words: plasma microwave electronics, plasma-filled, kinetic theory, gyrotron, electron cyclotron maser

Abstract– Building upon the theoretical foundations presented in Part I of this paper [1], the kinetic theory of electron-beam-wave interactions in a magnetized plasma-filled waveguide (MPW) is presented in this second part. This kinetic theory treatment is more generally applicable to cases of less-intense electron-beams [2]. The dispersion relations for longitudinal and transverse interactions, in both smooth and corrugated waveguides, are all derived by using kinetic theory to model the e-beam dynamics. This includes kinetic theory treatments of the plasma filled electron cyclotron resonance maser (ECRM) and a combination of Cherenkov-Cyclotron resonance phenomena. It is important to note that in an MPW (magnetized plasma waveguide), transverse interactions (eg. – ECRM interactions) are always coupled with longitudinal interactions.

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I. Introduction

In Part I of this paper, the general theory of electron-beam-wave interactions in and dispersion relations for a waveguide filled with plasma immersed in a finite magnetic field (MPW) have been presented. In this second part, kinetic theory is used for analyzing the electron beam, while the plasma background is still treated using fluid theory. In what follows, the kinetic theory of beam-wave interactions in a smooth-walled MPW is given in Section II, while that for a corrugated MPW is given in Section III. The kinetic theory of a plasma-filled Electron Cyclotron Resonance Maser (ECRM) is given in Section IV. Section V deals briefly with the combination of Cherenkov and Cyclotron (TWT/BWO-Cyclotron) resonance interactions. Section VI deals with transverse beam-wave interactions with an ion-channel taken into account. In Section VII, sample numerical calculations for a plasma-filled ECRM are given. Detailed discussions and conclusions are presented in Section VIII. The most cumbersome portions of these calculations are relegated to Appendices A, B, and C in order to streamline the flow of the paper.

II. Kinetic Theory of E-Beam/Wave Interactions in a Smoothwalled MPW

The dispersion equations for electron-beam-wave interactions in a smoothwalled MPW have been derived in Part I of this paper using fluid models for both beam and plasma. Now beam kinetic theory is used to obtain the dispersion equations.

A. For Longitudinal Interactions

The dispersion equation for longitudinal interactions in an MPW is given by (66) in Part I [1] as:

$$\varepsilon_3(k_z^2 - k_{z0}^2)P_E + j\omega\mu_0\varepsilon_2(k_z - k_{z0})P_{HE} = -j\omega\mu_0\varepsilon_1 \int J_z \cdot E_{z0}^* ds + \frac{jk_z}{\varepsilon_0}\int \rho_1 E_{z0}^* ds \tag{1}$$

where P_E and $P_{//E}$ are defined by (70) in Part I [1]. J_z and ρ_1 are now calculated using kinetic theory, as follows:

$$J_z = -e\int f_1 \frac{p_z}{m\gamma_0} d\bar{p} \tag{2}$$

$$\rho_1 = -e\int f_1 d\bar{p} \tag{3}$$

where f_1 is the perturbed distribution function [3]:

$$f_1 = -e\int_{-\infty}^{t} dt' \left[(E'_\rho + v_z B'_r)\frac{\partial f_0}{\partial p_\perp} + (E'_z - v_\theta B'_r)\frac{\partial f_0}{\partial p_z} \right] \tag{4}$$

where f_0 is the equilibrium distribution function. If there is no transverse motion, f_0 can be chosen to take the form [4]:

$$f_0 = \frac{n_b}{2\pi}\delta(p_{//} - p_0)\Theta(R_c - R) \tag{5}$$

where:

$$\Theta(x) = \begin{cases} 0 & (x \le 0) \\ 1 & (x > 0) \end{cases}$$

Thus f_1 can be rewritten as:

$$f_1 = -e\int_{-\infty}^{t} E'_z \frac{\partial f_0}{\partial p_{//}} dt' = -\frac{jeE_z}{(\omega - k_z v_z)}\frac{\partial f_0}{\partial p_{//}} \tag{6}$$

After integration in momentum space, we get:

$$J_z = -e\int_{-\infty}^{\infty} dp_z \int_0^{2\pi} \frac{p_z}{m\gamma_0\gamma} f_1 d\phi = -\frac{j\omega\varepsilon_0\omega_b^2}{(\omega - k_z v_{z0})^2}E_z \tag{7}$$

$$\rho_1 = -e\int_{-\infty}^{\infty} dp_z \int_0^{2\pi} d\phi f_1 = \frac{jk_z\varepsilon_0\omega_b^2}{(\omega - k_z v_{z0})^2}E_z \tag{8}$$

where ω_b is defined by (71) in Part I [1]. The dispersion relation for longitudinal interactions (1) then becomes:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = \frac{\omega_b^2(k^2\varepsilon_1 - k_z^2)P_E}{(\omega - k_z v_{z0})^2} \tag{9}$$

This is identical to the dispersion equation derived using fluid theory {Part I, (79)}.

B. For Transverse Interactions

For transverse interactions, the dispersion equation is given by (82) in Part I as [1]:

$$\begin{aligned} &\varepsilon_3(k_z^2 - k_{z0}^2)P_{E_\perp} + j\omega\mu_0\varepsilon_2(k_z - k_{z0})P_{HE_\perp} + \\ &\omega\mu_0(\varepsilon_3 - \varepsilon_1)(k_z - k_{z0})P_{EH_\perp} = -j\omega\mu_0\varepsilon_1\int J_\phi \cdot E_{\phi 0}^* ds - \frac{1}{\varepsilon_0}\int(\nabla_\perp\rho)_\phi \cdot E_{\phi 0}^* ds \end{aligned} \tag{10}$$

where:

$$J_\phi = -e\int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \frac{p_\phi}{m_0\gamma} \tag{11}$$

$$\rho = -e\int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \tag{12}$$

$$\begin{aligned} f_1 = -e\int_{-\infty}^{t} dt\Bigg[&\left(E_\phi^{'} + v_z B_R^{'}\right)\left(\frac{\partial f_0}{\partial p_\phi} + \frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial p_\phi}\right) \\ &+\left(E_z^{'} - v_\phi B_R^{'}\right)\frac{\partial f_0}{\partial p_z} - \left(E_R^{'} + v_\phi^{'} B_z^{'} - v_z^{'} B_\phi^{'}\right)\frac{1}{p_z}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial \phi}\Bigg] \end{aligned} \tag{13}$$

The integration should be taken along an unperturbed orbit, and f_0 is the equilibrium distribution function.

For a hollow gyrating beam, a proper f_0 function can be written as [4]:

$$f_0 = \frac{n_b}{2\pi p_{\perp 0}}\delta(p_z - p_{z0})\delta(p_\perp - p_{\perp 0}) \tag{14}$$

Since they are linearly dependent on the field components, f_1 and J_ϕ can be divided into four parts:

$$f_1 = f_{11} + f_{12} + f_{11p} + f_{12p}$$
$$J_\phi = J_{\phi 1} + J_{\phi 2} + J_{\phi 1p} + J_{\phi 2p} \tag{15}$$

where:

$$f_{1j} = -e \int_{-\infty}^{t} dt \left[(E'_{\phi j} + v_z B'_{Rj}) \frac{\partial f_0}{\partial p_\perp} - v_\phi B'_{\phi j} \frac{\partial f_0}{\partial p_{//}} \right] \tag{16}$$

$$f_{12} = -e \int_{-\infty}^{t} dt \left[(E'_{\phi 2} + v_z B'_{R2}) \frac{\partial f_0}{\partial p_\perp} - (E'_z - v_\phi B'_{R2}) \frac{\partial f_0}{\partial p_{//}} \right] \tag{17}$$

$$f_{1jp} = -e \int_{-\infty}^{t} dt \left[(E'_{\phi 1p} + v_z B'_{R1p}) \frac{\partial f_0}{\partial p_\perp} - v_\phi B'_{R1p} \frac{\partial f_0}{\partial p} \right] \tag{18}$$

$$f_{12p} = -e \int_{-\infty}^{t} dt \left[(E'_{\phi 2p} + v_z B'_{R2p}) \frac{\partial f_0}{\partial p_\perp} - v_\phi B'_{R2p} \frac{\partial f_0}{\partial p_{//}} \right] \tag{19}$$

$$J_{\phi j} = -e \int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_j \frac{p_\phi}{m_0 \gamma} = -2\pi e \int_{-\infty}^{+\infty} dp_z \int_0^{\infty} \frac{p_\perp p_{//}}{m_0 \gamma} f_{11} dp_\perp \tag{20}$$

$$J_{\phi 2} = -2\pi e \int_{-\infty}^{+\infty} dp_z \int_0^{\infty} \frac{p_\perp p_{//}}{m_0 \gamma} f_{12} dp_\perp \tag{21}$$

$$J_{\phi 1p} = -2\pi e \int_{-\infty}^{+\infty} dp_z \int_0^{\infty} \frac{p_\perp p_{//}}{m_0 \gamma} f_{11p} dp_\perp \tag{22}$$

$$J_{\phi 2p} = -2\pi e \int_{-\infty}^{+\infty} dp_z \int_0^{\infty} \frac{p_\perp p_{//}}{m_0 \gamma} f_{12p} dp_\perp \tag{23}$$

Substituting the field component expressions (A-3)-(A-6) of Part I [1] and integrating in momentum space, we obtain:

$$J_{\phi j} = 2\pi e^2 \sum_i \int \frac{A_i}{D} \mu_0 h_i p_i K^2 v_\perp \left\{ \frac{(\omega - k_z v_z)}{\Omega} \left(2 J'_{mi} + p_i R_c J''_{mi} \right) - \frac{v_\perp^2 J'_{mi}}{\Omega^2} \left(k^2 - k_z^2 \right) \right\} f_0 d\bar{p} \tag{24}$$

$$J_{\theta 1r} = 2\pi e^2 \sum_i \int \frac{A_i}{D} \mu_0 h_i k_g^2 m\omega_c \left\{ \frac{(\omega - k_z v_z)}{\Omega} \left(2J_{mi} + p_i R_c J'_{mi}\right) - \frac{\beta_\perp^2 J_{mi}}{\Omega^2} \left(\omega^2 - k_z^2 c^2\right) \right\} f_0 d\bar{p} \quad (25)$$

$$\begin{aligned} J_{\theta 2} = -j2\pi e^2 \sum_i \int \frac{A_i}{D} \Bigg\{ & \frac{K^2 m\omega_c}{\Omega} \left(J_{mi} + p_i R_c J'_{mi}\right)\left(k_z - k\beta_{//}\varepsilon_1\right) \\ & - \frac{\beta_\perp^2 J_{mi}}{\Omega^2} \left(k_z \omega K^2 m\omega_c (\varepsilon_1 - 1) + D(\omega v_z - k_z c^2)\right) \Bigg\} f_0 d\bar{p} \end{aligned} \quad (26)$$

$$\begin{aligned} J_{\theta 2r} = -j2\pi e^2 \sum_i \int \frac{A_i}{D} \Bigg\{ & \frac{1}{\Omega} \Big[k_z v_\perp (k_g^2 - \varepsilon_2 k_z k \beta_{//}) \left(2J'_{mi} + p_i R_c J''_{mi}\right) \\ & + \varepsilon_2 k_g^2 \beta_{//} K^2 \omega_c (J_{mi} + p_i R_c J'_{mi}) \Big] + \frac{\beta_\perp^2}{\Omega^2} k_g \omega\omega_c \beta_\perp \left[\left(k^2 - k_z^2\right) p_i R_c J'_{mi} - m k_g^2 J_{mi} \right] \Bigg\} f_0 d\bar{p} \end{aligned} \quad (27)$$

where K and D are defined by (27) in Part I [1]. When the effect of space charge is omitted, the dispersion equation for transverse interactions given by (82) in Part I can be rewritten as:

$$\begin{aligned} & \varepsilon_3\left(k_z^2 - k_{z0}^2\right) P_{E\perp} + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right) P_{HE\perp} + \omega\mu_0\left(\varepsilon_3 - \varepsilon_1\right) \cdot \\ & \left(k_z - k_{z0}\right) P_{EH\perp} = 2\pi \frac{\omega_b^2}{c^2} \frac{\omega\varepsilon_1}{\gamma v_{\perp 0}} \sum_i \frac{A_i}{D^2} R_0 \left[\frac{Q_1}{\Omega} - \frac{Q_2 \beta_\perp^2}{\Omega^2} \right] J_{mi}^2 \end{aligned} \quad (28)$$

$$\Omega = \omega - k_z v_z - \omega_c \quad (29)$$

$$Q_1 = Q_{11} + Q_{12} + Q_{11r} + Q_{12r} \quad (30)$$

$$Q_2 = Q_{21} + Q_{22} + Q_{21r} + Q_{22r} \quad (31)$$

where the detailed expressions for all eight Q-components may be found in (A-1) – (A-8) in Appendix A.

III. Kinetic Theory of E-Beam/Wave Interactions in a Corrugated MPW

For a corrugated waveguide, the dispersion equation for longitudinal interactions is given by (78) in Part I as:

$$\sum_s \varepsilon_3\left(k_{z,s}^2 - k_{z0,s}^2\right) P_{E,s} + j\omega\mu_0\varepsilon_2\left(k_{z,s} - k_{z0,s}\right) P_{HE,s} = \sum_s \frac{\omega_p^2 (k^2\varepsilon_1 - k_{z,s}^2) P_{E,s}}{(\omega - k_{z,s} v_{z0})^2} \quad (32)$$

The dispersion equation for transverse interactions is seen from (28) above to be:

$$\sum_s \varepsilon_3\left(k_{z,s}^2 - k_{z0,s}^2\right)P_{E\perp,s} + j\omega\mu_0\varepsilon_2\left(k_{z,s} - k_{z0,s}\right)P_{HE\perp,s} + \omega\mu_0\left(\varepsilon_3 - \varepsilon_1\right)\cdot$$
$$\left(k_{z,s} - k_{z0,s}\right)P_{EH\perp,s} = 4\pi^2 e^2 n_0 \omega\mu_0\varepsilon_1 \sum_{i,s}\frac{A_{i,s}}{D^2}R_e\left[\frac{Q_{1,s}}{\Omega} - \frac{Q_{2,s}\beta_\perp^2}{\Omega^2}\right]J_{mi,s}^2 \tag{33}$$

where

$$Q_{1,s} = \sum_s\left(Q_{11,s} + Q_{12,s} + Q_{11p,s} + Q_{12p,s}\right) \tag{34}$$

$$Q_{2,s} = \sum_s\left(Q_{21,s} + Q_{22,s} + Q_{21p,s} + Q_{22p,s}\right) \tag{35}$$

and $P_{E,s}$, $P_{HE,s}$, $P_{EH,s}$ are defined by (B-8) - (B-10) in Part I [1]. To obtain $Q_{1,s}$ and $Q_{2,s}$ we need only replace k_z by $k_{z,s}$ and k_{z0} by $k_{z0,s}$ in the relevant equations. In fact, we know that:

$$k_{z,s}^2 - k_{z0,s}^2 = (k_z - k_{z0})\left(2k_{z0} + \frac{4\pi s}{L}\right) = 2k_{z0,s}(k - k_{z0}) \tag{36}$$

$$k_{z,s} - k_{z0,s} = k_z - k_{z0} \tag{37}$$

This dispersion equation can be simplified further if desired.

IV. Plasma -Filled Electron Cyclotron Maser (ECRM)

For the plasma-filled electron cyclotron maser, electrons gyrate around guiding centers with small cyclic orbits. The dispersion equation is given by (82) in Part I as:

$$\varepsilon_1(k_z^2 - k_{z0}^2)P_{E\perp} + j\omega\varepsilon_0\varepsilon_2(k_z - k_{z0})P_{HE_\perp} +$$
$$\mu_0(\varepsilon_3 - \varepsilon_1)(k_z - k_{z0})P_{EH_\perp} = -j\omega\mu_0\varepsilon_1\int J_\theta \cdot E_{\theta 0}^* ds - \frac{1}{\varepsilon_0}\int(\nabla_\perp\rho)_\theta \cdot E_{\theta 0}^* ds \tag{38}$$

Neglecting the space-charge term, the main problem is to calculate the term, $\iint \bar{J}_\theta \cdot \bar{E}_\theta^* ds$. By means of kinetic theory [2], we have:

$$\bar{J} = -e\int d\bar{p}\left(f_1 \frac{\bar{p}}{m_0\gamma}\right) \tag{39}$$

which yields:

$$J_\theta = -e\int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \frac{p_\theta}{m_0\gamma} \tag{40}$$

$$\rho = -e\int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \tag{41}$$

where the perturbed distribution function is [4], [5]:

$$\begin{aligned} f_1 = -e\int_{-\infty}^{t} dt' \left(\bar{E}_1' + \bar{v}' \times \bar{B}_1'\right)\cdot \nabla_p f_0 = -e\int_{-\infty}^{t} dt' \Bigg[\left(E_\theta' + v_z B_r'\right)\left(\frac{\partial f_0}{\partial p_\theta} + \frac{\partial f_0}{\partial R_g}\right) + \\ \left(E_z' - v_\theta' B_r'\right)\frac{\partial f_0}{\partial p_z} - \left(E_r' + v_\theta' \quad B_\theta'\right)\frac{1}{p_\theta}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial \phi} \Bigg] \end{aligned} \tag{42}$$

The integration should be taken along an unperturbed orbit, and f_0 is the equilibrium distribution function.

$$f_0 = \frac{n_b}{4\pi^2 p_{\perp 0} R_0}\delta(p_z - p_{z0})\delta(p_\perp - p_{\perp 0})\delta(R_g - R_0) \tag{43}$$

where R_0 is the radius of the guiding center of the electron beam. Since the field can be split into four parts, we get:

$$E_\theta = E_{\theta 1} + E_{\theta 2} + E_{\theta 1 r} + E_{\theta 2 r}, etc. \tag{44}$$

$$f_1 = f_{11} + f_{12} + f_{11r} + f_{12r} \tag{45}$$

$$J_\theta = J_{\theta 1} + J_{\theta 2} + J_{\theta 1r} + J_{\theta 2r} \tag{46}$$

Thus,

$$\begin{aligned} \iint J_\theta \cdot E_\theta^* ds = \iint \big(J_{\theta 1}E_{\theta 1}^* + J_{\theta 1}E_{\theta 2}^* + J_{\theta 1}E_{\theta 1r}^* + J_{\theta 1}E_{\theta 2r}^* + J_{\theta 1r}E_{\theta 1}^* + J_{\theta 1r}E_{\theta 2}^* + J_{\theta 1r}E_{\theta 1r}^* + \\ J_{\theta 1r}E_{\theta 2r}^* + J_{\theta 2}E_{\theta 1}^* + J_{\theta 2}E_{\theta 2}^* + J_{\theta 2}E_{\theta 1r}^* + J_{\theta 2}E_{\theta 2r}^* + J_{\theta 2r}E_{\theta 1}^* + J_{\theta 2r}E_{\theta 2}^* + J_{\theta 2r}E_{\theta 1r}^* + J_{\theta 2r}E_{\theta 2r}^* \big) \end{aligned} \tag{47}$$

Equations (44)-(47) indicate that the electric field of the wave is split into four components: $E_{\theta 1}, E_{\theta 2}, E_{\theta 1p}$, and $E_{\theta 2p}$. $E_{\theta 1}$ and $E_{\theta 2}$ are the TE-like and TM-like field components, respectively, while $E_{\theta 1p}$ and $E_{\theta 2p}$ are the 'TE-like' and 'TM-like' field components due to the plasma background respectively. f_{11}, f_{12}, f_{11p} and f_{12p} are the four components of the perturbed distribution function corresponding to $E_{\theta 1}, E_{\theta 2}, E_{\theta 1p}$, and $E_{\theta 2p}$, respectively. $J_{\theta 1p}$ and $J_{\theta 2p}$ are excited by the field components due to the plasma background for TE-like and TM-like fields, respectively. It should be remembered that when plasma is absent, $J_{\theta 1p}$ and $J_{\theta 2p}$ disappear while $J_{\theta 1}$ and $J_{\theta 2}$ remain but are decoupled and can exist independently.

When the plasma is absent, $\omega_p = 0$. For TE modes, only $E_{\theta 1} \neq 0$ and (47) thus becomes:

$$\iint J_\theta \cdot E_\theta^* ds = \iint J_{\theta 1} \cdot E_{\theta 1}^* ds \tag{48}$$

With $P_{HF} = P_{EH} = 0, \varepsilon_3 = 1$; (38) thus becomes:

$$k_z^2 - k_{z0}^2 = -j\frac{\omega\mu_0}{P_E}\iint J_{\theta 1} \cdot E_{\theta 1}^* ds \tag{49}$$

This is the same as that for the vacuum ECRM case [6] and (83) of [1]. Correspondingly, for TM modes, only $J_{\theta 2} \neq 0$ and (38) then becomes:

$$k_z^2 - k_{z0}^2 = -j\frac{\omega\mu_0}{P_E}\iint J_{\theta 2} \cdot E_{\theta 2}^* ds \tag{50}$$

Equation (50) is exactly that given for TM modes in [4]. In (49) and (50):

$$k_{z0}^2 = \frac{\omega^2}{c^2} - k_{cR}^2 \tag{51}$$

while $k_{cR}^2 = \frac{\mu_{mn}^2}{R_c^2}$ for TE, but $k_{cR}^2 = \frac{\nu_{mn}^2}{R_c^2}$ for TM. μ_{mn} and ν_{mn} are the roots of $J_m(x) = 0$ and $J_m^{'}(x) = 0$, respectively.

In order to solve (38), we must first calculate (47) which requires all the listed field and current density components. These are presented in Appendix B. After substituting all those terms into (47) and then inserting the resultant into (38), we can then rewrite the general dispersion relation for a plasma-filled ECRM as:

$$\begin{aligned} &\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_{E\perp} + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE\perp}^{(1)} + \omega\mu_0(\varepsilon_3 - \varepsilon_1)(k_z - k_{z0})P_{HE\perp}^{(2)} \\ &= 2\pi\frac{\omega_b^2}{c^2\gamma}\frac{\omega\varepsilon_1}{v_{\perp 0}}\sum_{i,l}\frac{A_{il}^2}{D^2}R_0\cdot\left[\frac{T_1}{\Omega_1} - \frac{T_2\beta_\perp^2}{\Omega_1^2}\right]\cdot\left|J_{m-li}\right|^2 \end{aligned} \tag{52}$$

where:

$$\left.\begin{aligned} T_1 &= T_{\theta 11} + T_{\theta 1p1} + T_{\theta 21} + T_{\theta 2p1} \\ T_2 &= T_{\theta 12} + T_{\theta 1p2} + T_{\theta 22} + T_{\theta 2p2} \end{aligned}\right\} \tag{53}$$

$$\Omega_1 = \omega - k_z v_z - l\omega_c \tag{54}$$

$$\left.\begin{aligned} J_{li} = J_l(p_i r) \qquad J_{li}^{'} = \frac{d}{dr}J_l(p_i r) \\ J_{m-l,i} = J_{m-l}(p_i R_0) \end{aligned}\right\} \quad i = 1,2 \tag{55}$$

$$A_i = \begin{cases} A & i = 1 \\ -\dfrac{J_m(p_2 R_c)}{J_m(p_1 R_c)}A & i = 2 \end{cases} \tag{56}$$

where k_{z0} can be found from the dispersion equation given in Appendix A of Part I. All eight of the tedious T coefficients may be found in Appendix A at the end of this paper. Equations (52) through (56) are the general dispersion equations for a plasma-filled ECRM. They are also valid for the vacuum case. In fact, when $\omega_p = 0$, we get the same dispersion equation as that given in [4].

A. Coupled Longitudinal Interactions

As mentioned above for an MPW, the TE and TM modes are always coupled. Transverse and longitudinal interactions, therefore, always coexist with each other. Thus the dispersion equation for longitudinal interactions in a plasma-filled ECRM should be taken into account simultaneously. The dispersion equation for such longitudinal interactions is given by (66) in Part I as:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = -j\omega\mu_0\varepsilon_1 \iint J_z E_z^* ds + \frac{jk_z}{\varepsilon_0}\iint \rho E_z^* ds \tag{57}$$

Here, J_z may be calculated using kinetic theory as.

$$J_z = -e\int_{-\infty}^{\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \frac{p_z}{m_0\gamma} = -2\pi e\int_{-\infty}^{\infty} dp_z \int_0^{\infty} \frac{p_\perp p_z}{m_0\gamma}(f_{11} + f_{12} + f_{11p} + f_{12p})dp_\perp \tag{58}$$

By integrating through momentum space, we obtain the full expressions for:

$$J_z = J_{z1} + J_{z2} + J_{z1p} + J_{z2p} \tag{59}$$

These are given in (B-29) – (B-32) in Appendix B. Substituting (B-29)-(B-32) into (57) and neglecting the space charge term, we get the dispersion equations for the accompanying longitudinal interactions as:

$$\begin{aligned}&\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = \\ &2\pi\frac{\omega_b^2}{c^2\gamma}\frac{\omega\varepsilon_1}{v_{\perp 0}}\cdot\sum_{i,l}\frac{A_{li}^2}{D^2}R_n\cdot\left[\frac{\Pi_1}{\Omega_1} - \frac{\Pi_2\beta_\perp\beta_{\parallel}}{\Omega_1^2}\right]\cdot\left|J_{m-li}\right|^2\end{aligned} \tag{60}$$

where Π_1, Π_2 are given in Appendix B, (B-34), (B-35). Equation (60) is always accompanied by (52).

From the above formulation, we can see that although the dispersion equations of a plasma-filled ECRM are very complicated, the structure remains unchanged. First, there are still two main terms: $(\omega - k_z v_z - l\omega_c)^{-1}$ and $(\omega - k_z v_z - l\omega_c)^{-2}$, just as in the vacuum ECRM case. Second, since there are four parts of the wave field and four parts of the RF current, the term $\int \vec{J}\cdot\vec{E}ds$ has sixteen terms. This makes the equations very tedious, but the physical lines are still clear. Third, we can see clearly that, looking at the TM-like and TE-like parts separately, the structures of the dispersion equations are similar to that of the vacuum case. Also when the plasma is absent, the equations reduce to that for the vacuum case. Fourth, the most important difference between the plasma-filled and the vacuum case is that some of the terms associated with the parts of the field produced by the plasma background are imaginary. These terms not only make the dispersion equations complicated but also cause an instability different from that in the vacuum case. Finally, the most essential difference is that the ECRM interactions are always accompanied by and coupled with the corresponding longitudinal interactions.

B. Interactions in a Corrugated MPW

The above equations are for a smooth waveguide. For a periodic structure and neglecting the space-charge term, (38) and (40) simply become:

$$\begin{aligned}&\sum_s\left[\varepsilon_3\left(k_{z,s}^2-k_{z0}^2\right)P_{E\perp,s}+j\omega\mu_0\varepsilon_2\left(k_{z,s}-k_{z0}\right)P_{HE\perp,s}+\omega\mu_0(\varepsilon_3-\varepsilon_1)(k_{z,s}-k_{z0})P_{EH\perp,s}\right]\\&=-\sum_s\left[j\omega\mu_0\varepsilon_1\iint\bar{J}_{\theta,s}\cdot\bar{E}_{\theta,s}^{*}ds\right]\end{aligned}\tag{61}$$

and:

$$J_{\theta,s}=-e\int_{-\infty}^{\infty}dp_z\int_0^{\infty}p_\perp dp_\perp\int_0^{2\pi}d\phi f_{1,s}\frac{p_\theta}{m_0\gamma}\tag{62}$$

where:

$$\begin{aligned}f_{1,s}&=-e\int_{-\infty}^{t}dt'\left(\bar{E}_{1,s}'+\bar{v}'\times\bar{B}_{1,s}'\right)\cdot\nabla_p f_0=-e\int_{-\infty}^{+\infty}dt'\left[\left(E_{\theta,s}'+v_z'B_{r,s}'\right)\left(\frac{\partial f_0}{\partial p_\theta}+\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial p_\theta}\right)+\right.\\&\left.\left(E_{z,s}'-v_\theta' B_{r,s}'\right)\frac{\partial f_0}{\partial p_z}-\left(E_{r,s}'+v_\theta' B_{z,s}'-v_z' B_{\theta,s}'\right)\frac{1}{p_\theta}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial\phi}\right]\end{aligned}\tag{63}$$

Similarly:

$$E_{\theta,s} = E_{\theta 1,s} + E_{\theta 2,s} + E_{\theta 1p,s} + E_{\theta 2p,s} \tag{64}$$

$$f_{1,s} = f_{11,s} + f_{12,s} + f_{11p,s} + f_{12p,s} \tag{65}$$

$$J_{\theta,s} = J_{\theta 1,s} + J_{\theta 2,s} + J_{\theta 1p,s} + J_{\theta 2p,s} \tag{66}$$

$$\begin{aligned}\sum_s \iint J_{\theta,s} \cdot E_{\theta,s}^{*} ds = \sum_s \iint \Big(& J_{\theta 1,s}E_{\theta 1,s}^{*} + J_{\theta 1,s}E_{\theta 2,s}^{*} + J_{\theta 1,s}E_{\theta 1p,s}^{*} + J_{\theta 1,s}E_{\theta 2p,s}^{*} + J_{\theta 1p,s}E_{\theta 1,s}^{*} + \\ & J_{\theta 1p,s}E_{\theta 2,s}^{*} + J_{\theta 1p,s}E_{\theta 1p,s}^{*} + J_{\theta 1p,s}E_{\theta 2p,s}^{*} + J_{\theta 2,s}E_{\theta 1,s}^{*} + J_{\theta 2,s}E_{\theta 2,s}^{*} + J_{\theta 2,s}E_{\theta 1p,s}^{*} + \\ & J_{\theta 2,s}E_{\theta 2p,s}^{*} + J_{\theta 2p,s}E_{\theta 1,s}^{*} + J_{\theta 2p,s}E_{\theta 2,s}^{*} + J_{\theta 2p,s}E_{\theta 1p,s}^{*} + J_{\theta 2p,s}E_{\theta 2p,s}^{*} \Big) ds\end{aligned} \tag{67}$$

If only seeking the resonance term, one need simply delete the $\sum_s$ in the equations and replace the arguments in $J_{li}, J_{li}', J_{m-li}$ by $p_{\perp,s}r_c$ and $p_{\perp,s}, R_g$. These procedures parallel those we followed to obtain (52). Therefore, it is not surprising that the dispersion relation that emerges is similar to (52) where we replace all k_z by $k_{z,s} = k_z + \frac{2\pi s}{L}$ and replace A_{il} by $A_{il,s}$, the s-th harmonic, and obtain:

$$\begin{aligned}&\varepsilon_3 \sum_s \left\{ \left(k_{z,s}^2 - k_{z0,s}^2\right) P_{E\perp,s} + j\omega\mu_0\varepsilon_2\left(k_{z,s} - k_{z0,s}\right) P_{HE\perp,s}^{(1)} + \omega\mu_0\left(\varepsilon_3 - \varepsilon_1\right)\left(k_{z,s} - k_{z0,s}\right) P_{HE\perp,s}^{(2)} \right\} \\ &= 4\pi^2 e^2 \omega\mu_0\varepsilon_1 \sum_s \left\{ \sum_{i,l} \frac{A_{il,s}^2}{D_s^2} \cdot \left[\frac{T_{1,s}}{\Omega_l} - \frac{T_{2,s}\beta_\perp^2}{\Omega_l^2} \right] \right\}\end{aligned} \tag{68}$$

For the accompanying longitudinal interactions, we have:

$$\begin{aligned}&\sum_s \left[\varepsilon_3\left(k_{z,s}^2 - k_{z0,s}^2\right) P_{E,s} + j\omega\mu_0\varepsilon_2\left(k_{z,s} - k_{z0,s}\right) P_{HE,s} \right] = \\ &4\pi^2 e^2 \omega\mu_0\varepsilon_1 \cdot \sum_{i,l}\sum_s \frac{A_{il,s}^2}{D_s^2} R_0 \left[\frac{H_{1,s}}{\Omega_{l,s}} - \frac{H_{2,s}\beta_\perp\beta_\parallel}{\Omega_{l,s}^2} \right] \cdot \left|J_{m-li}\right|^2\end{aligned} \tag{69}$$

$$J_z = \sum_s J_{z,s} \tag{70}$$

$$J_{z,s} = -e\int_{-\infty}^{\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_{1,s} \frac{p_z}{m_0\gamma} \tag{71}$$

$$E_z = \sum_s E_{z,s} = \sum_s \left(E_{z1,s} + E_{z2,s} \right) \tag{72}$$

$$f_{1,s} = f_{11,s} + f_{12,s} + f_{11p,s} + f_{12p,s} \tag{73}$$

where again A_{ls} is the amplitude of s-th harmonic and $k_{z,s} = k_z + \frac{2\pi s}{L}$. For simplicity, in the above equations one need only take the s-harmonic. It should be noted here that when $r_c \to R_0$, all the results given here are still valid.

V. On the Combination of Transverse and Longitudinal Interactions

The combination of Cherenkov radiation and the electron cyclotron resonance instability is presented in [7]. It is important to mention this kind of wave instability here also. From the kinetic theory of the ECRM and accompanying longitudinal interactions given in the above section, we can see that, if there is transverse electron motion, then both transverse and longitudinal interactions can co-exist under the condition:

$$\Omega_l = \omega - k_z v_z - l\omega_c \approx 0 \qquad \text{or} \quad \Omega_s = \omega - k_{z,s} v_z - l\omega_c \approx 0 \tag{74}$$

It should be emphasized here that the coexistence (or combination) of transverse and longitudinal interactions (ECRM and Cherenkov or ECRM and BWO, for example) is one of the most significant features for electron beam-wave interactions in an MPW. Now, since there is transverse electron motion, the singularity is also at $\Omega = \omega - k_z v_z - l\omega_c \approx 0$ for the coupled longitudinal interactions.

VI. Dispersion Equations For Transverse Interactions with an Ion-Channel Taken into Account

We now turn our attention to the case of transverse interactions with an ion-channel taken into consideration. We refer to the simplified model depicted in Fig. 1b of Part I with a defined as the MPW radius, R_i the ion channel radius, and b the radius of the thin hollow e-beam [1]. Here, we only consider the simple condition when $n_b >> n_p R_i^2/b^2$. For this case, the ion-channel completely fills the MPW outside the beam so that the radius of the ion-channel, R_i, is equal to the radius of the waveguide, a. This leaves only two regions to consider. Region I is the ion-channel volume outside the hollow beam ($b<R<a$) and Region II is the plasma volume inside the hollow beam ($R<b$). The field components in Region I and Region II are presented in Appendix C. As is given in (11)-(15) in Part I, the field components in Region I can be expressed as follows:

$$E_R = E_{R1} + E_{R2} + E_{Rp1} + E_{Rp2} \tag{75}$$

$$E_\varphi = E_{\varphi 1} + E_{\varphi 2} + E_{\varphi p1} + E_{\varphi p2} \tag{76}$$

$$H_R = H_{R1} + H_{R2} + H_{Rp1} + H_{Rp2} \tag{77}$$

$$H_\varphi = H_{\varphi 1} + H_{\varphi 2} + H_{\varphi p1} + H_{\varphi p2} \tag{78}$$

$$E_{Ri} = \frac{A_i}{D} T_{1i}; E_{Rpi} = \frac{A_i}{D} P_1 \tag{79}$$

$$E_{\varphi i} = \frac{A_i}{D} T_{2i}; E_{\varphi pi} = \frac{A_i}{D} P_{2i} \tag{80}$$

$$H_{Ri} = \frac{A_i}{D} T_{3i}; H_{Rpi} = \frac{A_i}{D} P_{3i} \tag{81}$$

$$H_{\varphi i} = \frac{A_i}{D} T_{4i}; H_{\varphi pi} = \frac{A_i}{D} P_{4i} \tag{82}$$

where the T and P coefficients are defined by (C-3) in Appendix C.

From the above equations, we can see that the field components are also split into four types: TE-like, TM-like, and plasma-produced TE-like and TM-like parts. There are important differences. The amplitude of each component is connected with the others through the T and P coefficients (see Appendix C). This shows that the influence of the plasma on the wave field is strong when an ion-channel is formed. The beam-wave interactions, therefore, become much more complicated.

Now we present the kinetic theory treatment of the ECRM filled with magnetized plasma with an ion-channel taken into account. Since the interactions take place in the ion-channel region, simple mathematics yields the dispersion equation:

$$\left(k_z^2 - k_{z0}^2\right)P_E = -j\omega\mu_0 \int J_\varphi E_\varphi^* dS \tag{83}$$

for large orbit transverse interactions, and

$$\left(k_z^2 - k_{z0}^2\right)P_E = -j\omega\mu_0 \sum_l \int J_{\theta l} E_{\theta}^* dS \tag{84}$$

for the ECRM instability. By means of kinetic theory, we can obtain:

$$f_1 = f_{11} + f_{12} + f_{13} + f_{14} \tag{85}$$

where $f_{1i} = e\int_{-\infty} dt' \left\{\left[E_{\theta i}' + \upsilon_z B_{\theta i}'\right]\frac{\partial f_0}{\partial p_\perp} + \left(E_{zi}' + \upsilon_\theta B_{ri}'\right)\frac{\partial f_0}{\partial p_\parallel}\right\}$

$$= \frac{-je}{\Omega_l}\left\{\left[E_{\theta i} + \upsilon_z B_{ri}\right]\frac{\partial f_0}{\partial p_\perp} + \left(E_{zi} + \upsilon_\theta B_{ri}\right)\frac{\partial f_0}{\partial p_\parallel}\right\} \tag{86}$$

$$\left(i = 1,2,3,4 \; for E_{\theta i}, B_{ri}; i = 1,2 \; for E_{zi}\right)$$

We then get:

$$J_\theta = \sum_l J_{\theta l} = \sum_i \sum_l -\frac{je^2}{m_0\gamma}\left\{\frac{1}{\Omega_l}\left[2\left(E_{\theta il} + v_z B_{ril}\right)\right] + p_\perp\left(\frac{\partial E_{\theta il}}{\partial p_\perp} + v_z \frac{\partial B_{ril}}{\partial p_\perp}\right) - \frac{v_\perp}{\Omega_l^2}\left[k\beta_\perp E_{\theta il} + B_{ril}k_z v_\perp + k\beta_{//}E_{zil} - k_z E_{zil}\right]\right\} \tag{87}$$

Finally we can substitute this into (83) and get the dispersion equation for the ECRM as follows:

$$(k_z^2 - k_{z0}^2) = \frac{\omega\omega_p^2}{c^2\gamma}\sum_l\left[\frac{Q}{\Omega_l} - \frac{\beta_\perp^2 W}{\Omega_l^2}\right]M \tag{88}$$

where

$$Q = Q_1 + Q_2 ; W = W_1 + W_2 \tag{89}$$

$$Q_1 = \frac{A_1 l}{r_c}(k\beta_1 - k_z)(J_l + pr_c J_l')J_{m-l} + \frac{A_2 l}{r_c}(k\beta_1 - k_z)(N_l + pr_c N_l')N_{m-l} \tag{90}$$

$$Q_2 = jA_3\mu_0 p(\omega - k_z\upsilon_{z0})(2J_l' + pr_c J_l'')J_{m-l} + jA_4\mu_0 p(\omega - k_z\upsilon_{z0})(2N_l' + pr_c N_l'')N_{m-l} \tag{91}$$

$$W_1 = p^2\left(\frac{\omega\beta_1}{\beta_\perp} - \frac{k_z c}{\beta_\perp}\right)\left(A_1 J_l J_{m-l} + A_2 N_l N_{m-l}\right) \tag{92}$$

$$W_2 = j\mu_0 p(\omega^2 - k_z^2 c^2)(A_3 J_l' J_{m-l} + A_4 N_l' N_{m-l}) \tag{93}$$

$$M = \frac{lk_z}{r_c}\left(A_1^* J_l J_{m-l} + A_2^* N_l N_{m-l}\right) + j\omega\mu_0 p\left(A_3^* J_l' J_{m-l} + A_4^* N_l' N_{m-l}\right) \tag{94}$$

$$J_l = J_l(pr_c); J_{m-l} = J_{m-l}(pR_0) \tag{95}$$

where the A coefficients are derived in terms of the field and current density components in Appendix C. As already mentioned, there exist coupled longitudinal interactions. The dispersion equations can be derived by a similar procedure. With the use of a simplified MPW model, we can very efficiently study transverse interactions in an MPW with an ion-channel.

VII. Sample Numerical Calculations of the Transverse Interactions

We have calculated the characteristics of a plasma-filled ECRM in Section IV. This case serves as a good example for illustrating the nature of transverse interactions. The dispersion relation for the plasma-filled ECRM is given by (52) in Section IV using kinetic theory. Ancillary equations are given in (53)-(56) and the related cumbersome coefficients are found in Appendix A.

Several interesting points are worth noting about the plasma filled ECRM. First, for the HE_{01} mode, the dispersion curve is split into two branches; one below $\omega=\omega_h$ (cyclotron mode), the other in the frequency range higher than $\omega=\omega_h$ (waveguide mode). Since no wave can propagate above the $\omega=\omega_h$ line [as was indicated in Part I] Therefore, for the HE_{01} mode, we must do two separate calculations to characterize these two branches.

Second, for the branch below $\omega=\omega_h$, the frequency band is quite narrow and very close to the cut-off line $\omega=\omega_h$. Thus the calculation must be done very carefully. For the branch above $\omega=\omega_h$, we found that only the line of higher harmonics can touch the dispersion curve. That means that for this branch the ECRM can only work at higher cyclotron harmonics. Our sample calculation is given for the second harmonic.

One other special feature should be pointed out before presenting the sample calculations. For the plasma-filled ECRM, the field depends on two eigenvalues, p_1 and p_2. Both p_1 and p_2 are functions of ω. Therefore, the calculations based on ω seem rather complicated. It will be much easier to do the same calculations based on k_z. Since we know the exact relation between ω and k_z, once we get one we can calculate the other. In the following, we deal with k_z ($k_z = \mathrm{Re}(k_z) + j\,\mathrm{Im}(k_z)$) rather than ω. The results of these calculations are shown in Figs. 1 through 6.

Figures 1a and 1b show the main features of beam-wave interactions in a plasma-filled ECRM. The specific parameters used in this example are given in the figure captions. Fig. 1a shows the typical dispersion curves and the beam-wave interaction plot. (See Figure A-1 in Appendix A of Part I for more complete details.) It shows that the beam-wave interaction happens at the second harmonic with the waveguide mode HE_{01}. On the other hand, Fig. 1b shows the beam-wave interaction happens at the first harmonic with one of the cyclotron modes. It is clear from Figs. 1a and 1b that the plasma filled ECRM prefers to operate at higher harmonics. Fig. 2 shows a sample plot of the dispersion equation (for the imaginary parts of k_z). It can be seen in Figs. 3 through 6 (for the imaginary part of k_z) that for higher ω_c or higher ω_p, the dispersion curves go up rapidly. This is because the group velocity of the wave approaches zero near the cut-off frequency. Once again, the specific parameters used for each figure are shown in the figure captions. Figure 4 shows beam-wave interactions for the second cyclotron harmonic (l=2), for the waveguide wave (HE_{01} mode). Figs. 3 and 4 are for different values of the magnetic field, B_0 (as reflected in different values of ω_c). Figs. 5 and 6 show the effect of different plasma background densities (as reflected in different values of ω_p). It can be seen that increasing the magnetic field strength may cause an increase of the bandwidth and the growth rate of the interaction as well. Increasing the density of the plasma background, on the other hand, may at first cause an increase of both growth rate and bandwidth of the interaction, but then may cause the decay of both the growth rate and band width when ω_p is large. It is interesting to see that the vacuum case, ω_p=0, is intermediate.

The figures show that the growth rate of the plasma-filled case is comparable to that for the vacuum case. It should be noted that all the calculations were for the same value of current density. It is obvious that in the plasma-filled case, a much higher current density can be used than that for the vacuum case.

As was mentioned above, the longitudinal interactions are always coupled with transverse interactions. Longitudinal interactions may occur, in general for slow waves, therefore, for a corrugated MPW, coupled longitudinal interactions are always accompanied by transverse interaction waves. However, for a smooth waveguide, the coupled longitudinal interactions may occur either with the slow cyclotron mode or with plasma waves (T-G modes) and with phase velocity less than the velocity of light. Therefore, for a smooth waveguide, the operating condition for coupled longitudinal interactions is:

$$\omega - k_z v_{z0} - \omega_c \approx 0 \tag{96}$$

for plasma waves. The frequency range is between 0 and ω_p ($0<\omega<\omega_p$). However, for the ECRM, the operating condition for a second harmonic waveguide wave is:

$$\omega - k_z v_{z0} - 2\omega_c \approx 0 \tag{97}$$

with frequency range ($\omega > \omega_h$). This shows that the coupled longitudinal interactions occur at a different frequency (much lower) than that for transverse interactions. It can be shown, however, that for a corrugated waveguide, the cyclotron frequency for both longitudinal as well as for transverse interactions can be the same. Sample calculations of coupled longitudinal interactions may be carried out by using (1) and (9) for a plasma wave.

VIII. Conclusions

A kinetic theory treatment of electron-beam/wave interactions in an MPW, for both longitudinal and transverse interactions in both smooth and corrugated waveguides, has been given in this paper. At least six points from this presentation are worthy of special emphasis.

First, although the mathematical manipulations are tedious, the structures of all the dispersion relations are simple. They consist of two parts, one is proportional to $\left(\frac{1}{\Omega_m}\right)$ or $\left(\frac{1}{\Omega_l}\right)$ (or $\left(\frac{1}{\Omega_{m,s}}\right)\cdot\left(\frac{1}{\Omega_{l,s}}\right)$), the other to $\left(\frac{1}{\Omega_m^2}\right)$ or $\left(\frac{1}{\Omega_l^2}\right)$ (or $\left(\frac{1}{\Omega_{m,s}^2}\right)\cdot\left(\frac{1}{\Omega_{l,s}^2}\right)$) , where: $\begin{array}{l}\Omega_l = \omega - k_z v_z - l\omega_c \\ \Omega_m = \omega - k_z v_z - m\omega_c\end{array}$

This is just like that for the familiar vacuum case.

The second most important feature of beam-wave interactions in an MPW is the coexistence or combination of instabilities of both transverse and longitudinal interactions. This is because in such an

MPW the TE and TM modes are always coupled so that the E_z and $\vec{E}_\perp$ field components always exist together. Due to the transverse motion in the magnetic field, the singularity of the accompanied longitudinal interaction is at $\Omega = \omega - k_z v_z - l\omega_c \approx 0$.

The third most important feature is that in the dispersion equations there are special imaginary terms. These terms are generated by the field parts that are produced by the plasma background. These imaginary terms may influence the instability.

Fourth, only when there is no transverse motion can we have purely longitudinal interactions. In that case the singularity is at $\Omega = \omega - k_z v_z$, just as that for the TM mode in the vacuum case.

Fifth, for the plasma-filled ECRM, the most significant feature is that the beam-wave interactions are split into two regimes: either with cyclotron modes or with waveguide modes, due to the characteristics of wave propagation along an MPW. For cyclotron modes, the interaction seems difficult since there are so many dense modes. For the waveguide modes, the ECRM prefers to operate at higher harmonics.

Finally, with the coexistent instabilities (ECRM and Cherenkov, for example), there must be some difference in the frequency response between the two kinds of interactions. Therefore the spectral purity of the output of a device based on an MPW will not be as good as that for a corresponding vacuum device.

Appendix A: Cumbersome Coefficients

1. The eight Q coefficients for the dispersion equations (28)-(31) for transverse interactions in a smooth-walled MPW

$$Q_{11} = -j\mu_0 h_i p_i K^2 v_\perp (\omega - k_z v_z)(2J'_{mi} + p_i R_c J''_{mi}) \cdot$$
$$\left[k_g (k_g^2 p_i J'_{mi} + K^2 \frac{m}{R_c} J_{mi}) - j\omega\mu_0 h_i (K^2 p_i^* J'^*_{mi} + k_g^2 \frac{m}{R_c} J^*_{mi}) \right] \tag{A-1}$$

$$Q_{12} = j\mu_0 h_i p_i K^2 v_\perp (\omega^2 - c^2 k_z^2) J'_{mi} \cdot$$
$$\left[k_g \left(k_g^2 p_i^* J'^*_{mi} + K^2 \frac{m}{R_c} J^*_{mi} \right) - j\omega\mu_0 h_i^* (K^2 p_i^* J'^*_{mi} + k_g^2 \frac{m}{R_c} J^*_{mi}) \right] \tag{A-2}$$

$$Q_{11p} = -j\omega\mu_0 h_i k_g^2 (\omega - k_z v_z)(J_{mi} + p_i R_c J'_{mi}) \cdot$$
$$\left[k_g \left(k_g^2 p_i^* J^*_{mi} + K^2 \frac{m}{R_c} J^*_{mi} \right) - j\omega\mu_0 h_i^* (K^2 p_i^* J'^*_{mi} + k_g^2 \frac{m}{R_c} J^*_{mi}) \right] \tag{A-3}$$

$$Q_{12p} = j\mu_0 h_i k_g^2 J_{mi} (\omega^2 - k_z^2 c^2) \cdot \left[k_z \left(k_g^2 p_i^* J'^*_{mi} + K^2 \frac{m}{R_c} J^*_{mi} \right) \right.$$
$$\left. - j\omega\mu_0 h_i^* \left(K^2 p_i^* J^*_{mi} + k_g^2 \frac{m}{R_c} J^*_{mi} \right) \right] \tag{A-4}$$

$$Q_{21} = -K^2 l \omega_c (J_{mi} + p_i R_c J'_{mi})(k_g - k\beta_z \varepsilon_1) \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J'^*_{mi} + K^2 \frac{m}{R_c} J^*_{mi} \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J^*_{mi} + k_g^2 \frac{m}{R_c} J^*_{mi} \right) \right] \tag{A-5}$$

$$Q_{22} = -J_{mi} \left[k_g \omega K^2 m \omega_c (\varepsilon_1 - 1) - D(\omega v_z - k_z c^2) \right] \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J'^*_{mi} + K^2 \frac{m}{R_c} J^*_{mi} \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J'^*_{mi} + k_g^2 \frac{m}{R_c} J^*_{mi} \right) \right] \tag{A-6}$$

$$Q_{21p} = -\left[k_z v_\perp p_i \left(k_g^2 - \varepsilon_2 k_z k \beta_{//} \right) \left(2J'_{mi} + p_i R_c J''_{mi} \right) + \varepsilon_2 k_g^2 \beta_\perp k l \omega_c \left(J_{mi} + p_i R_c J'_{mi} \right) \right] \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J^*_{mi} + K^2 \frac{m}{R_c} J^*_{mi} \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J'^*_{mi} + k_g^2 \frac{m}{R_c} J^*_{mi} \right) \right] \tag{A-7}$$

$$Q_{22p} = -k_z \omega \omega_c \varepsilon_2 \left[\left(k_z^2 - k^2 \right) p_i R_c J'_{mi} - m k_g^2 J_{mi} \right] \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{mi}^{**} + K^2 \frac{m}{R_c} J_{mi}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{mi}^{**} + k_z \frac{m}{R_c} J_{mi}^* \right) \right] \tag{A-8}$$

2. The eight T coefficients for the dispersion equations (52)-(53) for a plasma-filled ECRM.

$$T_{\theta 11} = -j\mu_0 h_i p_i K^2 v_\perp (\omega - k_z v_z)(2J'_{li} + p_i r_c J''_{li}) \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{li}^{**} + K^2 \frac{l}{r} J_{li}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{li}^{**} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-9}$$

$$T_{\theta 12} = j\mu_0 h_i p_i K^2 v_\perp (\omega^2 - c^2 k_z^2) J'_{li} \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{li}^{**} + K^2 \frac{l}{r} J_{li}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{li}^{**} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-10}$$

$$T_{\theta 1p1} = -j\mu_0 h_i p_i k_g^2 (\omega - k_z v_z)(J_{li} + p_i r_c J'_{li}) \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{li}^{**} + K^2 \frac{l}{r} J_{li}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{li}^{**} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-11}$$

$$T_{\theta 1p2} = j\mu_0 h_i k_g^2 J_{li} (\omega^2 - c^2 k_z^2) \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{li}^{**} + K^2 \frac{l}{r} J_{li}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{li}^{**} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-12}$$

$$T_{\theta 21} = -K^2 l \omega_c (J_{li} + p_i r_c J'_{li})(k_z - k\beta_{//} \varepsilon_1) \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{li}^{**} + K^2 \frac{l}{r} J_{li}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{li}^{**} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-13}$$

$$T_{\theta 22} = -J_{li} \left[k_z \omega K^2 l \omega_c (\varepsilon_1 - 1) - D(\omega v_z - k_z c^2) \right] \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{li}^{**} + K^2 \frac{l}{r} J_{li}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{li}^{**} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-14}$$

$$T_{\theta 2p1} = -\left[k_z p_i v_\perp (k_g^2 - \varepsilon_2 k_z k \beta_{//})(2J'_{li} + p_i r_c J''_{li}) + \varepsilon_2 k_g^2 \beta_{//} k l \omega_c (J_{li} + p_i r_c J'_{li}) \right] \cdot$$
$$\left[k_z \left(k_g^2 p_i^* J_{li}^{**} + K^2 \frac{l}{r} J_{li}^* \right) - j\omega\mu_0 h_i^* \left(K^2 p_i^* J_{li}^{**} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-15}$$

$$T_{\theta 2p2} = -k_z \omega \omega_c \varepsilon_2 \left[\left(k_z^2 - k^2 \right) p_i r_c J_{li}^{'} - l k_g^2 J_{li} \right] \cdot$$

$$\left[k_z \left(k_g^2 p_i^* J_{li}^{'*} + K^2 \frac{l}{r} J_{li}^* \right) - j \omega \mu_0 h_i^* \left(K^2 p_i^* J_{li}^{'*} + k_g^2 \frac{l}{r} J_{li}^* \right) \right] \tag{A-16}$$

Appendix B: Detailed Terms for Dispersion Equations for a Plasma-Filled ECRM

1. Expressions for the Field Components in a Plasma-Filled ECRM.

According to [4], the wave field may be expanded in the guiding center coordinate system as follows:

$$E_z = \sum_{i=1}^{2} \sum_{l=-\infty}^{\infty} A_{il} J_{li}(p_i r) J_{m-li}(p_i R_g) \tag{B-1}$$

$$H_z = \sum_{i=1}^{2} \sum_{l=-\infty}^{\infty} A_{il} h_i J_{li}(p_i r) J_{m-li}(p_i R_g) \tag{B-2}$$

$$E_r = \sum_{i=1}^{2} \sum_{l=-\infty}^{\infty} \frac{A_{il}}{D} \left[-jk_z \left(K^2 p_i J_{li}^{'} + k_g^2 \frac{l}{r} J_{li} \right) + \omega \mu_0 h_i \left(k_g^2 p_i J_{li}^{'} + K^2 \frac{l}{r} J_{li} \right) \right] J_{m-li} \tag{B-3}$$

$$E_\theta = \sum_{i=1}^{2} \sum_{l=-\infty}^{\infty} \frac{A_{il}}{D} \left[k_z \left(k_g^2 p_i J_{li}^{'} + K^2 \frac{l}{r} J_{li} \right) + j \omega \mu_0 h_i \left(K^2 p_i J_{li}^{'} + k_g^2 \frac{l}{r} J_{li} \right) \right] J_{m-li} \tag{B-4}$$

$$B_r = \sum_{i=1}^{2} \sum_{l=-\infty}^{\infty} \frac{A_{il} \mu_0}{D} \left\{ \omega \varepsilon_0 \left[-\varepsilon_g k_z^2 p_i J_{li}^{'} - \left(\varepsilon_1 K^2 - \varepsilon_2 k_g^2 \right) \frac{l}{r} J_{li} \right] - jk_z h_i \left(K^2 p_i J_{li}^{'} + k_g^2 \frac{l}{r} J_{li} \right) \right\} J_{m-li} \tag{B-5}$$

$$B_\theta = \sum_{i=1}^{2} \sum_{l=-\infty}^{\infty} \frac{A_{il} \mu_0}{D} \left\{ - j \omega \varepsilon_0 \left[\left(\varepsilon_1 K^2 - \varepsilon_2 k_g^2 \right) p_i J_{li}^{'} + \varepsilon_2 k_z^2 \frac{l}{r} J_{li} \right] + k_z h_i \left(k_g^2 p_i J_{li}^{'} + K^2 \frac{l}{r} J_{li} \right) \right\} J_{m-li} \tag{B-6}$$

Equations (B-1)-(B-6) can be simplified as:

$$E_z = \sum_{i,l} A_{il} J_{li} J_{m-li} \tag{B-7}$$

$$H_z = \sum_{i,l} A_{il} h_i J_{li} J_{m-li} \tag{B-8}$$

$$\left.\begin{aligned} E_r &= E_{r1} + E_{r2} + E_{r1p} + E_{r2p} \\ E_\theta &= E_{\theta 1} + E_{\theta 2} + E_{\theta 1p} + E_{\theta 2p} \\ H_r &= H_{r1} + H_{r2} + H_{r1p} + H_{r2p} \\ H_\theta &= H_{\theta 1} + H_{\theta 2} + H_{\theta 1p} + H_{\theta 2p} \end{aligned}\right\} \tag{B-9}$$

where:

$$\left.\begin{aligned} E_{r1} &= \sum_{i,l} \frac{A_{il}}{D} \omega\mu_0 h_i K^2 \frac{l}{r} J_{li} J_{m-li} \\ E_{r2} &= \sum_{i,l} \frac{A_{il}}{D} \left(-jk_z K^2 p_i J_{li}^{\prime}\right) J_{m-li} \\ E_{r1p} &= \sum_{i,l} \frac{A_{il}}{D} \omega\mu_0 h_i k_g^2 p_i J_{li}^{\prime} J_{m-li} \\ E_{r2p} &= -\sum_{i,l} \frac{A_{il}}{D} jk_z k_g^2 \frac{l}{r} J_{li} J_{m-li} \end{aligned}\right\} \tag{B-10}$$

$$\left.\begin{aligned} E_{\theta 1} &= \sum_{i,l} \frac{A_{il}}{D} j\omega\mu_0 h_i K^2 p_i J_{li}^{\prime} J_{m-li} \\ E_{\theta 2} &= \sum_{i,l} \frac{A_{il}}{D} k_z K^2 \frac{l}{r} J_{li} J_{m-li} \\ E_{\theta 1p} &= \sum_{i,l} \frac{A_{il}}{D} j\omega\mu_0 h_i k_g^2 \frac{l}{r} J_{li} J_{m-li} \\ E_{\theta 2p} &= \sum_{i,l} \frac{A_{il}}{D} k_z k_g^2 p_i J_{li}^{\prime} J_{m-li} \end{aligned}\right\} \tag{B-11}$$

$$
\left.
\begin{aligned}
B_{r1} &= \sum_{i,l} \frac{A_{il}\mu_0}{D}\left(-jk_z h_i K^2 p_i J_{li}^{'}\right) J_{m-li} \\
B_{r2} &= \sum_{i,l} \frac{A_{il}\mu_0}{D}\left(-\omega\varepsilon_0\varepsilon_1 K^2 \frac{l}{r} J_{li}\right) J_{m-li} \\
B_{r1p} &= \sum_{i,l} \frac{A_{il}\mu_0}{D}\left(-jk_z h_i k_g^2 \frac{l}{r} J_{li}\right) J_{m-li} \\
B_{r2p} &= \sum_{i,l} \frac{A_{il}\mu_0}{D}(-\omega\varepsilon_0)\varepsilon_2\left(k_z^2 p_i J_{li}^{'} - k_g^2 \frac{l}{r} J_{li}\right) J_{m-li}
\end{aligned}
\right\} \tag{B-12}
$$

$$
\left.
\begin{aligned}
B_{\theta 1} &= \sum_{i,l} \frac{A_{il}\mu_0}{D} k_z h_i K^2 \frac{l}{r} J_{li} J_{m-li} \\
B_{\theta 2} &= \sum_{i,l} \frac{A_{il}\mu_0}{D}\left(-j\omega\varepsilon_0\varepsilon_1 K^2 p_i J_{li}^{'}\right) J_{m-li} \\
B_{\theta 1p} &= \sum_{i,l} \frac{A_{il}\mu_0}{D} k_z h_i k_g^2 p_i J_{li}^{'} J_{m-li} \\
B_{\theta 2p} &= \sum_{i,l} \frac{A_{il}\mu_0}{D} j\omega\varepsilon_0\varepsilon_2\left(k_g^2 p_i J_{li}^{'} - k_z^2 \frac{l}{r} J_{li}\right) J_{m-li}
\end{aligned}
\right\} \tag{B-13}
$$

where: $$A_i = \begin{cases} 1 & i = 1 \\ -\dfrac{J_m(p_2 R_c)}{J_m(p_1 R_c)} & i = 2 \end{cases}$$

Equations (B-7)-(B-13) clearly show that the plasma fill completely changes the wave field. The TE and TM modes are always coupled. Furthermore, there are additional parts of the wave field associated with $k_g^2 = k^2\varepsilon_2$ that are produced by the plasma. The wave field, therefore, may be divided into four parts: TM-like, TE-like (E_{θ_1}, E_{θ_2} , for example), and the corresponding parts produced by the plasma ($E_{\theta_{1p}}$, $E_{\theta_{2p}}$, for example).

2. The Perturbed Transverse Current Density Components

Since f_1 is linearly dependent on the wave field, we can write:

$$f_1 = f_{11} + f_{12} + f_{11p} + f_{12p} \tag{B-14}$$

Thus, the perturbed distribution function also consists of four parts: f_{11} due to the TM-like part of wave field, f_{12} due to the TE-like part of wave field, and f_{11p} and f_{12p} due to the plasma-produced parts of the wave field that correspond to TM-like and TE-like, respectively.

$$\left.\begin{aligned}
f_{11} &= -e\int_{-\infty}^{t} dt'\left(\vec{E}_1' + \vec{v}' \times \vec{B}_1'\right)_{TE-like} \cdot \nabla_p f_0 \\
f_{12} &= -e\int_{-\infty}^{t} dt'\left(\vec{E}_1' + \vec{v}' \times \vec{B}_1'\right)_{TM-like} \cdot \nabla_p f_0 \\
f_{11p} &= -e\int_{-\infty}^{t} dt'\left(\vec{E}_1' + \vec{v}' \times \vec{B}_1'\right)_{plasma-TE-like} \cdot \nabla_p f_0 \\
f_{12p} &= -e\int_{-\infty}^{t} dt'\left(\vec{E}_1' + \vec{v}' \times \vec{B}_1'\right)_{plasma-TM-like} \cdot \nabla_p f_0
\end{aligned}\right\} \tag{B-15}$$

Therefore, we have:

$$\mathrm{J}_{\theta} = \mathrm{J}_{\theta 1} + \mathrm{J}_{\theta 2} + \mathrm{J}_{\theta 1p} + \mathrm{J}_{\theta 2p} \tag{B-16}$$

$$\left.\begin{aligned}
J_{\theta 1} &= -e\int d^3p \cdot f_{11} \frac{p_\perp}{m_0\gamma} \\
J_{\theta 2} &= -e\int d^3p \cdot f_{12} \frac{p_\perp}{m_0\gamma} \\
J_{\theta 1p} &= -e\int d^3p \cdot f_{11p} \frac{p_\perp}{m_0\gamma} \\
J_{\theta 2p} &= -e\int d^3p \cdot f_{12p} \frac{p_\perp}{m_0\gamma}
\end{aligned}\right\} \tag{B-17}$$

Substituting (B-9)-(B-13) into (B-14) and (B-15), we can find:

$$\begin{aligned}
f_{11} = -\sum_{i,l}\frac{je}{\Omega_l}\frac{A_{il}}{D}\Bigg\{ & jh_i p_i K^2 \mu_0 J_{li}'\left[\left(\omega - k_z v_z\right)\frac{\partial f_0}{\partial p_\perp} + k_z v_\perp \frac{\partial f_0}{\partial p_z}\right] - \\
& \left[\frac{l}{r}K^2\left(\omega - k_z v_z\right) + Dv_\perp\right] h_i \mu_0 J_{li} \frac{1}{p_\theta}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial \phi}\Bigg\} J_{m-li}
\end{aligned} \tag{B-18}$$

$$f_{12} = -\sum_{i,l}\frac{je}{\Omega_l}\frac{A_{il}\mu_0}{D}\left\{K^2\frac{l}{r}J_{li}\left[\left(k_z - k\beta_{//}\varepsilon_1\right)\frac{\partial f_0}{\partial p_\perp} + k\beta_\perp\varepsilon_1\frac{\partial f_0}{\partial p_{//}}\right]+ DJ_{li}\frac{\partial f_0}{\partial p_{//}} + jK^2 p_i J_{li}^{\cdot}\left(k_z - k\beta_{//}\varepsilon_1\right)\frac{1}{p_\theta}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial\phi}\right\}J_{m-li} \quad \text{(B-19)}$$

$$f_{11p} = -\sum_{i,l}\frac{je}{\Omega_l}\frac{A_{il}\mu_0 h_i k_g^2}{D}\left\{j\frac{l}{r}J_{li}\left[\left(\omega - k_z v_z\right)\frac{\partial f_0}{\partial p_\perp} + k_z v_\perp\frac{\partial f_0}{\partial p_{//}}\right]- p_i J_{li}^{\cdot}\left(\omega - k_z v_z\right)\frac{1}{p_\theta}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial\phi}\right\}J_{m-li} \quad \text{(B-20)}$$

$$f_{12p} = -\sum_{i,l}\frac{je}{\Omega_l}\frac{A_{il}}{D}\left\{\left[k_g k_z^2 p_i J_{li}^{\cdot} + \varepsilon_2 k\beta_{//}\left(k_g^2\frac{l}{r}J_{li} - k_z^2 p_i J_{li}^{\cdot}\right)\right]\frac{\partial f_0}{\partial p_\perp} - k\varepsilon_2\beta_\perp\left(k_g^2\frac{l}{r}J_{li} - p_i J_{li}^{\cdot}\right)\frac{\partial f_0}{\partial p_{//}} + j\frac{1}{p_\theta}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial\phi}\cdot \left[k_g^2 k_z\frac{l}{r}J_{li} - \varepsilon_2 k\beta_{//}\left(k_z^2\frac{l}{r}J_{li} - k_g^2 p_i J_{li}^{\cdot}\right)\right]\right\}J_{m-li} \quad \text{(B-21)}$$

where:

$$\Omega_l = \omega - k_z v_z - l\omega_c \quad \text{(B-22)}$$

The further derivation is quite complicated and tedious. In order to simplify the mathematical procedure without sacrificing the accuracy and completenession of the theory, in the next step of the calculation, the term connected with $\frac{1}{p_\theta}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial\phi}$ will be neglected. As was pointed out and checked in [4], the contribution of this term is very small. Therefore, after some mathematical manipulation, we can obtain:

$$J_{\theta 1} = 2\pi e^2\sum_{i,l}\int\frac{A_{il}}{D}\mu_0 h_i p_i K^2 v_\perp\left\{\frac{\left(\omega - k_z v_z\right)}{\Omega_l}\left(2J_{li}^{\cdot} + p_i r_c J_{li}^{\cdot\cdot}\right) - \frac{v_\perp^2 J_{li}^{\cdot}}{\Omega_l^2}\left(k^2 - k_z^2\right)\right\}J_{m-li}f_0 d\bar{p} \quad \text{(B-23)}$$

$$J_{\theta 1p} = 2\pi e^2\sum_{i,l}\int\frac{A_{il}\mu_0 h_i k_g^2}{D}\left\{\frac{1}{\Omega_l}\left(\omega - k_z v_z\right)\left(J_{li} + p_i r_c J_{li}^{\cdot}\right) - \frac{\beta_\perp^2 J_{li}}{\Omega_l^2}\left(\omega^2 - k_z^2 c^2\right)\right\}J_{m-li}f_0 d\bar{p} \quad \text{(B-24)}$$

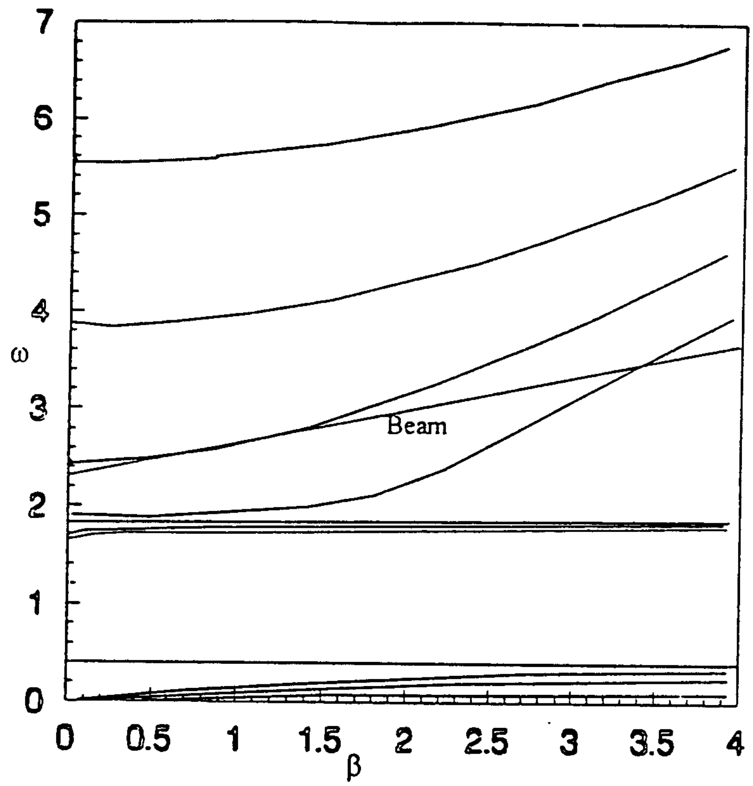

Figure 1a. Interaction Plots for the beam with the waveguide mode *HE01*. The following parameters were used:

$V_0 = 300KV \quad \bar{\omega}_c = 3.79 \quad \bar{R}_0 = 0.6$

$V_\perp / V_{//} = 1.5 \quad \bar{\omega}_p = 0.41 \quad \bar{\omega}_b = 0.2$

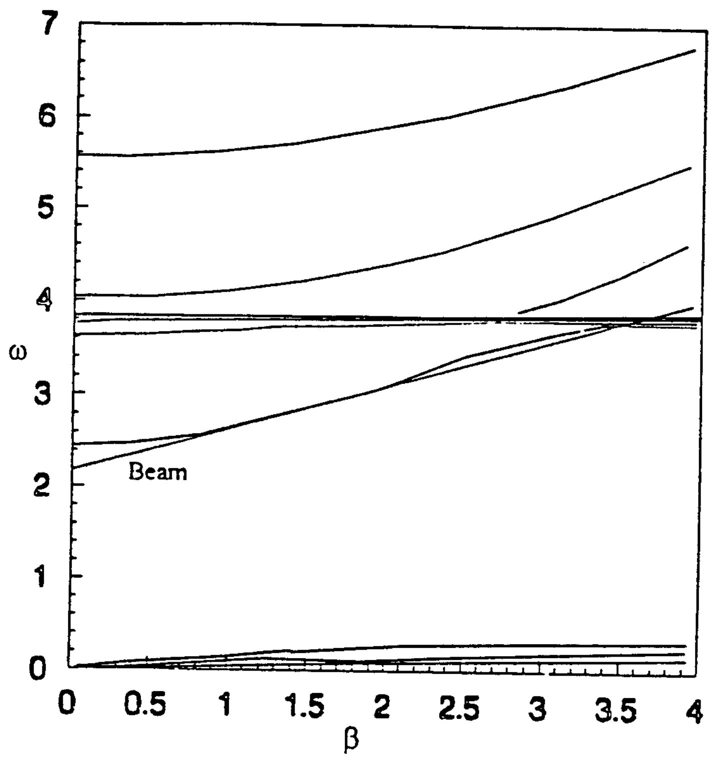

Figure 1b. Interaction Plot for the beam with the Cyclotron mode. The following parameters were used:

$V_0 = 300\,KV$ $\quad \bar{\omega}_c = 3.79$ $\quad \bar{R}_0 = 0.6$

$V_\perp / V_\parallel = 1.5$ $\quad \bar{\omega}_p = 0.41$ $\quad \bar{\omega}_b = 0.2$

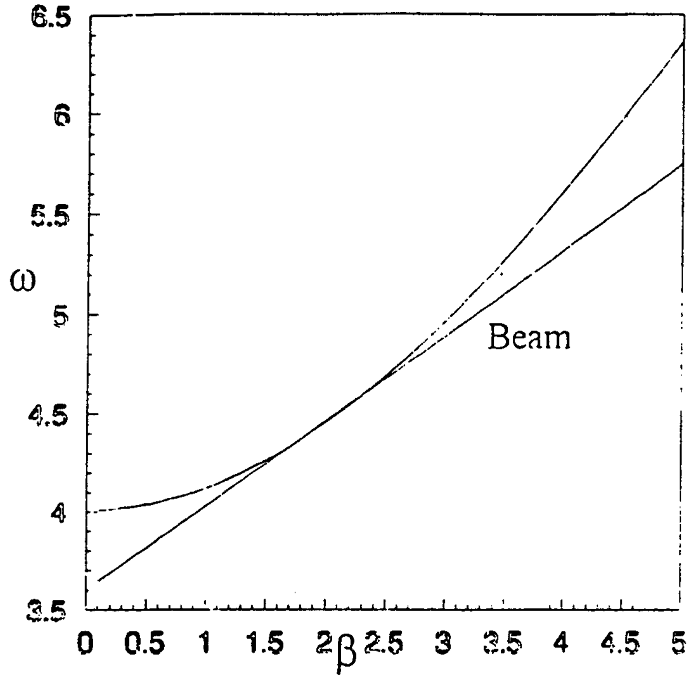

Figure 2. Interaction of the beam with the waveguide mode for the following sample parameters:

$V_0 = 300KV \quad \omega_c = 1.80 \quad R'_0 = 0.6$

$v_\perp / v_{//} = 1.5 \quad \bar{\omega}_p = 0.81 \quad \omega_b = 0.2$

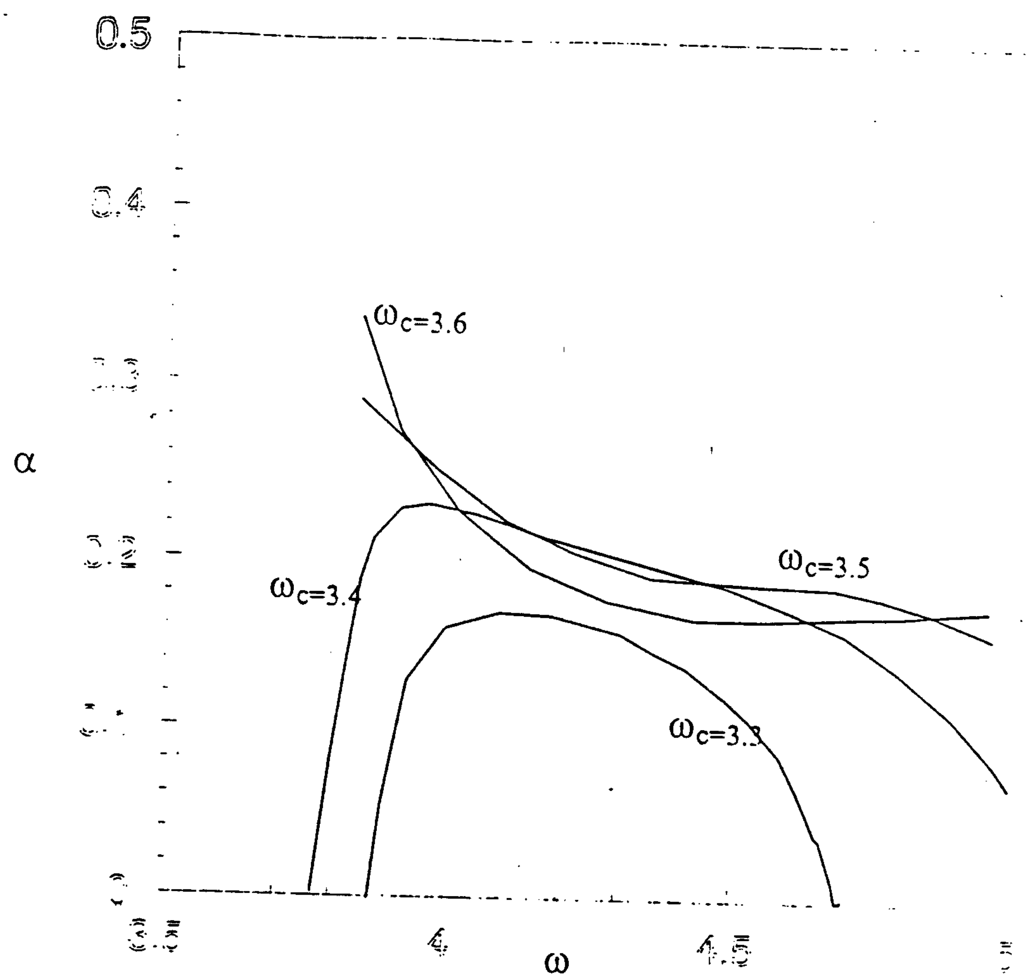

Figure 3. Growth rate versus ω_c for the cyclotron mode for the following sample parameters:

$V_0 = 300\,KV$ $\bar{\omega}_p = 0.41$ $\bar{R}_0 = 0.6$

$V_\perp / V_{//} = 1.5$ $\bar{\omega}_b = 0.2$

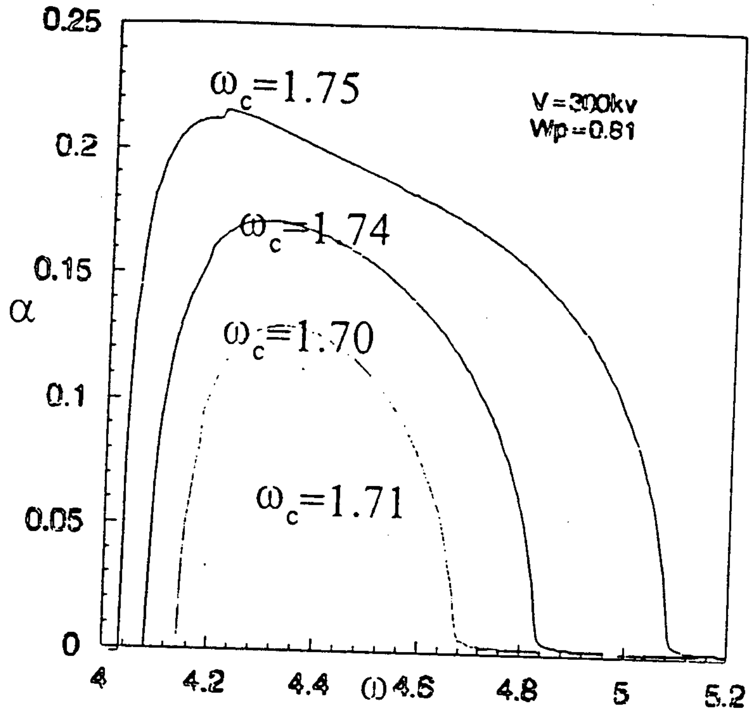

Figure 4. Growth rate versus ω_c of the waveguide mode for the following sample parameters:

$V_0 = 300 KV$ $\quad \bar{\omega}_p = 0.81$ $\quad \bar{R}_0 = 0.6$

$V_\perp / V_{//} = 1.5$ $\quad \bar{\omega}_b = 0.2$

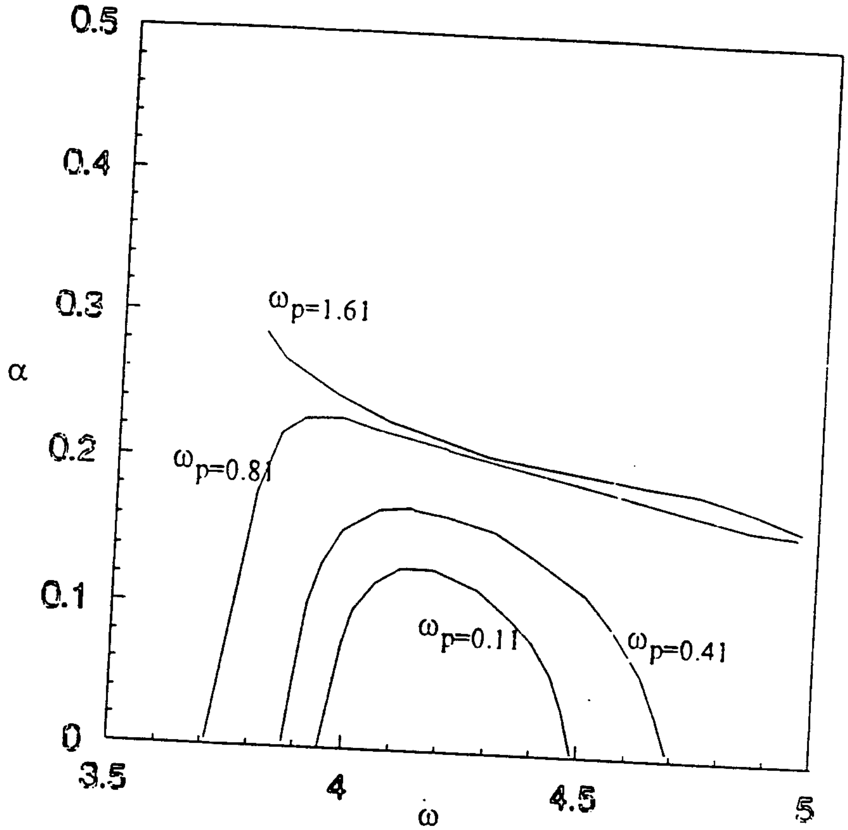

Figure 5. Growth rate versus ω_p of the cyclotron mode for the following sample parameters:

$V_0 = 300KV$ $\quad \bar{\omega}_c = 3.79$ $\quad R_0 = 0.6$

$V_\perp / V_{//} = 1.5$ $\quad \bar{\omega}_b = 0.2$

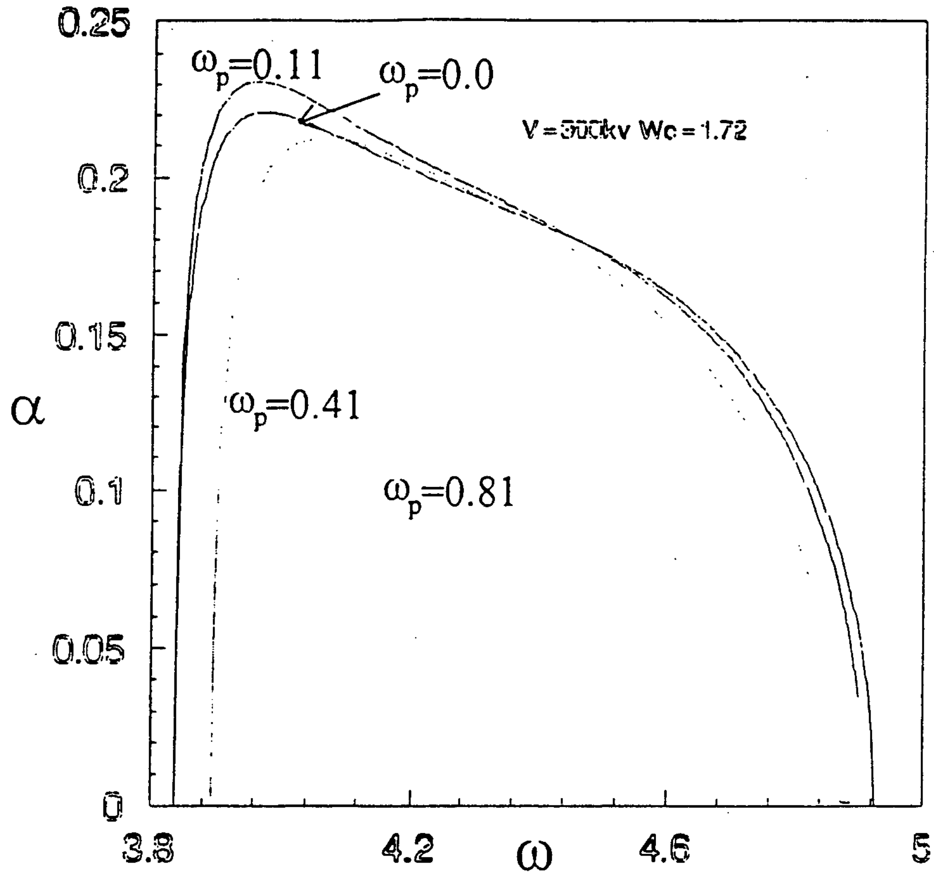

Figure 6. Growth rate versus ω_p of the waveguide mode for the following sample parameters:

$V_0 = 300KV$ $\quad \bar{\omega}_c = 1.80$ $\quad \bar{R}_0 = 0.6$

$V_\perp / V_{//} = 1.5$ $\quad \bar{\omega}_b = 0.2$

3. Shenggang Liu, Robert J. Barker, Zhu Dajun, Yan Yang, and Gao Hong, "Basic Theoretical Formulations of Microwave Plasma Electronics".

APFA'98 and APPTC'98

5

Basic Theoretical Formulations of Microwave Plasma Electronics

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(Invited talk)

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Abstract

Basic theoretical formulations for electron beam-wave interactions in a plasma-filled waveguide immersed in a finite magnetic field are presented in this two-part paper.

1. The general interaction and dispersion equations of the longitudinal and transverse interactions in both smooth and corrugated magnetized plasma-filled waveguides are formulated. The resultant equations are then applied to examine the specifics of plasma Cherenkov radiation, the plasma-filled travelling-wave-tube/backward-wave-oscillator (TWT/BWO), the plasma-filled electron cyclotron resonance maser(ECRM) and many types of beam-wave interactions including those involving ion-channels.

2. Some possible new interactions in a magnetized plasma-filled waveguide that do not appear in previously published papers are proposed.

3. A detailed discussion and analysis of the physics of the important role of the plasma background are given in the paper.

4. It is pointed out that in a magnetized plasma-filled waveguide, there are a lot of interesting features of beam-wave interactions, two of them are most essential. One is that the transverse interactions are always accompanied by the longitudinal interactions. The other is that the magnetized plasma itself is strongly involved in the interaction mechanisms via an additional component of the field.

This paper consists of two parts. Part I presents general theoretical formulations of electron beam-wave interactions in a magnetized plasma waveguide using only a fluid model for both the plasma and beam. Part II extends the analyses of these interaction by retaining a fluid treatment for the plasma-fill but substituting a kinetic theory treatment for the electron beam. It continues further to include a detailed treatment of the physical effects of the ion channel that is formed in the plasma by an intense electron beam. In both parts of the paper, sample numerical calculations are presented in both parts in order to illustrate the physics.

Part I: Fluid Model of Electron Beam-Wave Interactions in a Magnetized Plasma-Filled Waveguide

I. Introduction

The goal of Microwave Electronics: **To find ways to create improved microwave devices which can provide higher output power with higher efficiency.**

Some fundamental limits: One of the most important limits is the space charge effect. Busian Instability.

The most promising approach: Plasma fill. Pierce instability

New results given in this paper:

(1). Magnetized plasma filling strongly changes the behaviour of wave propagation in waveguide, TE and TM modes can no longer exist independently, instead, there are EH and HE hybrid modes. Also, there are two eigen-values and two corresponding eigenfunctions instead of only the one that exists for the vacuum case or for the cases where magnetic field $B_0=0$ and $B_0 \to \infty$), so the field components are:

$$E_z = A_1 J_m(p_1 R) + A_2 J_m(p_2 R) \tag{1}$$

$$H_z = A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R) \tag{2}$$

Therefore, the field pattern, the cut-off frequency (thus the wave number) and the dispersion relations are all changed. It is clear, that we could not use formulations based on the TE mode or TM mode alone, if we want to know the contribution of magnetized plasma fill.

(2). The magnetized plasma background not only strongly changes the behaviour of wave propagation but also strongly changes the character of the beam-wave interactions. Since the TE and TM modes are always coupled, E_z and $\vec{E}_\perp$ always exist simultaneously. If the electron beam has both longitudinal and transverse components of motion, it is inherent that in a magnetized plasma filled waveguide, the longitudinal interaction (J_z~E_z) and the transverse interaction ($\vec{J}_\perp - \vec{E}_\perp$) are always present together. This implies that Cherenkov type and TWT/BWO type interactions are always accompanied by ECRM and/or gyro-peniotron type interactions.

(3). The magnetized plasma background itself is deeply involved in the beam-wave interactions. Moreover since there are varieties of waves that can propagate along a magnetized plasma waveguide, it is obvious that there must exist some kind of coupling among waves through the electron beam. This coupling may lead to some instabilities. For example, parametric coupling may result in instabilities. The study of this kind of instability has been presented in [30], however, it was again based on the TM mode without taking into consideration the fact that the TM and TE modes are always coupled together.

(4). The dispersion characteristics of wave propagation in circular cylindrical magnetized plasma waveguide is shown in Fig.1. The Fig.1 shows that there are at least three kinds of waves that can propagate through the magnetized plasma waveguide: plasma waves; cyclotron waves; waveguide waves. All these waves are electromagnetic waves, all of them can interact with electron beam.

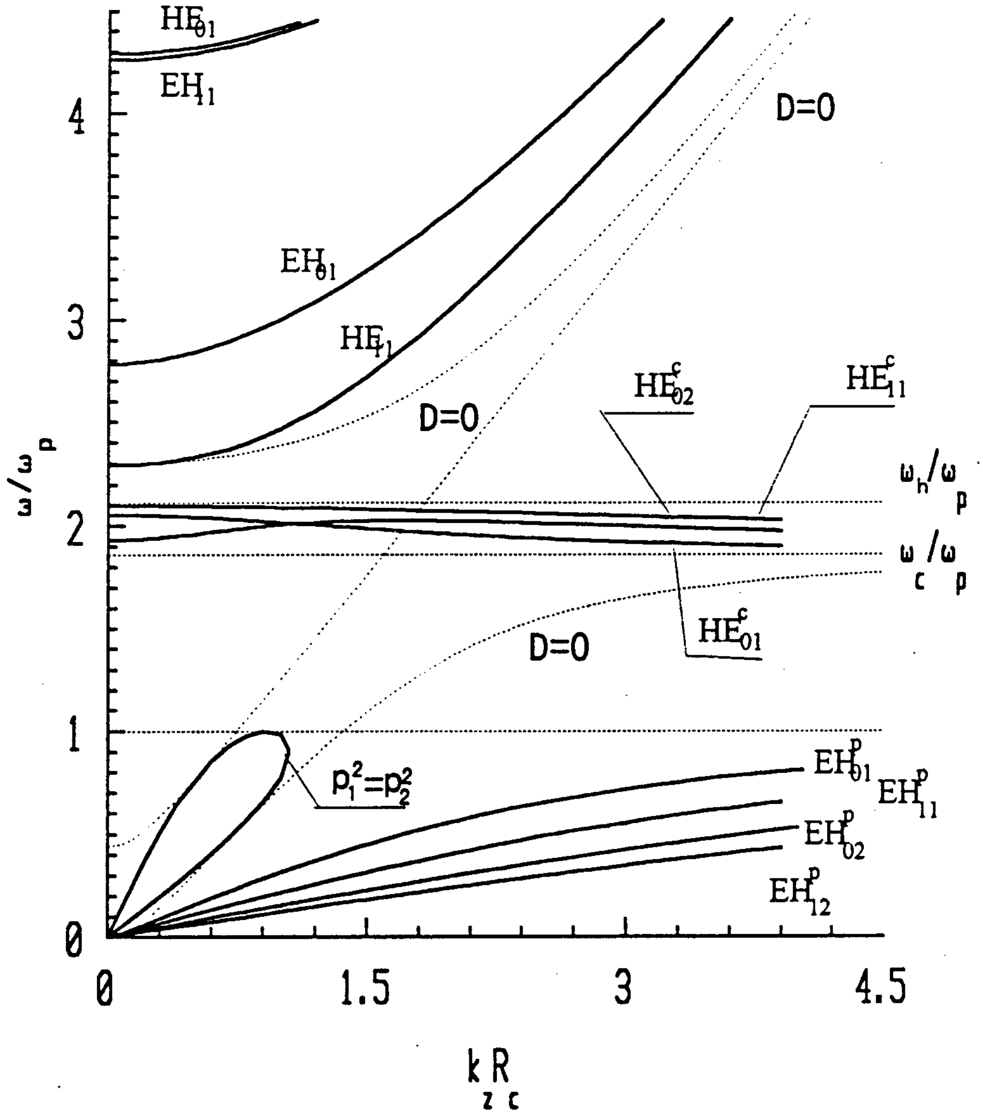


Fig. 1 The typical dispersion curves with B_0=0.175T (ω_c/ω_p=1.86),
$\omega_p=1.85\times10^{10}s^{-1}$, R=1.5cm.

II. Field Equations in Magnetized Plasma Waveguide

Permittivity tensor:

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_1 & j\varepsilon_2 & 0 \\ -j\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \tag{6}$$

The field components:

$$\left.\begin{aligned} E_z &= \sum_i A_i J_{mi} \\ H_z &= \sum_i A_i h_i J_{mi} \end{aligned}\right\} \quad (i=1,2) \tag{10}$$

$$\left.\begin{aligned} E_R &= E_{R1} + E_{R2} + E_{R1p} + E_{R2p} \\ E_\varphi &= E_{\varphi 1} + E_{\varphi 2} + E_{\varphi 1p} + E_{\varphi 2p} \\ H_R &= H_{R1} + H_{R2} + H_{R1p} + H_{R2p} \\ H_\varphi &= H_{\varphi 1} + H_{\varphi 2} + H_{\varphi 1p} + H_{\varphi 2p} \end{aligned}\right\} \tag{11}$$

The subscripts 1 and 2 denote the TE-like and TM-like part, respectively, and 1p and 2p denote the plasma produced TE-like and TM-like part, respectively.

For the corrugated waveguide:

$$\left.\begin{aligned} E_z &= \sum_s E_{z,s} e^{j\{\omega t - k_{z,s} z - m\varphi\}} \\ H_z &= \sum_s H_{z,s} e^{j\{\omega t - k_{z,s} z - m\varphi\}} \end{aligned}\right\} \tag{16}$$

So for the E_R, E_φ, B_R and B_φ. Where:

$$k_{z,s} = k_z + \frac{2\pi s}{L_p} \qquad (s = 0,\ \pm 1,\ \pm 2,\ \ldots) \tag{17}$$

All the plasma produced field parts are proportional to $k_g^2 = k^2 \varepsilon_2$. So, when the plasma is absent, or the magnetic field $B_0 \to 0$ or $B_0 \to \infty$, we have $\varepsilon_2 = 0$, these field parts are vanished. And for the vacuum case, $\varepsilon_2 = 0$, $\varepsilon_1 = \varepsilon_3 = 1$, we have TE and TM modes independently, and the TE-like and TM-like fields automatically become the fields of TE and TM modes, respectively.

III. General Equations Governing Electron Beam-wave Interactions in Magnetized Plasma Waveguide

Maxwell's equations can be written as:

$$\left.\begin{aligned} &\nabla \times \vec{E} = -j\omega\mu_0 \vec{H} \\ &\nabla \times \vec{H} = j\omega\varepsilon_0 \vec{D} + \vec{J} \\ &\nabla \cdot \vec{D} = \rho \\ &\vec{D} = \varepsilon_0 \bar{\varepsilon} \cdot \vec{E} \\ &\nabla \cdot \vec{B} = 0 \end{aligned}\right\} \tag{21}$$

Then from eq. (21), we can obtain the following two sets of equations:

A. Interaction equations expressed in terms of longitudinal fields:

From eq. (21), we get:

$$\nabla_\perp^2 E_z + aE_z = bH_z + j\omega\mu_0 J_z - \frac{jk_z}{\varepsilon_0\varepsilon_1}\rho \tag{22}$$

$$\nabla_\perp^2 H_z + cH_z = dE_z - \left(\nabla \times \vec{J}\right)_z - \frac{\omega\varepsilon_2}{\varepsilon_1}\rho \tag{23}$$

Where:

$$\left.\begin{aligned} &a = \left(-k_z^2 + k^2\varepsilon_1\right)\varepsilon_3 / \varepsilon_1 \\ &b = jk_z\omega\mu_0\varepsilon_2 / \varepsilon_1 \\ &c = -k_z^2 + k^2\left(\varepsilon_1^2 - \varepsilon_2^2\right)/\varepsilon_1 \\ &d = -jk_z\omega\varepsilon_0\varepsilon_2\varepsilon_3 / \varepsilon_1 \end{aligned}\right\} \tag{24}$$

The transverse field components may be found from E_z and H_z:

$$\begin{aligned} \vec{E}_\perp = \frac{1}{D}\Big[&-jk_z K^2 \nabla_\perp E_z + \omega\mu_0 k_g^2 \nabla_\perp H_z - k_z k_g^2 \nabla_\perp E_z \times \vec{e}_z \\ &-j\omega\mu_0 K^2 \nabla_\perp H_z \times \vec{e}_z + j\omega\mu_0 K^2 \vec{J}_\perp + \omega\mu_0 k_g^2 \vec{J}_\perp \times \vec{e}_z\Big] \end{aligned} \tag{25}$$

$$\begin{aligned} \vec{H}_\perp = \frac{1}{D}\Big[&-\omega\varepsilon_0\varepsilon_2 k_z^2 \nabla_\perp E_z - jk_z K^2 \nabla_\perp H_z - j\omega\varepsilon_0\left(k_g^2\varepsilon_2 - K^2\varepsilon_1\right)\nabla_\perp E_z \times \vec{e}_z \\ &-k_z k_g^2 \nabla_\perp H_z \times \vec{e}_z + k_z k_g^2 \vec{J}_\perp - jk_z K^2 \vec{J}_\perp \times \vec{e}_z\Big] \end{aligned} \tag{26}$$

Where:

$$K^2 = k^2\varepsilon_1 - k_z^2; \qquad k_g^2 = k^2\varepsilon_2; \qquad D = K^4 - k_g^4; \qquad k^2 = \omega^2\varepsilon_0\mu_0 \tag{27}$$

Eq. (22) can be used to study longitudinal interactions, plasma Cherenkov radiation devices for example, while eq. (23) can be used to study transverse interaction, electron cyclotron maser with plasma fill.

B. Interaction equations expressed in terms of transverse fields:

The beam-wave interaction equations can also be expressed in terms of the transverse field components:

$$\nabla_\perp^2 \vec{E}_\perp + \frac{k^2\left(\varepsilon_1^2 - \varepsilon_2^2\right) - k_z^2 \varepsilon_3}{\varepsilon_1}\vec{E}_\perp = \frac{jk_z\omega\mu_0\varepsilon_2}{\varepsilon_1}\vec{H}_\perp + k_z\omega\mu_0\frac{\left(\varepsilon_3 - \varepsilon_1\right)}{\varepsilon_1}\vec{e}_z \times \vec{H}_\perp$$
$$+ j\omega\mu_0\vec{J}_\perp - \frac{\omega\mu_0\varepsilon_2}{\varepsilon_1}\vec{e}_z \times \vec{J}_\perp + \frac{\nabla_\perp \rho}{\varepsilon_0\varepsilon_1} \tag{36}$$

and:

$$\nabla_\perp^2 \vec{H}_\perp + \left(k^2\varepsilon_3 - k_z^2\right)\vec{H}_\perp = -jk_z\omega\varepsilon_0\varepsilon_2\vec{E}_\perp + k_z\omega\varepsilon_0\left(\varepsilon_3 - \varepsilon_1\right)\vec{e}_z \times \vec{E}_\perp - \left(\nabla \times \vec{J}\right)_\perp \tag{37}$$

Then E_z and H_z can be written as:

$$E_z = -\frac{j}{\omega\varepsilon_0\varepsilon_3}\left[\left(\nabla \times \vec{H}_\perp\right)\cdot\vec{e}_z - J_z\right] \tag{38}$$

$$H_z = \frac{j}{\omega\mu_0}\left(\nabla \times \vec{E}_\perp\right)\cdot\vec{e}_z \tag{39}$$

Eq. (40) has been used for the kinetic theory of ECRM with space charge taken into consideration. Equations (38) and (39) remain unchanged.

The above obtained general interaction equations may cover almost all kinds of interactions in a smooth waveguide with and/or without plasma fill. Which interaction equation is preferred to be used depends on the specific case and the convenience of the author.

It should be pointed out at this point that, because the TE and TM modes are always coupled to each other, the longitudinal and the transverse interactions are also both present if the transverse electron motion is existent.

IV. Beam-wave interactions in A corrugated magnetized plasma waveguide

Wave propagation along a corrugated magnetized plasma waveguide has been studied[40],[41]. According to Floquet's theorem, the fields should be expanded:

$$\left.\begin{aligned} \vec{E} &= \sum_s \vec{E}_s e^{-jk_{z,s}z} \\ \vec{H} &= \sum_s \vec{H}_s e^{-jk_{z,s}z} \end{aligned}\right\} \tag{46}$$

Where:

$$k_{z,s} = k_{z0} + \frac{2\pi s}{L} \tag{47}$$

L is the spatial period, and s is the spatial harmonic number.

We can also find the interaction equations expressed in terms of longitudinal as well as transverse field components.

A. Longitudinal fields expressions:

$$\sum_s \left(\nabla_\perp^2 E_{z,s} + aE_{z,s} \right) = \sum_s \left(b_s H_{z,s} + j\omega\mu_0 J_{z,s} - \frac{jk_{z,s}}{\varepsilon_0 \varepsilon_1} \rho_s \right) \tag{48}$$

$$\sum_s \left(\nabla_\perp^2 H_{z,s} + c_s H_{z,s} \right) = \sum_s \left(d_s E_{z,s} - \left(\nabla \times \bar{J}_s \right)_z - \frac{\omega \varepsilon_2}{\varepsilon_1} \rho_s \right) \tag{49}$$

Where:

$$\left.\begin{aligned} a_s &= \left(-k_{z,s}^2 + k^2 \varepsilon_1 \right) \varepsilon_3 / \varepsilon_1 \\ b_s &= jk_{z,s} \omega \mu_0 \varepsilon_2 / \varepsilon_1 \\ c_s &= -k_{z,s}^2 + k^2 \left(\varepsilon_1^2 - \varepsilon_2^2 \right) / \varepsilon_1 \\ d_s &= -jk_{z,s} \omega \varepsilon_0 \varepsilon_2 \varepsilon_3 / \varepsilon_1 \end{aligned} \right\} \tag{50}$$

The transverse field components $\bar{E}_{\perp,s}$ and $\bar{H}_{\perp,s}$ may be found from eq. (25) and eq. (26) only need to replace k_z, K^2, D and $J_\perp$ by $k_{z,s}$, K_s^2, D_s and $J_{\perp,s}$, respectively.
Where:

$$K_{z,s}^2 = k^2 \varepsilon_1 - k_{z,s}^2; \qquad D_{z,s} = K_{z,s}^4 - k_g^4 \tag{51}$$

Eq. (48) can be used in plasma filled devices like the TWT and BWO, and eq. (49) can be used for a plasma filled ECRM in periodic system.

B. Transverse fields expressions:

$$\sum_s \left[\nabla_\perp^2 \bar{E}_{\perp,s} + \frac{k^2 \left(\varepsilon_1^2 - \varepsilon_2^2 \right) - k_{z,s}^2 \varepsilon_3}{\varepsilon_1} \bar{E}_{\perp,s} \right] = \sum_s \left[\frac{jk_z \omega \mu_0 \varepsilon_2}{\varepsilon_1} \bar{H}_{\perp,s} + j\omega\mu_0 \bar{J}_{\perp,s} + \right.$$

$$\left. k_{z,s} \omega \mu_0 \frac{\left(\varepsilon_3 - \varepsilon_1 \right)}{\varepsilon_1} \bar{e}_z \times \bar{H}_{\perp,s} - \frac{\omega \mu_0 \varepsilon_2}{\varepsilon_1} \bar{e}_z \times \bar{J}_{\perp,s} + \frac{\nabla_\perp \rho_s}{\varepsilon_0 \varepsilon_1} \right] \tag{58}$$

and:

$$\sum_s \left[\nabla_\perp^2 \bar{H}_{\perp,s} + \left(k^2 \varepsilon_3 - k_{z,s}^2 \right) \bar{H}_{\perp,s} \right] =$$

$$\sum_s \left[-jk_{z,s} \omega \varepsilon_0 \varepsilon_2 \bar{E}_{\perp,s} + k_z \omega \varepsilon_0 \left(\varepsilon_3 - \varepsilon_1 \right) \bar{e}_z \times \bar{E}_{\perp,s} - \left(\nabla \times \bar{J}_s \right)_\perp \right] \tag{59}$$

The longitudinal field components $E_{z,s}$ and $H_{z,s}$ can be found by the same replacement mentioned before.

The interaction equations obtained above can be used for linear and non-linear beam-wave interactions in general cases including both vacuum and plasma fill. It should be also pointed out that the longitudinal and transverse interactions are both present also in a periodic structure provided that transverse electron motion exists.

V. Dispersion equations of electron beam-wave interactions in magnetized plasma waveguide by means of fluid theory

Based on the interaction equations given in the last section, the dispersion equations of different kinds of beam-wave interactions in a magnetized plasma waveguide can be obtained.

A. Dispersion equations of longitudinal interactions:

From eq. (22), we can get the following dispersion equations:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = -j\omega\mu_0\varepsilon_1\iint J_z E_{z0}^* ds + \frac{jk_z}{\varepsilon_0}\iint \rho E_{z0}^* ds \tag{66}$$

Plasma Cherenkov radiation devices are typical longitudinal interactions. From eq. (22), making use of the continuity equation and by means of a fluid model for the electron beam, we obtain

$$J_z = j\frac{\omega\varepsilon_0\omega_b^2}{\left(\omega - k_z v_0\right)^2}E_z \tag{68}$$

We can write the dispersion equations (66) as:

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = \frac{\omega_b^2\left(\varepsilon_1 k^2 - k_z^2\right)}{\left(\omega - k_z v_0\right)^2}P_E \tag{69}$$

Where:

$$\left.\begin{aligned} P_E &= \iint E_z \cdot E_{z0}^* ds \\ P_{HE} &= \iint H_z \cdot E_{z0}^* ds \end{aligned}\right\} \tag{70}$$

The solutions of eq.(69) can be obtained easily:

$$k_z = \frac{1}{2\left[\varepsilon_3 + \dfrac{\omega_b^2}{\left(\omega - k_z v_0\right)^2}\right]P_E}\left\{-j\omega\mu_0\varepsilon_2 P_{HE} \pm \left\{-\omega^2\mu_0^2\varepsilon_2^2 P_{HE}^2 + 4\left[\varepsilon_3 + \frac{\omega_b^2}{\left(\omega - k_z v_0\right)^2}\right]P_E\left[k_{z0}^2\varepsilon_3 P_E + j\omega\mu_0\varepsilon_2 P_{HE}k_{z0} + \frac{\omega_b^2\varepsilon_1 k^2}{\left(\omega - k_z v_0\right)^2}P_E\right]\right\}^{1/2}\right\} \tag{72}$$

From eq. (72), we can get the instability criteria for plasma Cherenkov radiation:

$$4\left[\varepsilon_3 + \frac{\omega_b^2}{\left(\omega - k_z v_0\right)^2}\right]P_E^2\left[k_{z0}^2\varepsilon_3 + j\omega\mu_0\varepsilon_2\frac{P_{HE}}{P_E}k_{z0} + \frac{\omega_b^2\varepsilon_1 k^2}{\left(\omega - k_z v_0\right)^2}\right] < \omega^2\mu_0^2\varepsilon_2^2 P_{HE}^2 \tag{73}$$

B. The dispersion equations for TWT/BWO

In a TWT or BWO, a corrugated waveguide is often used. From eq. (48), we obtain:

$$\sum_s \left[\varepsilon_3 \left(k_{z,s}^2 - k_{z0,s}^2 \right) P_{E,s} + j\omega\mu_0 \varepsilon_2 \left(k_{z,s} - k_{z0,s} \right) P_{HE,s} \right] =$$

$$\sum_s \left[-j\omega\mu_0 \varepsilon_1 \iint J_z E_{z0}^* ds + \frac{jk_z}{\varepsilon_0} \iint \rho E_{z0}^* ds \right] \tag{77}$$

and eq. (77) can be momodified as

$$\sum_s \left[\varepsilon_3 \left(k_{z,s}^2 - k_{z0,s}^2 \right) P_{E,s} + j\omega\mu_0 \varepsilon_2 \left(k_{z,s} - k_{z0,s} \right) P_{HE,s} \right] = \sum_s \left[\frac{\omega_b^2 \left(\varepsilon_1 k^2 - k_{z,s}^2 \right)}{\left(\omega - k_{z,s} v_0 \right)^2} P_{E,s} \right] \tag{78}$$

C. The dispersion equations for transverse interaction:

A typical and very important transverse interaction is that found in the ECRM, now we deal with this interaction with a plasma fill. From eq. (36), the following dispersion equations can be obtained:

$$\varepsilon_3 \left(k_z^2 - k_{z0}^2 \right) P_{E_\perp} + j\omega\mu_0 \varepsilon_2 (k_z - k_{z0}) P_{HE_\perp} + \omega\mu_0 (\varepsilon_3 - \varepsilon_1)(k_z - k_{z0}) P_{EH_\perp} =$$

$$-j\omega\mu_0 \varepsilon_1 \iint \bar{J}_\perp \cdot \bar{E}_{\perp 0}^* ds - \frac{1}{\varepsilon_0} \iint (\nabla_\perp \rho)_\perp \cdot \bar{E}_{\perp 0}^* ds \tag{82}$$

In corrugated waveguide, eq. (82) should be modified as:

$$\sum_s \left[\varepsilon_3 \left(k_{z,s}^2 - k_{z0}^2 \right) P_{E_\perp,s} + j\omega\mu_0 \varepsilon_2 \left(k_{z,s} - k_{z0} \right) P_{HE_\perp,s} + \omega\mu_0 (\varepsilon_3 - \varepsilon_1) \left(k_{z,s} - k_{z0} \right) P_{EH_\perp,s} \right]$$

$$= -\sum_s \left[j\omega\mu_0 \varepsilon_1 \iint \bar{J}_{\perp,s} \cdot \bar{E}_{\perp 0,s}^* ds + \frac{1}{\varepsilon_0} \iint (\nabla_\perp \rho)_\perp \cdot \bar{E}_{\perp 0,s}^* ds \right] \tag{85}$$

Now some important points should be mentioned. These are:

1. According to eq. (9)-(14), each field component is split into four parts, and correspondingly, J_z and $\bar{J}_\perp$ are also divided into four parts.

2. Therefore, each of the intergration P_E, $P_{E\perp}$, P_{HE}, $P_{HE\perp}$, $\iint J_z \cdot E_{z0}^* ds$ and $\iint \bar{J}_\varphi \cdot \bar{E}_{\varphi 0}^* ds$ has 16 terms. It makes the interaction and dispersion equations very complicated.

3. Because of the coupling of TE and TM modes, the E_z and $E_\perp$ are always exist simultaneously. Therefore, we always have transverse and longitudinal interactions accompanied. This is the most inportant feature of the beam-wave interactions in magnetized plasma waveguide.

VI. Some new interactions that may occur in a magnetized plasma waveguide

A. A possible new interaction

There exists a special kind of wave family that can propagate in a magnetized plasma waveguide in the frequency region: ($\omega_p < \omega < \omega_c$ or $\omega_c < \omega < \omega_p$), called the cyclotron modes. In particular, some of these waves are inherently backward waves (negative dispersion). So we may possibly have discovered a new interaction mechanism with the cyclotron waves. Also with the backward wave, we can construct a new type of BWO without a periodic structure.

The formulations of beam-wave interactions with cyclotron waves are the same as those have been given in the above sections.

In principle, the slow cyclotron waves may also be used as pumping wave for parametric excitation.

B. A new hybrid ion-channel maser

$$\omega_0 = \frac{\omega_{c0}}{2\gamma_0}\left[1+\left(1+\frac{4\gamma_0\overline{\omega}_p^2}{\omega_{c0}^2}\right)^{1/2}\right]$$

C. Parametric coupling excitation

There are varieties of propagating waves in a magnetized plasma waveguide. It is natural that, when the driven electron beam is present, there must be some coupling between/among waves, and the coupling may lead to some instabilities. In [14], for example, a parametric coupling excitation was presented. It is suggested that the TG mode parametrically couples with a TM mode to excite a negative energy beam mode. The beam mode feeds energy into the positive energy TG and TM modes giving rise to an explosive instability.

In the magnetized plasma waveguide, actually, the mechanism is modified from that given in [14]. Here the TG mode should parametrically couple with either the EH_{mn} or HE_{mn} mode, or even with one of the cyclotron modes, and then excites the electron beam. Now there are two transverse wave numbers p_1 and p_2, so the excitation condition shoud be modified also.

Considering the varieties of waves that can propagate in a magnetized plasma waveguide, the formulations become much more complicated, some results will be given in another paper by the authors.

VIII. Discussion and analysis

1. Physically, the cyclotron motion of the background plasma electron plays a very important role. Because of it, we have $\varepsilon_2 = -\frac{\xi^2 \tau}{1-\tau^2} \neq 0$ (see eq. (5)), and it causes first, the coupling between TE and TM modes because of $b \neq 0$ and $d \neq 0$. Secondly it produces the additional parts of the wave field components associated with $k_g^2 = k^2 \varepsilon_2$. These parts of the wave field are directly involved in the beam-wave interactions.

2. The field patterns in a magnetized plasma waveguide become much more complicated than those in a vacuum waveguide. Due to the magnetized plasma fill, the field structure is completely changed. There are two eigenvalues and two corresponding eigenfunctions, the filling plasma produces the additional parts of the wave. Therefore, all field components for example, E_φ may be divided into four parts: $E_\varphi = E_{\varphi 1} + E_{\varphi 2} + E_{\varphi 1p} + E_{\varphi 2p}$. Where $E_{\varphi 1}$ is produced by $H_z = A_1 h_1 J_m(p_1 R) + A_2 h_2 J_m(p_2 R)$ and is called the TE-like part. $E_{\varphi 2}$ is produced by $E_z = A_1 J_m(p_1 R) + A_2 J_m(p_2 R)$ and is called the TM-like part. $E_{\varphi 1p}$ and $E_{\varphi 2p}$ are due to plasma background in addition to the TE-like part and TE-like part, respectively. All these parts are involved in the beam-wave interactions.

This field complexity certainly will strongly influence the beam-wave interactions. Correspondingly, we have the RF current density: $J_\varphi = J_{\varphi 1} + J_{\varphi 2} + J_{\varphi 1p} + J_{\varphi 2p}$, and $J_z = J_{z1} + J_{z2} + J_{z1p} + J_{z2p}$ (See part II of the paper).

The beam-wave interactions mainly depend on the term $\iint \bar{J} \cdot \bar{E}^* ds$. Therefore, the interactions are much more complicated as: $\iint \bar{J}_\perp \cdot \bar{E}_\perp^* ds = \iint J_\varphi \cdot E_\varphi^* ds = \iint \big(J_{\varphi 1} E_{\varphi 1}^* + J_{\varphi 1} E_{\varphi 2}^* + J_{\varphi 1} E_{\varphi 1p}^* + J_{\varphi 1} E_{\varphi 2p}^* + J_{\varphi 1p} E_{\varphi 1}^* + J_{\varphi 1p} E_{\varphi 2}^* + J_{\varphi 1p} E_{\varphi 1p}^* + J_{\varphi 1p} E_{\varphi 2p}^* + J_{\varphi 2} E_{\varphi 1}^* + J_{\varphi 2} E_{\varphi 2}^* + J_{\varphi 2} E_{\varphi 1p}^* + J_{\varphi 2} E_{\varphi 2p}^* + J_{\varphi 2p} E_{\varphi 1}^* + J_{\varphi 2p} E_{\varphi 2}^* + J_{\varphi 2p} E_{\varphi 1p}^* + J_{\varphi 2p} E_{\varphi 2p}^* \big) ds$. Likewise for $\iint J_z \cdot E_z^* ds$. Since the $\iint \bar{J}_\perp \cdot \bar{E}_\perp^* ds$ represents the transverse beam-wave interaction, and since $\iint J_z \cdot E_z^* ds$ represents the longitudinal beam-wave interaction, since both $\bar{E}_\perp$ and E_z are always existent, we can see clearly that the transverse interaction is always accompanied by the longitudinal interaction. So the beam-wave interactions in a magnetized plasma waveguide are much more complicated and richer than that in the vacuum case.

3. The instability of longitudinal interactions and that of transverse interactions may or may not occur in the same frequency. The instability will be enhanced in the case where they occur at same frequency. In the case they occur in a different frequency, a spurious spectrum will occur.

IX. Conclusion

The basic theory of electron beam-wave interactions in a waveguide filled with plasma immersed in a finite magnetic field has been presented in this paper. The interaction equations and dispersion reations for both longitudinal and transverse interactions in magnetized plasma have been formulated. These equations cover almost all kinds of beam-wave interactions. They can also be used for parametric excitations that may exist. The interaction equations can be used for both linear and non-linear waves, and the dispersion relations can only be used for the linear case. The theory given in this paper is valid as long ain property of the plasma background is not distorted, that is as ong as the background plasma can be described by the permittivity tensor as in equations (6)-(8). From the formulations given in this paper, the following results can be obtained:

1. The importance of the background plasma is: (1). The electron gyrating motion of the background plasma couples the TE modes and TM modes. They are always coupled to each other. (2). This coupling generates the hybrid HE mode and EH mode. Besides, because of the magnetized plasma, there are varieties of modes propagating along the waveguide. (3). The background plasma itself is involved in the electron beam-wave interactions by producing the additional parts of the wave that depend on the gyrating motion. Thus, the magnetized background plasma makes the electron beam-wave interactions much more complicated and rich.

2. Since the TE and TM modes are always coupled, in plasma-filled microwave devices, therefore, there are no pure transverse interactions. Likewise there is no pure longitudinal interaction, if there is any, even small, transverse electron motion, there must be some transverse interaction. It is inherent in a magnetized plasma-filled waveguide that the transverse and longitudinal interactions are couped.

3. Since there are varieties of waves in a magnetized plasma waveguide, when the electron beam is present, coupling between/among waves may happen. The low frequency plasma modes (TG modes or even cyclotron waves) may serve as the pumping wave, and parametric excitation may be obtained.

4. There is a special kind of wave family in the frequency range: ($\omega_p < \omega < \omega_c$ or $\omega_c < \omega < \omega_p$), called cyclotron modes. The waves in this family are all electromagnetic waves. They can interact directly with the electron beam, since their phase velocities may be less than the speed of light. In particular, some of the cyclotron waves are inherently negative; they are backward-wave naturally. The inherent backward-wave even may be used for building backward-wave oscillators without periodic structures.

5. The instabilities caused by longitudinal and transverse interactions may lead to two cases: (1). If two instability mechanisms occur at the same frequency or in the same frequency band, if it is properly adjusted, the instability will be dramatically enhanced. (2). If two instability mechanisms occur at different frequencies or frequency bands, then, spurious oscillations may occur.

6. The coupling between TE and TM modes in the waveguide, and the intensity of the interactions due to the participation of the plasma depend on the plasma electron gyrating motion and the plasma background density, and is proportional to the parameter $k_g^2 = k^2\varepsilon_2 = -k^2 \frac{\xi^2\tau}{1-\tau^2}$, $\xi = \frac{\omega_p}{\omega}$, $\tau = \frac{\omega_c}{\omega}$**. It is clear that adjusting the magnetic field and the density of the background plasma is important for the design and operation of the plasma filleddevices.**

7. For the plasma filled case, ECRM is prefer to operate at high harmonics. $\omega - k_z v_z - l\omega_c = 0$, $l \geq 2$.

8. Theoretical predictions show that: in general, the frequency spectrum and the spurious output of plasma filled devices may not be as good as that of vacuum devices. Perhaps, that it is the price that we must pay for enhancing the output power and efficiency of microwave devices by means of plasma filling.

Part II Kinetic theory of Electron Beam-wave interaction in magnetized Plasma Waveguide

I. Introduction

In this part of the paper, the kinetic theory is used for analyzing the electron beam, while the plasma background is still treated by fluid theory.

II Kinetic Theory of Electron-beam-wave interactions in uniform magnetized plasma waveguide

A. longitudinal interaction

The dispersion equation for longidudinal interaction is :

$$\varepsilon_3(k_z^2 - k_{z0}^2)P_E + j\omega\mu_0\varepsilon_2(k_z - k_{z0})P_{HE} = -j\omega\mu_0\varepsilon_1\int J_z \cdot E_{z0}^{*}ds + \frac{jk_z}{\varepsilon_0}\int \rho_1 E_{z0}^{*}ds \tag{1}$$

Where J_z and ρ_1 are going to be calculated by using the kinetic theory:

$$J_z = -e\int f_1 \frac{p_z}{m\gamma_0}d\bar{p} \tag{2}$$

$$\rho_1 = -e\int f_1 d\bar{p} \tag{3}$$

f_1 is the perturbed distribution function. The integrations should be completed in momentum space, according to vlasov theory. In this case it can be written as:

$$f_1 = -e\int_{-\infty}^{t} E_z' \frac{\partial f_0}{\partial p_{//}}dt' = -\frac{jeE_z}{\omega - k_z v_0}\frac{\partial f_0}{\partial p_{//}} \tag{4}$$

Where f_0 is equilibrium distribution function, if the transverse motion is not taken into consideration, f_0 can be chosen as:

$$f_0 = \frac{n_b}{2\pi}\delta(p_z - p_0)\Theta(R_c - R) \tag{5}$$

After integration in momentum space, the dispersion equation for longitudinal interactions can be obtained :

$$\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE} = \frac{\omega_b^2(k^2\varepsilon_1 - k_z^2)P_E}{(\omega - k_z v_{z0})^2} \tag{9}$$

Which is the same as the dispersion equation derived through fluid theory.

B. Transverse interactions

For transverse interactions, we have:

$$J_{\phi 1} = 2\pi e^2 \sum_i \int \frac{A_i}{D} \mu_0 h_i p_i K^2 v_\perp \left\{ \frac{(\omega - k_z v_z)}{\Omega} \left(2J'_{mi} + p_i R_c J''_{mi}\right) - \frac{v_\perp^2 J'_{mi}}{\Omega^2} \left(k^2 - k_z^2\right) \right\} f_0 d\bar{p} \tag{10}$$

$$J_{\phi 1p} = 2\pi e^2 \sum_i \int \frac{A_i}{D} \mu_0 h_i k_g^2 m \omega_c \left\{ \frac{(\omega - k_z v_z)}{\Omega} \left(2J_{mi} + p_i R_c J'_{mi}\right) - \frac{\beta_\perp^2 J_{mi}}{\Omega^2} \left(\omega^2 - k_z^2 c^2\right) \right\} f_0 d\bar{p} \tag{11}$$

$$J_{\phi 2} = -j2\pi e^2 \sum_i \int \frac{A_i}{D} \left\{ \frac{K^2 m \omega_c}{\Omega} \left(J_{mi} + p_i R_c J'_{mi}\right)\left(k_z - k\beta_{//}\varepsilon_1\right) - \frac{\beta_\perp^2 J_{mi}}{\Omega^2} \left(k_z \omega K^2 m \omega_c (\varepsilon_1 - 1) + D(\omega v_z - k_z c^2)\right) \right\} f_0 d\bar{p} \tag{12}$$

$$J_{\phi 2p} = -j2\pi e^2 \sum_i \int \frac{A_i}{D} \left\{ \frac{1}{\Omega} \left[k_z v_\perp (k_g^2 - \varepsilon_2 k_z k \beta_{//}) \left(2J'_{mi} + p_i R_c J''_{mi}\right) + \varepsilon_2 k_g^2 \beta_{//} K l \omega_c (J_{mi} + p_i R_c J'_{mi}) \right] + \frac{\beta_\perp^2}{\Omega^2} k_g \omega \omega_c \beta_\perp \left[\left(k^2 - k_z^2\right) p_i R_c J'_{mi} - m k_g^2 J_{mi} \right] \right\} f_0 d\bar{p} \tag{13}$$

Where the space charge effect is neglected. The dispersion equation for transverse interaction can be obtained as follows:

$$\varepsilon_3 \left(k_z^2 - k_{z0}^2\right) P_{E\perp} + j\omega\mu_0 \varepsilon_2 \left(k_z - k_{z0}\right) P_{HE\perp} + \omega\mu_0 \left(\varepsilon_3 - \varepsilon_1\right) \cdot \left(k_z - k_{z0}\right) P_{EH\perp} = 2\pi \frac{\omega_b^2}{c^2} \frac{\omega \varepsilon_1}{\gamma v_{\perp 0}} \sum_i \frac{A_i}{D^2} R_0 \left[\frac{Q_1}{\Omega} - \frac{Q_2 \beta_\perp^2}{\Omega^2} \right] J_{mi}^2 \tag{14}$$

Here the dispersion equations (14) can be used for large orbit cyclotron masers .

III. Kinetic theory of Electron Beam-wave interactions in a corrugated plasma-filled waveguide

Longitudinal interaction:

$$\sum_s \varepsilon_3\left(k_{z,s}^2 - k_{z0,s}^2\right)P_{E,s} + j\omega\mu_0\varepsilon_2\left(k_{z,s} - k_{z0,s}\right)P_{HE,s} = \sum_s \frac{\omega_b^2(k^2\varepsilon_1 - k_{z,s}^2)P_{E,s}}{(\omega - k_{z,s}v_{z0})^2} \tag{26}$$

Transverse interaction:

$$\sum_s \varepsilon_3\left(k_{z,s}^2 - k_{z0,s}^2\right)P_{E\perp,s} + j\omega\mu_0\varepsilon_2\left(k_{z,s} - k_{z0,s}\right)P_{HE\perp,s} + \omega\mu_0\left(\varepsilon_3 - \varepsilon_1\right)\cdot$$

$$\left(k_{z,s} - k_{z0,s}\right)P_{EH\perp,s} = 4\pi^2 e^2 n_0 \omega\mu_0\varepsilon_1 \sum_{l,s}\frac{A_{l,s}}{D^2}R_c\left[\frac{S_{1,s}}{\Omega} - \frac{S_{2,s}\beta_\perp^2}{\Omega^2}\right]J_{m,s}^2 \tag{27}$$

$$\left.\begin{aligned} S_{1,s} &= \sum_s\left(S_{11,s} + S_{12,s} + S_{11p,s} + S_{12p,s}\right) \\ S_{2,s} &= \sum_s\left(S_{21,s} + S_{22,s} + S_{21p,s} + S_{22p,s}\right) \end{aligned}\right\} \tag{28}$$

IV. Plasma -filled Electron Cyclotron Maser(ECRM)

For the plasma- filled electron cyclotron maser, with electrons are gyrating around a guiding center with a small cyclice orbits, the dispersion equation should be:

$$\varepsilon_3(k_z^2 - k_{z0}^2)P_{E\perp} + j\omega\varepsilon_0\varepsilon_2(k_z - k_{z0})P_{HE_\perp} +$$

$$\omega\mu_0(\varepsilon_3 - \varepsilon_1)(k_z - k_{z0})P_{EH_\perp} = -j\omega\mu_0\varepsilon_1\int J_\theta \cdot E_{\theta0}^* ds - \frac{1}{\varepsilon_0}\int\left(\nabla_\perp\rho\right)_\theta \cdot E_{\theta0}^* ds \tag{32}$$

where:

$$J_\theta = -e\int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \frac{p_\theta}{m_0\gamma} \tag{33}$$

$$\rho = -e\int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \tag{34}$$

where the petrubed distribution function is:

$$f_1 = -e\int_{-\infty}^{+\infty} dt'\left[\left(E_\theta' + v_z B_r'\right)\left(\frac{\partial f_0}{\partial p_\theta} + \frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial p_\theta'}\right)\right.$$

$$\left. + \left(E_z' - v_\theta' B_r'\right)\frac{\partial f_0}{\partial p_z} - \left(E_r' + v_\theta' B_z' - v_z' B_\theta'\right)\frac{1}{p_z}\frac{\partial f_0}{\partial R_g}\frac{\partial R_g}{\partial \phi'}\right] \tag{35}$$

The integration should be taken along an unperturbed orbit, and f_0 is equilibrium distribution function.

$$f_0 = \frac{n_b}{4\pi^2 p_{\perp0}R_0}\delta(p_z - p_{z0})\delta(p_\perp - p_{\perp0})\delta(R_g - R_0) \tag{36}$$

where R_0 is the radius of guiding center of electron beam. Since the field can be splited into four parts. we get:

$$E_\theta = E_{\theta 1} + E_{\theta 2} + E_{\theta p} + E_{\theta 2p}, etc. \tag{37}$$

$$f_1 = f_{11} + f_{12} + f_{11p} + f_{12p} \tag{38}$$

$$J_\theta = J_{\theta 1} + J_{\theta 2} + J_{\theta 1p} + J_{\theta 2p} \tag{39}$$

Thus, we get:

$$\begin{aligned}\iint J_\theta \cdot E_\theta^* ds = \iint \big(& J_{\theta 1}E_{\theta 1}^* + J_{\theta 1}E_{\theta 2}^* + J_{\theta 1}E_{\theta 1p}^* + J_{\theta 1}E_{\theta 2p}^* + J_{\theta 1p}E_{\theta 1}^* + J_{\theta 1p}E_{\theta 2}^* + J_{\theta 1p}E_{\theta 1p}^* + \\ & J_{\theta 1p}E_{\theta 2p}^* + J_{\theta 2}E_{\theta 1}^* + J_{\theta 2}E_{\theta 2}^* + J_{\theta 2}E_{\theta 1p}^* + J_{\theta 2}E_{\theta 2p}^* + J_{\theta 2p}E_{\theta 1}^* + J_{\theta 2p}E_{\theta 2}^* + J_{\theta 2p}E_{\theta 1p}^* + J_{\theta 2p}E_{\theta 2p}^* \big)\end{aligned} \tag{40}$$

Equation(51)-(54) indicates that the electric field of the wave is splited into four components: $E_{\theta 1}, E_{\theta 2}, E_{\theta 1p}$ and $E_{\theta 2p}$etc., $E_{\theta 1}$ and $E_{\theta 2}$ are the TE-like and TM-like field components, respectively. $E_{\theta 1p}$ and $E_{\theta 2p}$ are the field components due to plasma background corresponding to TE-like and TM-like, respectively. The f_{11}, f_{12}, f_{11p} and f_{12p} are the four components of perturbed distribution functions corresponding to $E_{\theta 1}, E_{\theta 2}, E_{\theta 1p}$ and $E_{\theta 2p}$etc.,respectively, and $J_{\theta 1}, J_{\theta 2p}$ are those excited by the field components corresponding to that due to plasma background for TE-like and TM-like fields, respectively. It can be remembered that when plasma is absent, $J_{\theta 1p}$ and $J_{\theta 2p}$ disappear, $J_{\theta 1}$ and $J_{\theta 2}$ are left but they are separated and can be exist independently.

After a proper and long mathematical manipulation, we can obtain the dispersion equations of the plasma-filled ECRM as:

$$\begin{aligned}&\varepsilon_3\left(k_z^2 - k_{z0}^2\right)P_{E\perp} + j\omega\mu_0\varepsilon_2\left(k_z - k_{z0}\right)P_{HE\perp}^{(1)} + \omega\mu_0(\varepsilon_3 - \varepsilon_1)(k_z - k_{z0})P_{HE\perp}^{(2)} \\ &= 2\pi \frac{\omega_b^2}{c^2\gamma}\frac{\omega\varepsilon_1}{v_{\perp 0}}\sum_{i,l}\frac{A_{il}^2}{D^2}R_0 \cdot \left[\frac{T_1}{\Omega_1} - \frac{T_2\beta_\perp^2}{\Omega_1^2}\right]\cdot \left|J_{m-li}\right|^2\end{aligned} \tag{45}$$

where:

$$\left.\begin{aligned}T_1 &= T_{\theta 11} + T_{\theta 1p1} + T_{\theta 21} + T_{\theta 2p1} \\ T_2 &= T_{\theta 12} + T_{\theta 1p2} + T_{\theta 22} + T_{\theta 2p2}\end{aligned}\right\} \tag{46}$$

and:

$$\Omega_1 = \omega - k_z v_z - l\omega_c \tag{47}$$

B.Coupled longitudinal interactions

As mentioned above in a magnetized plasma wave guide, the TE and TM modes are always coupled, the transverse and longitudinal interaction, therefore, are always accompanied by each other. So the longitudinal dispersion equation should be taken into account simultaneously. Now, the J_z should be calculated by means of kinetic theory as.

$$\begin{aligned} J_z &= -e\int_{-\infty}^{\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_0^{2\pi} d\phi f_1 \frac{p_z}{m_0\gamma} \\ &= -2\pi e\int_{-\infty}^{\infty} dp_z \int_0^{\infty} \frac{p_\perp p_z}{m_0\gamma}(f_{11}+f_{12}+f_{11p}+f_{12p})dp_\perp \end{aligned} \tag{58}$$

After integration along momentum space, we can obtain the dispersion equations:

$$\begin{aligned} &\varepsilon_3\left(k_z^2-k_{z0}^2\right)P_E + j\omega\mu_0\varepsilon_2\left(k_z-k_{z0}\right)P_{HE} = 2\pi\frac{\omega_b^2}{c^2\gamma}\frac{\omega\varepsilon_1}{v_{\perp 0}}\cdot \\ &\sum_{l,J}\frac{A_{li}^2}{D^2}R_0\cdot\left[\frac{\Pi_1}{\Omega_1}-\frac{\Pi_2\beta_\perp\beta_{//}}{\Omega_1^2}\right]\cdot\left|J_{m-li}\right|^2 \end{aligned} \tag{65}$$

Π_1,Π_2 are given in Appendix D.

From the above formulation, we can see that:

1. Although the dispersion equations of plasma-filled ECRM are very complicated, the structure remains unchanged. There are also two main terms: $(\omega-k_zv_z-l\omega_c)^{-1}$ and $(\omega-k_zv_z-l\omega_c)^{-2}$ terms, which are very familiar in the theory of ECRM in vacuum case.
2. Since there are four parts of wave field and four parts of RF current, so the term $\int\vec{J}\cdot\vec{E}ds$ has 16 terms. This makes the equations very tedious, but the physical lines are still clear.
3. We can see clearly that, looking at the TM-like and TE-like parts separately, the structures of the dispersion equations are similar with that in vacuum case. And when the plasma is absent, the equations reduce to that for vacuum case .
4. The most important differences between the plasma-filled one and the vacum case are that in some of the terms associated with the parts of field produced by plasma background, there is the imaginary sign **j**, These terms not only make the dispersion equations complicated but also cause an instability different from that in the vacuum case.
5. The most essential difference is that the ECRM is always accompanied by and coupled with corresponding longitudinal interactions.

C. Corrugated waveguide

The above equations are for a smooth waveguide. For a periodical structure, we have:

$$\sum_s\left[\varepsilon_3\left(k_{z,s}^2-k_{z0}^2\right)P_{E\perp,s}+j\omega\mu_0\varepsilon_2\left(k_{z,s}-k_{z0}\right)P_{HE\perp,s}+\omega\mu_0(\varepsilon_3-\varepsilon_1)(k_{z,s}-k_{z0})P_{EH\perp,s}\right]$$
$$=-\sum_s\left[j\omega\mu_0\varepsilon_1\iint\vec{J}_{\theta,s}\cdot\vec{E}_{\theta,s}^{*}ds\right] \tag{66}$$

and:

$$J_{\theta,s}=-e\int_{-\infty}^{\infty}dp_z\int_0^{\infty}p_\perp dp_\perp\int_0^{2\pi}d\phi f_{1,s}\frac{p_\theta}{m_0\gamma} \tag{67}$$

Similarly:

$$E_{\theta,s}=E_{\theta1,s}+E_{\theta2,s}+E_{\theta1p,s}+E_{\theta2p,s} \tag{69}$$

$$f_{1,s}=f_{11,s}+f_{12,s}+f_{11p,s}+f_{12p,s} \tag{70}$$

$$J_{\theta,s}=J_{\theta1,s}+J_{\theta2,s}+J_{\theta1p,s}+J_{\theta2p,s} \tag{71}$$

$$\sum_s\iint J_{\theta,s}\cdot E_{\theta,s}^{*}ds=\sum_s\iint\left(J_{\theta1,s}E_{\theta1,s}^{*}+J_{\theta1,s}E_{\theta2,s}^{*}+J_{\theta1,s}E_{\theta1p,s}^{*}+J_{\theta1,s}E_{\theta2p,s}^{*}+J_{\theta1p,s}E_{\theta1,s}^{*}+\right.$$
$$J_{\theta1p,s}E_{\theta2,s}^{*}+J_{\theta1p,s}E_{\theta1p,s}^{*}+J_{\theta1p,s}E_{\theta2p,s}^{*}+J_{\theta2,s}E_{\theta1,s}^{*}+J_{\theta2,s}E_{\theta2,s}^{*}+J_{\theta2,s}E_{\theta1p,s}^{*}+$$
$$\left.J_{\theta2,s}E_{\theta2p,s}^{*}+J_{\theta2p,s}E_{\theta1,s}^{*}+J_{\theta2p,s}E_{\theta2,s}^{*}+J_{\theta2p,s}E_{\theta1p,s}^{*}+J_{\theta2p,s}E_{\theta2p,s}^{*}\right)ds \tag{72}$$

If only taking the resonance term, we just need to delete the $\sum_s$ in the equations and replace the arguments in $J_{li},J_{li}^{'},J_{m-li}$ by $p_{i,s}r_c$ and $p_{i,s},R_g$.

For the accompanied longitudinal interaction, we have:

$$\sum_s\left[\varepsilon_3\left(k_{z,s}^2-k_{z0,s}^2\right)P_{E,s}+j\omega\mu_0\varepsilon_2\left(k_{z,s}-k_{z0,s}\right)P_{HE,s}\right]=\frac{\omega_b^2}{c^2}\frac{\omega\varepsilon_1}{\gamma v_{\perp0}}\cdot$$
$$\sum_{i,l}\sum_s\frac{A_{il,s}^2}{D_s^2}R_0\left[\frac{\Pi_{1,s}}{\Omega_{1,s}}-\frac{\Pi_{2,s}\beta_\perp\beta_{//}}{\Omega_{1,s}^2}\right]\cdot\left|J_{m-li}\right|^2 \tag{73}$$

It should be noted here that when $r_c\to R_0$, all the results given here reduce to that given in section II and III.

V. On the combination of Transverse and longitudinal Interactions

From the kinetic theory of ECRM and accompanied longitudinal interactions given in above section, we can see that, if there is electron transverse motion, both transverse and longitudinal interactions can co-exist under the condition:

$$\Omega_l = \omega - k_z v_z - l\omega_c \approx 0$$

$$\text{or} \qquad \Omega_s = \omega - k_{z,s} v_z - l\omega_c \approx 0$$

It should be emphased here that the coexistence (or combination) of transverse and longitudinal interactions (ECRM and Cherenkov or ECRM and BWO, for example) is one of the most significant features for electron beam-wave interactions in magnetized plasma waveguide. Now, since thre is electron transverse motion, the singularity is also $\Omega = \omega - k_z v_z - l\omega_c \approx 0$, rather than that given in [8]($\omega - k_z v_z \approx 0$) for coupled longitudinal interactions.

VI Summary

Kinetic theory of Electron-Beam-Wave interactions in magnetized plasma waveguide, for both longitudinal and transverse interactions in both smooth and corrugated waveguide have been given in this part of paper, the following points are significant:

1.Althrough the mathematical manipulations are tedious, the structures of all the dispersions are simple, they consist of two parts, one is proportional to $\left(\frac{1}{\Omega_m}\right)$ or $\left(\frac{1}{\Omega_l}\right)$(or $\left(\frac{1}{\Omega_{m,s}}\right),\left(\frac{1}{\Omega_{l,s}}\right)$), another one is to $\left(\frac{1}{\Omega_m^2}\right)$ or $\left(\frac{1}{\Omega_l^2}\right)$(or $\left(\frac{1}{\Omega_{m,s}^2}\right),\left(\frac{1}{\Omega_{l,s}^2}\right)$) , It is just like that is familiar in vacuum .

2. One of the most important features of the beam-wave interactions in magnetized plasma waveguide is coexistence (simulate existence) or combination of both instabilities of transverse and longitudinal interactions, it is because that in such plasma waveguide the TE and TM modes are always coupled so the Ez and $\vec{E}_\perp$ field components are always exist together. Because of the transverse motion in magnetic field, the singularity of the accompanied longitudinal interaction is $\Omega = \omega - k_z v_z - l\omega_c \approx 0$.

3. Another one of the most important features is that in the dispersion equations there are special terms that contain imaginary sign(j) , These terms are generated by the field parts that are produced by the plasma background., these imaginary terms may bring influence on the instability.

4. Only when there is no transverse motion, we can have pure longitudinal interaction, and the singularity is now $\Omega = \omega - k_z v_z$, as that for TM mode in vacuum case.

5.With the coexistent instabilities(ECRM and Cherenkov, for example),there must be some difference of frequency response between the two kind of interactions, spectrum purity of the output of the device base on the magnetized plasma filled waveguide may not as good as vacuum ones.

6. For a plasma filled case, the ECRM prefers to operate at higher harmonics (l≥2).

Longitudinal Interaction:

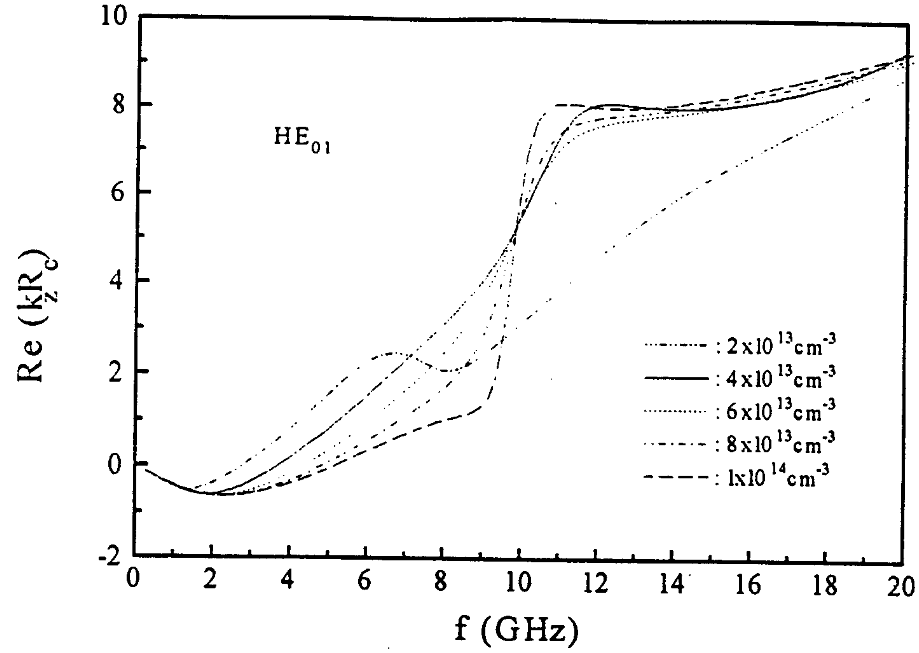


Fig.IV.4 The real part of k_zR_c of HE$_{01}$ mode *vs* the operation frequency.

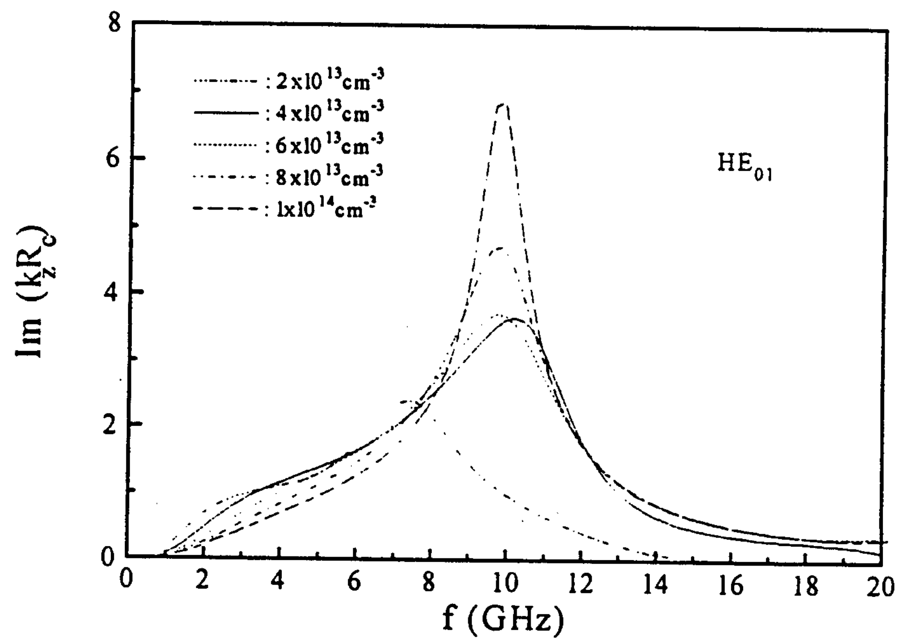
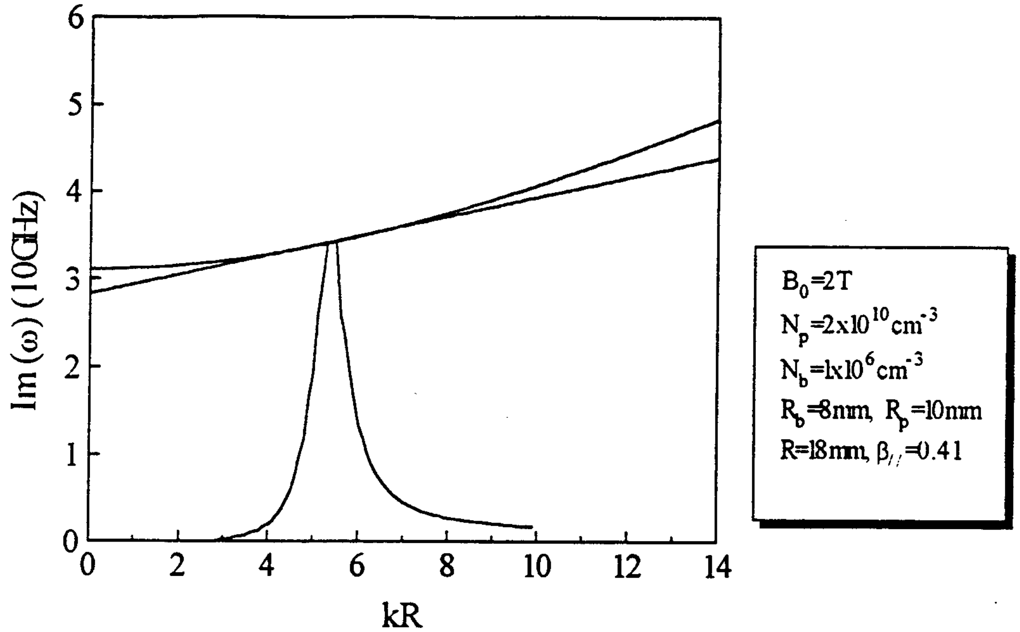
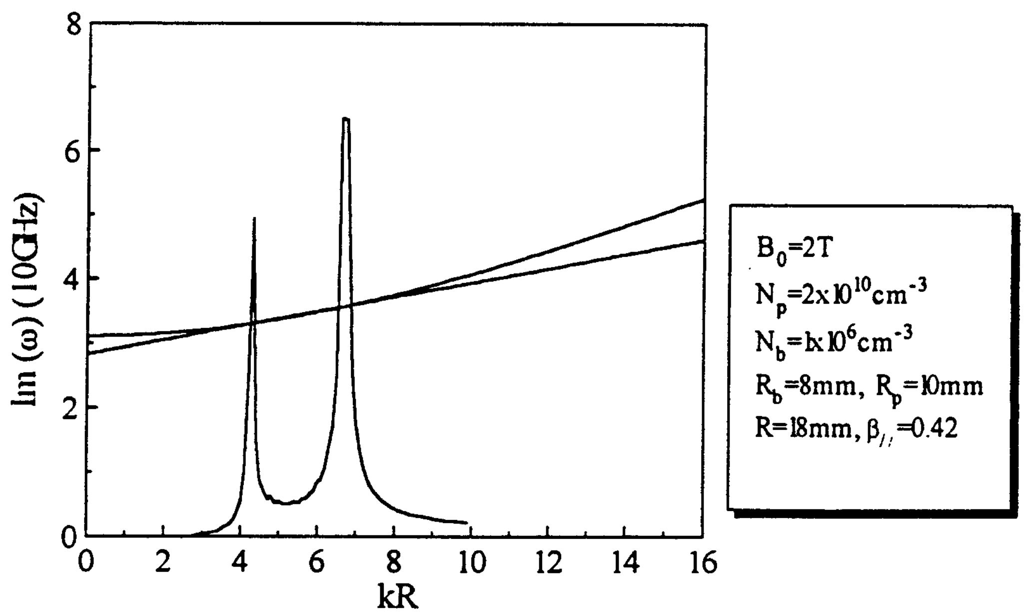


Fig.IV.5 The imaginary part of k_zR_c or the space growth rate of HE$_{01}$ mode *vs* the operation frequency.

Hybrid Ion-channel Maser:



The gain of HE_{11} mode. Some important parameters are list.



The gain of HE_{11} mode.

Plasma Filled ECR Maser:

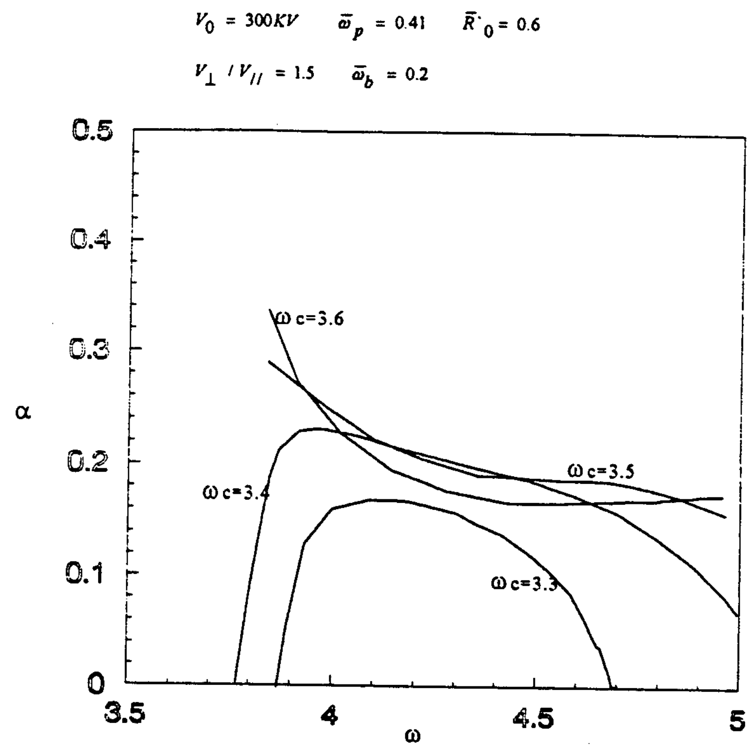


Fig.IV.10 Growth rate versus ω_c of Cyclotron mode

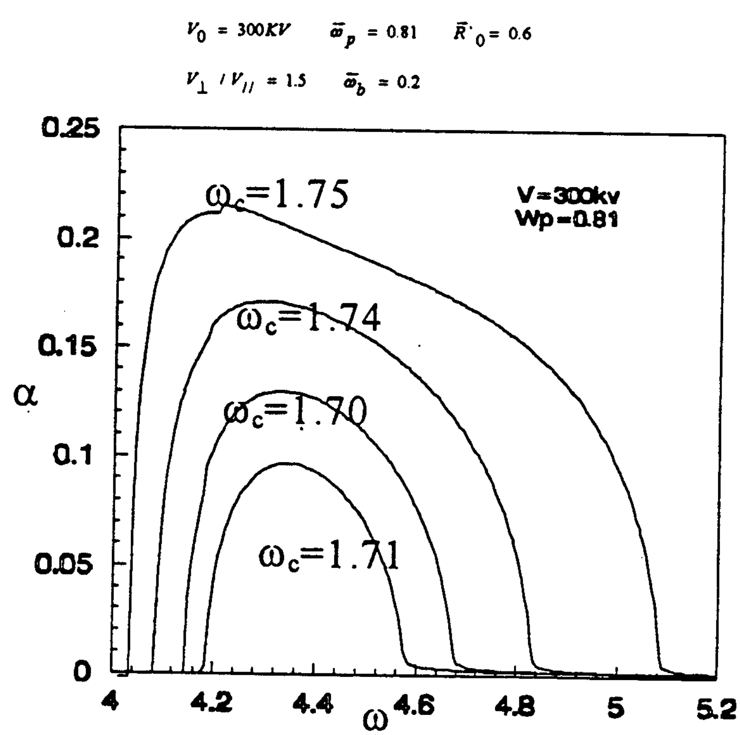


Fig.IV.11 Growth rate versus ω_c of Waveguide mode

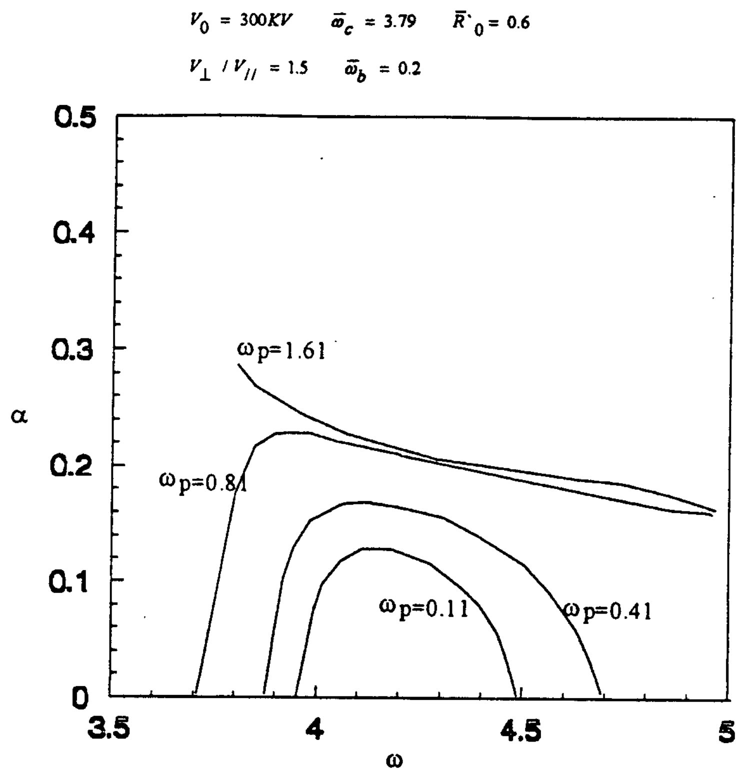


Fig.IV.12 Growth rate versus ω_p of cyclotron mode

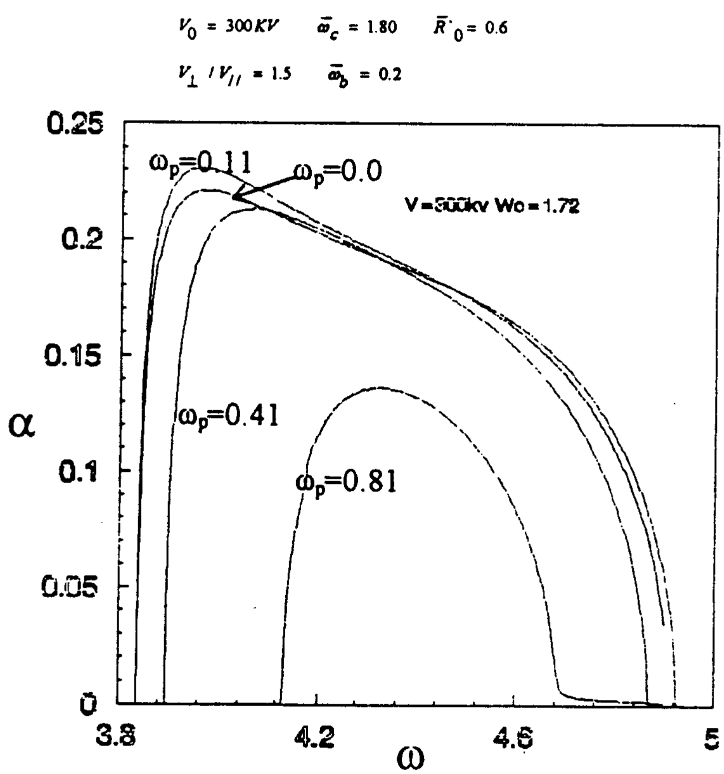


Fig.IV.13 Growth rate versus ω_p of waveguide mode

4. **Robert J. Barker and Shenggang Liu, "A New Hybrid Ion-Channel Maser Instability".**

1

A New Hybrid Ion-Channel Maser Instability ***

Robert J. Barker *, Liu Shenggang**

Abstract: A new hybrid ion-channel maser instability, in which the electron cyclotron maser instability mechanism and the ion-channel instability maser instability mechanism are combined, is proposed and studied. The features and the dispersion equation of the new maser are given in the paper with detailed discussion.

I. Introduction

An ion-channel can be formed due to either intens laser beam or relativistic electron beam injection. Based on this effect, varieties of ion-channel lasers' and ion-channel masers' instabilities have been presented and studied [1]-[5].Now a new instability scheme is proposed, in which the ion-channel maser instability and the electron cyclotron maser instability are combined. Theoretical analysis and the dispersion equation of the new instability mechanism are given in the paper. It shows that this new hybrid ion-channel maser instability has some interesting features.

II. Analysis

In an ion-channel, the force affecting the electron motion is

$$\bar{F}_i = -\frac{|e|^2 n_p}{2\varepsilon_0} R\bar{e}_R \qquad (1)$$

It is a centripetal force, we get the cyclotron frequency as

$$\omega_0^2 = \omega_p^2 / 2\gamma_0, \qquad \omega_p^2 = \frac{|e|^2 n_p}{m_0 \varepsilon_0} \qquad (2)$$

n_p is the ion density and γ_0 is the relativistic factor of the electron beam. Almost all electromagnetic instabilities of ion-channel laser' and maser' are based on equ.(2).
If there exists an axial magnetic field B, we have a combined force:

$$\bar{F} = \bar{F}_i + \bar{F}_m = -\left[\frac{|e|^2 n_p}{2\varepsilon_0} R + |e| B_0 v_\theta\right]\bar{e}_R \qquad (3)$$

It shows that both F_i and F_m are centripetal force. Then we get the electron cyclotron frequency:

*** presented at 24th Int. Conf. on IRMMW, Montery. CA. USA. Sept. 3-9, 1999.

$$\omega_0 = \frac{\omega_{c0}}{2\gamma_0}\left(1+\left(1+\frac{2\gamma_0\omega_p^2}{\omega_{c0}}\right)^{1/2}\right) \qquad (4)$$

Where $\omega_{c0} = \dfrac{|e|B_0}{m_0}$

When $\omega_{c0} >> \omega_p$, we get $\omega_0 = \omega_c = \omega_{c0}/\gamma_0$, when $\omega_p^2 >> \omega_{c0}^2$, we get $\omega_0 = \omega_p/\sqrt{2\gamma_0}$.

The former one is the base of Electron Cyclotron Maser instability, the $\gamma_0^{-1/2}$ energy dependence of which is γ_0^{-1} , while the second one is the base of ion-channel maser instability, the energy dependence is $\gamma_0^{-1/2}$ [3].Now equ.(4) shows that for the new hybrid one , the energy dependence is not γ_0^{-1}, nor $\gamma_0^{-1/2}$, it is a new interaction scheme.

III. Dispersion Equation

By using the fluid model the following dispersion equations of the hybrid ion-channel maser instability can be obtained:

$$\left(k_z^2 - k_{z0}^2\right) = -\frac{j\omega\mu_0}{P_E}\int J_\varphi \bullet E_\varphi^* ds = \frac{\omega\mu_0|e|n_b}{P_E}\left(j\frac{\omega_0^2}{\Omega}R_1 + (\omega_0+\Omega)R_0\varphi_1 + R_0\omega_0 k_{z0} z_1\right)\bullet E_\varphi^*\Big|_{R=R_0} . \qquad (5)$$

Where

$$\Omega = \omega - k_{z0}v_{z0} - \omega_0 \qquad (6)$$

$$R = \frac{\Delta_R}{\Delta} \qquad (7)$$

$$\bar{f} = -|e|\left(\bar{E}_1 + \bar{v}_0 \times \bar{B}_1\right) R_0\varphi_1 = \frac{\Delta_\varphi}{\Delta} \qquad (8)$$

$$z_1 = \frac{\Delta_z}{\Delta} \qquad (9)$$

$$\Delta = \gamma_0^3\Omega^4\left(\left(1+\gamma_0^2\beta^2\right)\left(\Omega^2 - \omega_0^2 - \frac{\omega_p^2}{\gamma_0}\right) - (\omega_0 - \omega_c)^2\left(1+\gamma_0^2\beta_\parallel^2\right)\right) \qquad (10)$$

Where $\bar{f}$ is the force due to the electromagnetic wave field:

$$\bar{f} = -|e|\left(\bar{E}_1 + \bar{v}_0 \times \bar{B}_1\right) \qquad (11)$$

Substituting the field expressions into the above equations we can get the complete form of the dispersion equation.

IV. Conclusion

A new kind of ion-channel maser instability is proposed and studied in the paper. This hybrid ion-channel maser is based on the combination of magnetic electron cyclotron maser and the ion-channel maser instabilities. The theoretical analysis given in the paper shows that the new hybrid ion-channel maser instability has the following features:

1. The new instability mechanism is based on the hybrid cyclotron frequency that has a special energy dependence (1/2<q<1). Accrding to [3],the energy dependence can be written as γ_0^{-q} , it showed in [3] that there are three kinds of the dependence: q= 1, is the electron cyclotron maser instability; q= 1/2 is the ion-channel maser instability and q=0 is the Free Electron Laser instability. Now we get a new one the q of which should be(1/2<q<1).. Actually , there is another one, the Electrostatic Electron Maser [5], the energy dependence is also q=1/2. Therefore, there are four different kinds of energy dependence for different instability mechanisms: q=0, q= ½, ½<q<1 and q=1. It shows that the instability mechanism is also negative mass effect, and the energy dependence is weaker than that of magnetic electron cyclotron maser and stronger than that for the ion-channel maser.

2.The character of the singularity in the dispersion equation of the new maser instability is different from that of electron cyclotron maser and the ion-channel maser. There are two singularities: $\Omega = 0$ and $\left(\left(1+\gamma_0^2\beta^2\right)\left(\Omega^2-\omega_0^2-\frac{\omega_p^2}{\gamma_0}\right)-(\omega_0-\omega_c)^2\left(1+\gamma_0^2\beta_1^2\right)\right)=0$. The last one was never seen in published papers.

3.Comparing with the electron cyclotron maser and the ion-channel maser , the new hybrid ion-channel maser may have some advantages: At first, because of the ion neutralization the beam density can be increased, secondly, since q<1, for the same electron energy, the operating frequency may be higher. In addition, since the interaction takes place in the ion-channel region, stronger interaction efficiency is expected.

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References

1. David H. Whilttom, Phys. Fluids, B4(3), 730, 1992,
2. David H. Whilttom, Andrew M. Sessler and J. M. Dawson. Phys. Rev. Letters, 64,21,2511,1990.
3. K. R. Chen. J.M.Dawson, A.T. Lin and T. Katsolear. Phys. Fluids, B(3),5, 1270, 1991,
4. Chang J. Tang, Pukun Liu and Liu, Shenggang. J. Phys. D. Appl. Phys. 29, 90, 1996,
5. Liu Shenggang. Robert J. Barker , et al. Unpublished paper,
6. Liu, Shenggang et al. J. Appl. Phys. 59,3621, 1986.

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7

A New Hybrid Ion-channel Maser Instability+

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Gao Hong*, Yan Yang* and Zhu Dajun*

Abstract-- **A new hybrid maser instability is described for the case of wave interactions for an electron-beam propagating through an ion channel in a plasma-filled waveguide immersed in a finite axial magnetic field. A complete linear theoretical formulation and sample numerical calculations are presented. The significant features of this new hybrid instability are discussed.**

INDEXING TERMS: Ion-channel, Hybrid Instability, Plasma-filled Waveguide, Perturbation Theory

I. INTRODUCTION

When a thin, annular, intense relativistic electron-beam (REB) of radius, R_0, and electron density, n_b, propagates through a plasma of density, n_p; an ion-channel may be formed of radius, $R_p \sim R_0(n_b / n_p)^{1/2}$. Based on this effect, varieties of ion channel laser and ion channel maser instabilities have been presented and studied. Whittum and Sessler first proposed the concept of an Ion-channel Laser in 1990 [1], [2]. Then Whittum studied the electromagnetic wave instability of the ion-focused regime in detail [3]. The Ion-ripple Laser was proposed by Chen and Dawson in 1992 [4], [5]. Tang *et al.* [6] studied electromagnetic wave instabilities in an ion-channel (IC) electron cyclotron maser (ECM) and proposed the concept of the ICECM. Recently, Parashar *et al.* [7] studied electromagnetic wave scattering in an ion-channel. Jha and Kumar [8] presented a

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linear theory to study the ion-channel-guiding, magnetic-wiggler FEL. Thus, research on the production of coherent radiation based on the ion channel effect has been very active in recent years.

An ion channel can be used to improve the quality of relativistic electron-beams (REBs) by helping to radially confine the beam current. The whole or partial charge-neutralization provided by the channel's ions permits devices operating at higher beam currents for a given structure radius which results in enhanced microwave output power. Not only the REBs themselves flowing through a plasma-filled waveguide, but also intense laser beam pulses can be used to form an ion-channel via ponderomotive force.

In this paper, a new hybrid ion-channel maser instability scheme is proposed, in which the ion channel maser instability and the electron cyclotron maser instability are combined. Theoretical analysis and sample numerical calculations of the new instability mechanism are also presented. The interesting features of this new hybrid ion channel maser instability are discussed.

II. THEORETICAL ANALYSIS

We begin with the idealized physical structure of a hollow electron-beam propagating through a magnetized plasma-filled waveguide as shown in Figure 1. The waveguide we consider here is an axially symmetric, cylindrical structure. For simplicity, we assume a *preformed* ion channel (created, for example, by a laser pulse) into which the hollow e-beam is injected. We examine the highly relativistic case for ion-channel formation where $n_p > n_b / \gamma_0^2$. It is easy to show that for this condition, the force acting on beam electrons due to the channel ions is much larger than the force due to the beam itself (i.e. - the self-field). Therefore, in the highly relativistic case, this force due to the ions in the ion-channel must be taken into account.

In the ion-channel the force due to the ions is:

$$\bar{F}_i = -\frac{|e|^2 n_p}{2\varepsilon_0} R\bar{e}_R \qquad (1)$$

where n_p is the ion density. Equation (1) takes the form of a centripetal force. It is clear that this force does not contribute to the longitudinal interactions where the beam electrons propagate in the z-direction along the axis. However, this ion force very strongly influences transverse interactions. If there were no magnetic field, this is the only force that acts on the beam electrons besides the rf-field force. This $B_0=0$ case corresponds to the ion-channel maser/laser case [1], [2], [9]-[12]. The electron betatron frequency is:

$$\omega_0^2 = \omega_p^2 \;\; 2\gamma_0 \text{ , where } \qquad \omega_p^2 = \frac{e^2 n_p}{m_0 \varepsilon_0} \tag{2}$$

where γ_0 is the relativistic factor of the electron beam. Almost all the electromagnetic instabilities of ion-channel lasers and masers are based on (2). If the system is now immersed in an axial magnetic field, the beam-wave interactions become more complex. In fact, a new kind of hybrid interaction emerges. For this case, the beam electrons are moving in a combined field that produces the force:

$$\bar{F} = \bar{F}_i + \bar{F}_m = -\left[\frac{e^2 n_p}{2\varepsilon_0} R + |e| B_0 v_p\right] \bar{e}_R \tag{3}$$

where $\bar{F}_i$ is the ion force and $\bar{F}_m$ is the force due to the magnetic field (Lorenz force: $(\bar{v} \times \bar{B}_0)_R$). Both of these force components are centripetal forces, that yield the electron cyclotron frequency as

$$\omega_0 = \frac{\omega_{c0}}{2\gamma_0}\left[1 + \left(1 + \frac{2\gamma_0 \omega_p^2}{\omega_{c0}^2}\right)^{1/2}\right] \tag{4}$$

where $\omega_{c0} = \dfrac{|e| B_0}{m_0}$ and B_0 is the applied axial magnetic field.

When $\omega_{c0} >> \omega_p$, we obtain $\omega_0 = \omega_c = \omega_{c0} \;\; \gamma_0$. However, when $\omega_p^2 >> \omega_{c0}^2$, we get: $\omega_0 = \omega_p / \sqrt{2\gamma_0}$. The former is the basis of the Electron Cyclotron Maser instability, and the latter is the basis of the Ion-Channel Maser instability. For the new hybrid ECM instability which we are presenting here, (4) shows that the energy dependence is different from the $\gamma_0^{-1/2}$ proportionality that was studied in [6] and [9].

In order to derive the dispersion relation for highly relativistic electron beam-wave interactions in a magnetized ion-channel, we begin with the following general equations of electron motion:

$$\gamma\left[\frac{d^2R}{dt^2} - R\left(\frac{d\varphi}{dt}\right)^2\right] + \frac{d\gamma}{dt}\frac{dR}{dt} = -\frac{eB_0}{m_0}R\frac{d\varphi}{dt} - \frac{\omega_P^2}{2}R + \frac{f_R}{m_0}$$

$$\gamma\left[2\frac{dR}{dt}\frac{d\varphi}{dt} + R\frac{d^2\varphi}{dt^2}\right] + R\frac{d\gamma}{dt}\frac{d\varphi}{dt} = \frac{f\varphi}{m_0} + \frac{eB_0}{m_0}\frac{dR}{dt} \tag{5}$$

$$\gamma\frac{d^2z}{dt^2} + \frac{d\gamma}{dt}\frac{dz}{dt} = \frac{f_z}{m_0}$$

and

$$\frac{\gamma^2}{c^2} = \left[c^2 - \left(\frac{dR}{dt}\right)^2 - R^2\left(\frac{d\varphi}{dt}\right)^2 - \left(\frac{dz}{dt}\right)^2\right]^{-1} \tag{6}$$

where $f_R\bar{e}_R + f_\varphi\bar{e}_\varphi + f_z\bar{e}_z = \bar{F}$ is the force acting on the electrons due to the rf-field. This force has the form:

$$\bar{F} = e(\bar{E} + \bar{v}\times\bar{B}) \tag{7}$$

In this formulation, the longitudinal components of the rf-radiation field in the magnetized plasma-filled waveguide have the forms, $E_z = A_1J_m(p_1R) + A_2J_m(p_2R)$ and $H_z = A_1h_1J_m(p_1R) + A_2h_2J_m(p_2R)$, where p_1 and p_2 are two eigenvalues while $J_m(p_1R)$ and $J_m(p_2R)$ are the two corresponding eigenfunctions. (Note that for a cylindrical waveguide, these eigenfunctions are Bessels functions.) The transverse rf-field components can be found by simply inserting these field components into Maxwell's Equation.

Invoking perturbation theory, we define:

$$\left.\begin{array}{l} R = R_0 + R_1\,;\ \varphi = \varphi_0 + \varphi_1;\ z = z_0 + z_1\,;\ \gamma = \gamma_0 + \gamma_1 \\ R_1 << R_0\ ;\quad \varphi_1 << \varphi_0\ ;\quad z_1 << z_0\ ;\quad \gamma_1 << \gamma_0 \end{array}\right\} \tag{8}$$

with the subsequent perturbation expansions performed in orders of the amplitude of the rf radiation field. Substituting (8) into (5) and (6), we find:

$$R_1 = \frac{\Delta_R}{\Delta};\quad R_0\varphi_1 = \frac{\Delta\varphi}{\Delta};\qquad z_1 = \frac{\Delta_z}{\Delta}; \tag{9}$$

and

$$c^2\gamma_1 = \gamma_0^3R_0\omega_0^2R_1 + j\gamma_0^3\omega_0R_0^2\Omega\varphi_1 + j\gamma_0^3\Omega v_{z0}z_1 \tag{10}$$

where Ω is the frequency of the electromagnetic wave and the determinants Δ, Δ_R, Δ_φ, Δ_z, are :

$$\Delta = \gamma_0^3\Omega^4\left[\left(1+\gamma_0^2\beta^2\right)\left(\Omega^2-\omega_0^2-\frac{\omega_p^2}{\gamma_0}\right)-\left(\omega_0-\omega_c\right)^2\left(1+\gamma_0^2\beta_{//}^2\right)\right] \tag{11}$$

$$\begin{aligned}\Delta_R = &-\gamma_0^2\Omega^4\left(1+\gamma_0^2\beta^2\right)f_R\ m_0\\ &+j\gamma_0^2\Omega^3\left[\left(1+\gamma_0^2\beta_{//}^2\right)\left(2\omega_0-\omega_c\right)+\gamma_0^2\beta_\perp^2\omega_0\right]f_\varphi\ m_0\\ &-j\gamma_0^4\Omega^3\beta_{//}\beta_\perp\left(\omega_0-\omega_c\right)f_z\ m_0\end{aligned} \tag{12}$$

$$\begin{aligned}\Delta_\varphi = &-j\gamma_0^2\Omega^3\left[\left(1+\gamma_0^2\beta^2\right)\left(2\omega_0-\omega_c\right)+\gamma_0^2\beta_\perp^2\omega_0\right]f_R\ m_0\\ &-\gamma_0^2\Omega^2\left[\Omega^2\left(1+\gamma_0^2\beta_{//}^2\right)+\gamma_0^2\beta_\perp^2\omega_0^2\right]f_\varphi\ m_0\\ &+\gamma_0^4\Omega^2\beta_{//}\beta_\perp\left(\Omega^2-2\omega_0^2+\omega_c\omega_c\right)f_z\ m_0\end{aligned} \tag{13}$$

$$\begin{aligned}\Delta_z = &\ j\gamma_0^4\Omega^3\beta\ \beta_\perp\left(\omega_0-\omega_c\right)f_R\ m_0\\ &+\gamma_0^4\Omega^2\beta_{//}\beta_\perp\left(\Omega^2-2\omega_0^2+\omega_c\omega_c\right)f_\varphi\ m_0\\ &-\gamma_0^2\Omega^2\left[\left(1+\gamma_0^2\beta_\perp^2\right)\left(\Omega^2-\omega_0^2-\omega_p^2/\gamma_0\right)-\left(\omega_0-\omega_c\right)^2\right]f_z\ m_0\end{aligned} \tag{14}$$

for the first harmonic of the electron cyclotron wave, $l=1$. In the above derivation, we followed the formalism presented in [12] and developed in more detail in [13].

According to the charge continuity equation and the above results, we can get the expression for the perturbed charge density:

$$\rho_1 = [\frac{(\omega-\Omega)}{R_0\Omega}R_1 - j\varphi_1 - jk_{z0}z_1]\sigma_0\delta(R-R_0) \tag{15}$$

where ω is the operating frequency and σ_0 is the surface charge density. Then the beam rf current density can be found as follows [5], [9]:

$$\begin{aligned}J &= e\sigma_0\delta(R-R_0)v_1 + ev_0\rho_1\\ &= J_\varphi + J_z\end{aligned} \tag{16}$$

where $\delta(R-R_0)$ is the Dirac δ-function, and

$$J_\varphi = -|e|\sigma_0[\frac{\omega_0^2}{\Omega}R_1 - j(\omega_0+\Omega)R_0\varphi_1 - jR_0\omega_0k_zz_1)]\delta(R-R_0) \tag{17}$$

$$J_z = |e|\sigma_0[\frac{(\omega-\Omega)v_{z0}}{R_0\Omega}R_1 - jv_{z0}\varphi_1 - j(\omega-\omega_0)z_1]\delta(R-R_0) \tag{18}$$

where $\Omega = \omega - k_{z0}v_{z0} - l\omega_0$, while R_1, $R_0\varphi_1$, and z_1 are the perturbed displacements in the R, φ, and z directions respectively due to the rf-field defined in (9) above. Then by using (16)-(18), we can obtain the following dispersion relation for this hybrid maser instability:

$$k_z^2 - k_{z0}^2 = -\frac{j\omega\mu_0}{P_E}\iint J \cdot E^* ds$$

$$= \frac{\omega\mu_0 e^2 n_0}{P_E}\{[j\frac{\omega_0^2}{\Omega}R_1 + (\omega_0 + \Omega)R_0\varphi_1 + R_0\omega_0 k_{z0} z_1] \cdot E_\varphi^* + \qquad (19)$$

$$[j\frac{(\omega - \Omega)v_{z0}}{R_0\Omega}R_1 + v_{z0}\varphi_1 + (\omega - \omega_0)z_1] \cdot E_z^*\}|_{R=R_0}$$

where we have defined the quantity:

$$P_E = \frac{1}{2}\iint E_1 \cdot E \ ds$$

The dispersion relation above is completely generalized for the physical system under consideration and is valid for any mode in that system.

III. SAMPLE NUMERICAL CALCULATIONS

It is instructive to begin by first numerically calculating the dispersion relation for the ion channel waveguide mode *without* the REB present. The dispersion relation for the waveguide mode, HE_{11}, for various values of the plasma density, n_p, are shown in Fig. 2. Here, the plasma was immersed in a 2-tesla axial magnetic field. We can see from Fig. 2 that the cutoff frequency of the waveguide modes increases with increasing n_p.

If one then injects an REB of $\beta_{//}$=0.06, the interaction region between the beam and the waveguide mode HE_{11} may be determined via the plot shown in Fig. 3(a). Fig. 3(b) is the linear growth rate of an electromagnetic wave for different beam densities, n_b, according to (19). The other operating parameters are listed in the figure. The variation of the maximum growth rate, G_{max}, with beam density is shown in Fig. 3(c).

Fig. (4) shows the variation of the maximum growth rate with the plasma density. We can see that there is a maximum growth rate for certain plasma densities. Fig.(4) also shows the variation of the frequency of the maximum growth rate for different plasma densities. We can see that that frequency is almost directly proportional to the density.

IV. DISCUSSION AND CONCLUSIONS

It is clear that, the highly relativistic case in which the ion force plays an important role deserves much attention. When $B_0 = 0$, we have the ion-channel maser (laser) instability. When $B_0 \neq 0$, the new hybrid instability mechanism described herein emerges. This paper shows that this new hybrid instability boasts at least five important and interesting features. First, for this hybrid maser, the electron cyclotron frequency is given by (4). This tells us that the energy dependence is neither $\gamma_0^{-1/2}$ nor γ_0^{-1}. It is rather *between* $\gamma_0^{-1/2}$ and γ_0^{-1}. This classifies this phenomenon as a special kind of negative mass effect.

According to reference [9], the energy dependence can be written as: γ_0^{-q}. That work defined the following three kinds of dependence: $q = +1$, the electron cyclotron maser instability; $q = 1/2$, the ion-channel maser instability; and $q = 0$, the Free Electron Laser instability. References [10] and [11] describe another case, the Electrostatic Electron Maser, for which, the energy dependence also has $q = 1/2$. This is due to the fact that both Ion-channel Lasers and Electrostatic ECR Masers are governed by a centripetal force. Now we have defined a new hybrid case, one for which $1/2 < q < 1$. Therefore, we have four different kinds of energy dependence for different instabilities: $q = +1$, $q = 1/2$, $q = 0$, and ($1/2 < q < 1$).

Second, (5) shows that for this hybrid instability, there are two singularities: $\Omega = \omega - k_{z0}v_{z0} - \omega_0$ and $\Omega^2 \cong (\omega_0 - \omega_C)^2 + (\omega_0^2 - (\omega_1^2)/\gamma_0)$. For the latter, the operating frequency, $\omega = \omega_c + k_{zo} v_{zo} + (\omega_c^2 + \omega_p^2 + (\omega_p^2)/\gamma_o)^{1/2}$, is very high and merits further study. Note also that for the ion-channel instability, there are three singularities:[6] $\Omega = 0$, $\Omega^2 - (4 - \beta_{\perp 0}^2)\omega_0^2 = 0$ and $\Omega^2 - \omega_e^2 = 0$ where $\omega_e^2 \equiv (1 + \gamma_0^2\beta^2)\omega_0^2$. While for the ECRM instability, there are also two singularities: $\Omega = 0$; $\Omega^2 - \omega_c^2 = 0$ (the Gyro-Peniotron instability).

A third interesting feature of this hybrid maser is that the working frequency is much higher than both ω_c and ω_0 as is shown in (4). Rough estimation shows that: $\omega \geq \dfrac{\omega_{c0}}{\gamma_0} + \dfrac{\omega_{c0}}{\gamma_0}(\dfrac{\overline{\omega}_p^2}{\omega_{c0}^2})$, or

$$\omega \approx \frac{\omega_{c0}}{\gamma_0} + \frac{\overline{\omega}_p}{\sqrt{\gamma_0}}, \text{ where } \overline{\omega}_p^2 = \frac{1}{2}\omega_p^2.$$

Fourth, in this hybrid case, the transverse interaction is always accompanied by the longitudinal interaction, since we must have both $\vec{E}_\perp$ and E_z, $\vec{J}_\perp$ and J_z [12].

Finally, it is obvious that since the ion-channel permits elevated electron-beam currents and densities, thus the output power of the maser can also be much elevated.

In conclusion, we have described a new scheme of electromagnetic wave generation by considering the effect of an ion channel in a magnetized plasma filled waveguide. The analysis yielded many important and interesting points. It is clear that this new hybrid instability should be studied in more detail.

REFERENCES

[1] D.H. Whittum, A.M. Sessler, and J.M. Dawson, "Ion-Channel Laser," *Phys. Rev. Lett.* , Vol. 64, pp. 2511-2514, 1990.

[2] K.R. Chen, T.C. Katsouleas, J.M. Dawson, "On the Amplification Mechanism of the Ion-Channel Laser," *IEEE Trans. Plasma Sci.* , Vol. 18, p. 837, 1990.

[3] D.H. Whittum, "Electromagnetic Instability of the Ion-Focused Regime," *Phys. Fluids B*, Vol. 4, p. 730, 1992.

[4] K.R. Chen and J.M. Dawson, "Ion-Ripple Laser," *Phys. Rev. Lett.*, Vol. 68, p. 29, 1992.

[5] K.R. Chen and J.M. Dawson, "Amplification Mechanism of Ion-Ripple Laser and its Possible Applications." *IEEE Trans. Plasma Sci.*, Vol. 21, p. 151, 1990.

[6] C.J. Tang, P.K. Lin, and Shenggang Liu, "Electromagnetic Instability in an Ion-Channel Electron Cyclotron Maser," *J. Phys. D:Appl. Phys.* , Vol. 29, pp. 90-94, 1996.

[7] J. Parashar, H.D. Pandey, A.K. Sharma, V.K. Tripathi, "Electromagnetic-Wave-Pumped Free-Electron Laser in a Plasma Channel," *J. Plasma Phys.*, Vol. 58, p. 613, 1997.

[8] Pallavi Jha and Punit Kumar, "Dispersion Relation and Growth in a Free-Electron Laser with Ion-Channel Guiding," *Phys. Rev. E* , Vol. 57, No. 2, p. 2256, 1998.

[9] K.R. Chen, J.M. Dawson, A.T. Lin, T. Katsouleas, "Unified Theory and Comparative Study of Cyclotron Masers, Ion-Channel Lasers, and Free-Electron Lasers," *Phys. Fluids. B*, Vol. 3, p. 1270, 1991.

[10] Liu Shenggang, "Electrostatic Electron Cyclotron Resonance Maser," *Scientic Sinica*, Vol. 6, p. 657, 1986.

[11] Liu Shenggang and Liu HongXue, "Electrostatic Free-Electron Laser," *Phys. Rev. A*, Vol. 36, p. 3837, 1987.

[12] Liu Shenggang, Zhu Dajun, Yan Yang, and Gao Hong, "Theory of Electron Beam Wave Interaction in Waveguide Filled with Plasma in Axial Magnetic Field," *Conf. Digest of the 22nd Int. Conf. On IR&MM-Waves*, H.P. Freund, Ed., Talk W-2.4, Wintergreen, VA. 20-25 July 1997.

[13] L. Shenggang, R.J. Barker, Z. Dajun, Y. Yang, and G. Hong, "Basic Theoretical Formulations of Plasma Microwave Electronics, Parts I & II", submitted to *IEEE Transactions on Plasma Science*, 1999.

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$\omega = \omega_p$, we also have $p_2 = 0$. But the curve **a** of $p_2 = 0$ is the line of equation $\beta^2 = k^2 \frac{\omega^2 + \omega\omega_c - \omega_p^2}{\omega(\omega + \omega_c)}$ (see section V and Appendix), it is one continuos line, not two lines as it was shown in [4]-[6], [8] (for example, Fig. 1 in [5]).

Fig. 5 shows rather complete dispersion curves. Dashed lines show three special critical lines: D=0 and $p_2 = 0$. Here again there is difference between our calculations with that given in Fig. 2 of [5]. Mainly, the cut-off frequency of HE_{11} mode is below ω_1, so the mode dispersion curve will go across the line of $p_2 = 0$. We have also found that there is no HE_1 mode, and the curve for HE_1 mode given in Fig.2 of [5] is actually the line of $p_2 = 0$.

From Fig. 5, we can see that there are mainly three kind of waves: the first is plasma modes their cut-off frequencies are zero. The second is the modes their cut-off frequencies are lower than ω_h, these modes, include EH modes and HE modes, are cyclotron wave modes. The dispersion curves of these cyclotron waves will be condensed between ω_h and $\mathrm{Max}(\omega_c, \omega_p)$. All these dispersion curves can not go through ω_h line. The third is the modes their cut-off frequencies are higher than ω_h. They are all waveguide modes. Fig. 6(a) and 6(b) show some waveguide waves, there is also difference with that given in Fig. 3 of [5], the mode HE_{01} should be below mode EH_{02}.

Fig. 7 and Fig. 8 show some waveguide waves the cut-off frequencies of which are close to ω_h. Again there are differences between our calculations with that given in Fig. 4(a) and Fig. 5 of [5]. It seems that in [5] Fig. 4(a) the curve EH_{11} was confused with the line of D=0, $p_2 = 0$, and the curve HE_1 is really the dispersion curve of HE_{11}.Fig. 8 shows that many EH modes are cyclotron modes.

It should be mentioned that in order to easily make comparison, all parameters used in our calculations are the same as that used in [5].

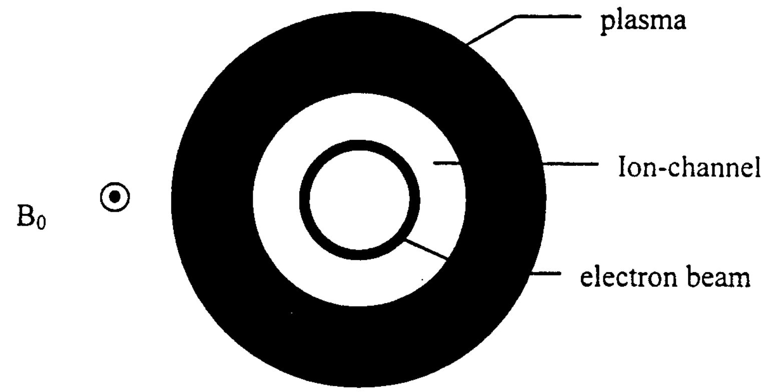

FIGURE 1 - Barker

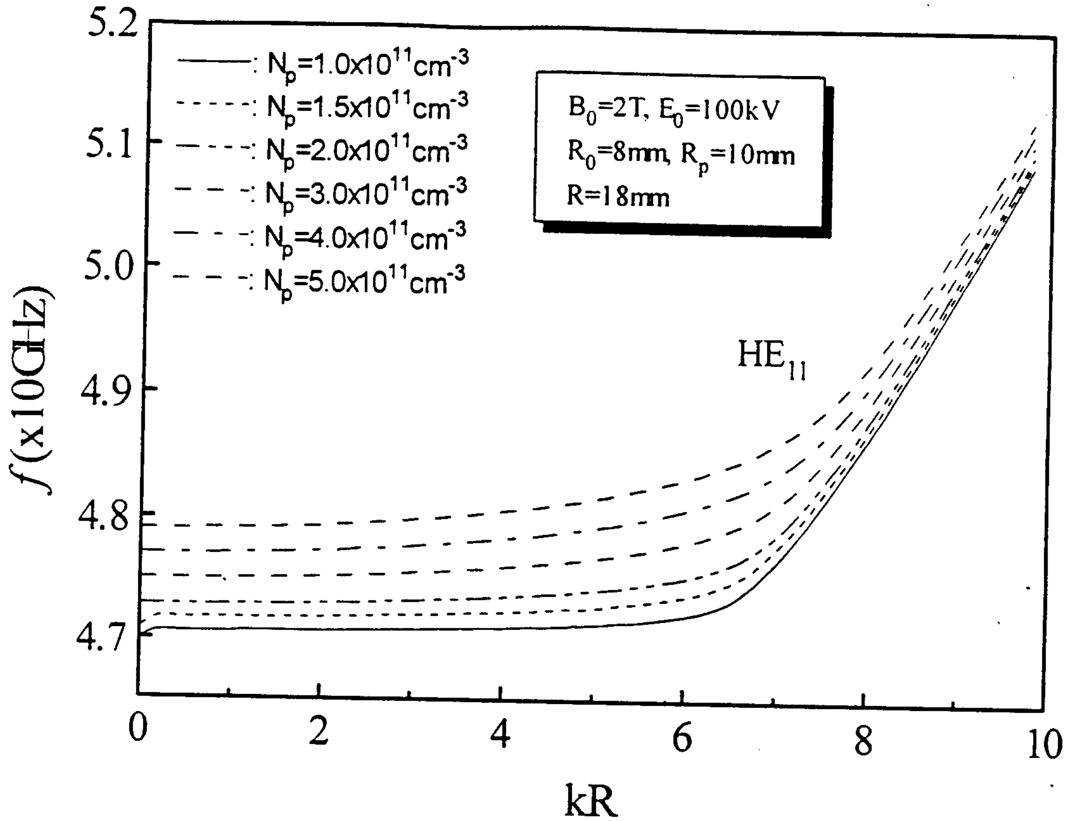

FIGURE 2 - Barker

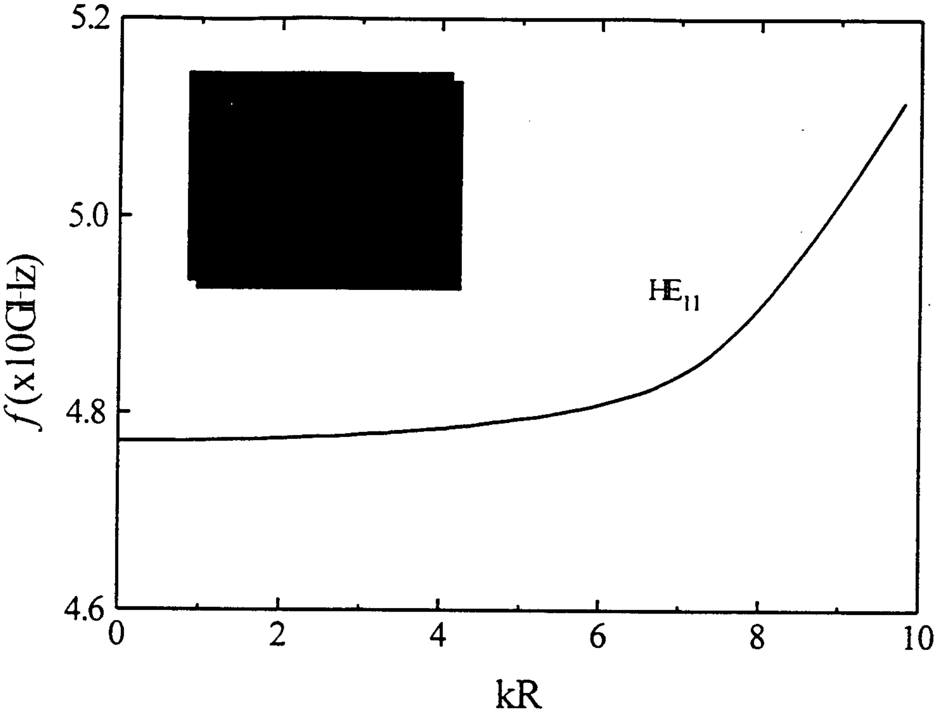

FIGURE 3(a) - Barker

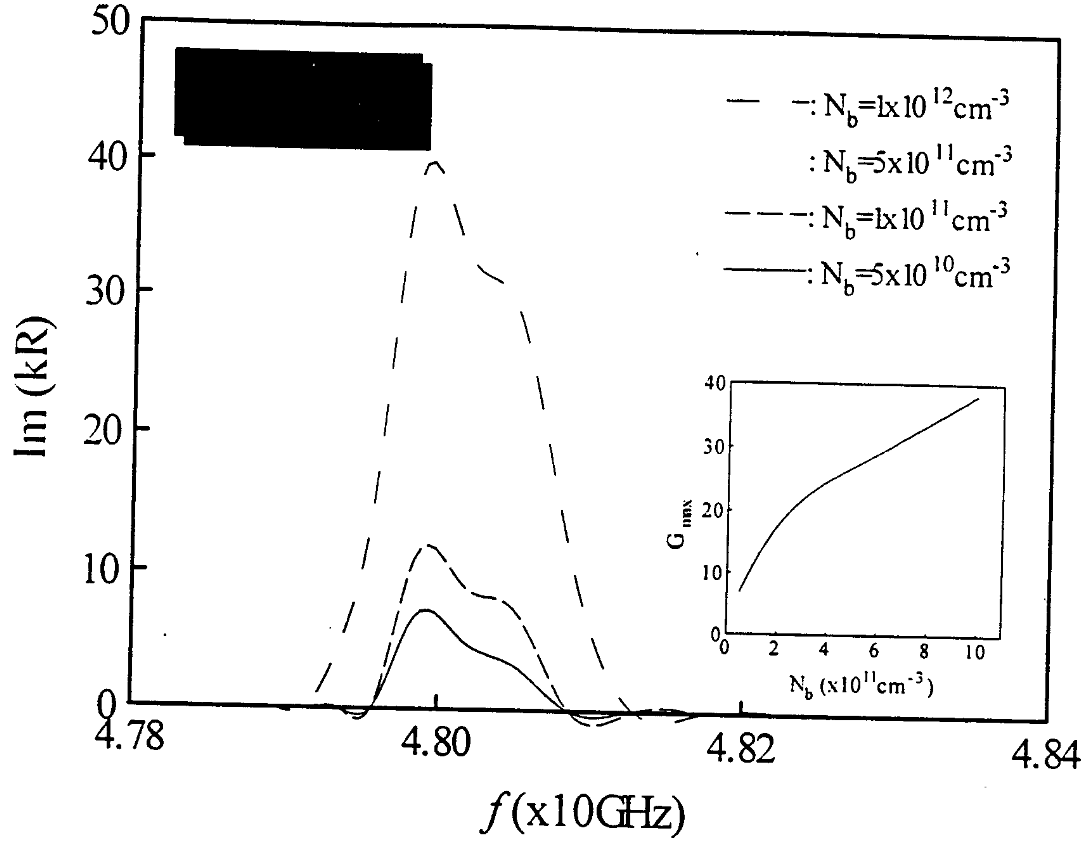

FIGURE 3 (b) & 3(c)inset - Barker

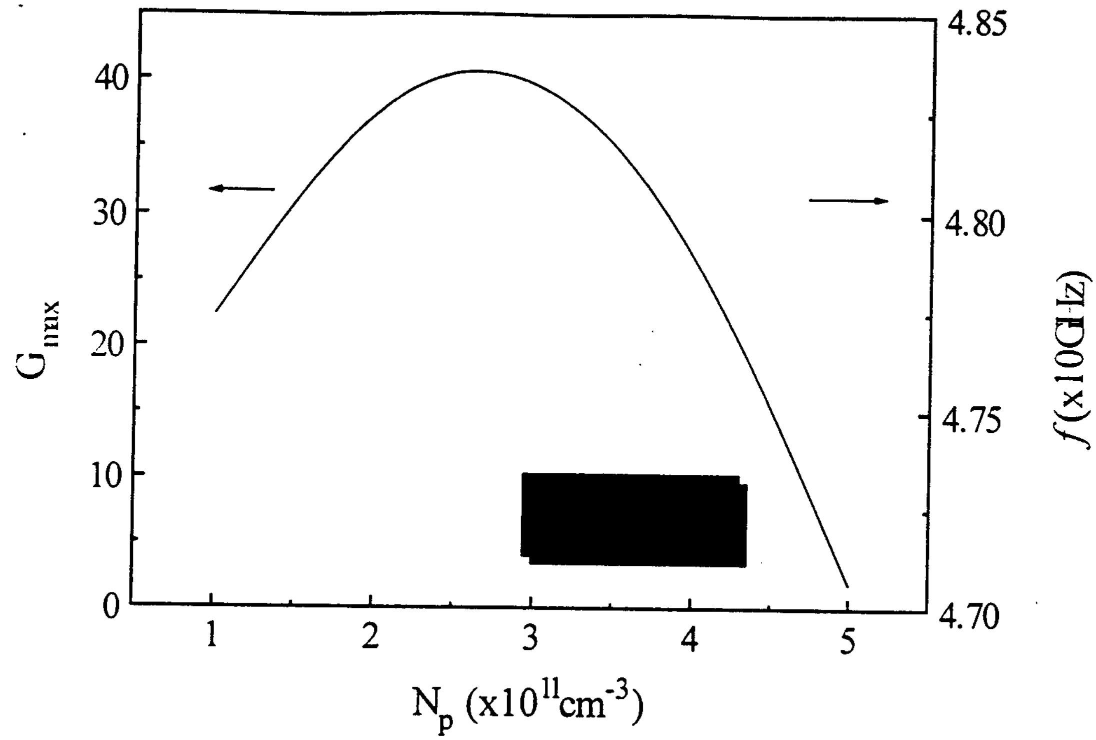

FIGURE 4. - Barker

5. Shenggang Liu, Robert J. Barker, Karl H. Schoenbach, Guofen Yu, and Chaoyu Liu, "Electromagnetic Characteristics of an Individual Spherical Biological Cell".

14

Electromagnetic Characteristics of an Individual Spherical Biological Cell[+,++]

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Theoretical study on electromagnetic characteristics of an individual spherical biological cell is given in the paper. The field in a cell induced by extra electrostatic field, the electromagnetic scattering for a plane wave due to a cell and the electromagnetic resonance condition of a cell, as an electromagnetic cavity, are analyzed. The possible application of the theory is discussed.

1. Introduction

Study on biological body and cells has become an attractive subject recently. The physics and electronics prove to be powerful and efficient in the research of this field. In this paper a theoretical study on the electromagnetic characteristics of a simplified cell is given. As shown in Fig.1 (a), the cell model is divided into two regions: region I, the inner part of the cell with radius b; region II, cell membrane with inner radius b and outer radius a. The region outside the cell is referred to as region III. The dielectric constants, permeability and conductivity of the three regions are indicated by ε_i, μ_i, and σ_i, respectively. The subscript "i" takes 1,2,3, which represent different regions. By using the Maxwell's equations the electromagnetic characteristics can be analyzed.

The paper is organized as below: Section 1 is a brief introduction. Section 2 deals with a cell in an electrostatic field. The analysis of the scattering wave by a cell to a plane E.M. wave is given in section 3. And in section 4, the cell is considered a E.M. cavity, the resonance condition is given. Section 5 deals with the possible applications of the theory. Section 6 is the conclusion.

2. A cell in electrostatic field

Consider an individual cell in an electrostatic field:

$$\vec{E} = E_0 \vec{e} \tag{1}$$

It is obvious that, in a spherical coordinate system (r, φ, θ), the field in each region

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should be axial symmetric, i.e., $\partial/\partial\varphi = 0$. The scalar potential of the applied electric field can be indicated as:

$$V_0 = -E_0 r\cos\theta = -E_0 r P_1(\cos\theta) \tag{2}$$

where $P_1(\cos\theta)$ is the Legendre function. Then, the electrostatic potential in the three regions can be written as:

$$V_1 = \sum_{n=0}^{\infty} A_n r^n P_n(\cos\theta) \qquad (0 \le r \le b) \tag{3}$$

$$V_2 = \sum_{n=0}^{\infty}\left[B_n r^{-(n+1)} + C_n r^n\right]P_n(\cos\theta) \qquad (b \le r \le a) \tag{4}$$

$$V_3 = \sum_{n=0}^{\infty} D_n r^{-(n+1)} P_n(\cos\theta) - E_0 r P_1(\cos\theta) \qquad (r \ge a) \tag{5}$$

The constants A_n, B_n, C_n and D_n can be determined by using the following boundary conditions:

$$E_\theta^{III} = E_\theta^{II} \quad (\sigma_3 + j\omega\varepsilon_3)E_r^{III} = (\sigma_2 + j\omega\varepsilon_2)E_r^{II} \qquad (r = a) \tag{6}$$

$$E_\theta^{II} = E_\theta^{I} \quad (\sigma_2 + j\omega\varepsilon_2)E_r^{II} = (\sigma_1 + j\omega\varepsilon_1)E_r^{I} \qquad (r = b) \tag{7}$$

The permitivity constants are assumed independent on any coordinates, we obtain:

[the matrix] (8)

|A|=
B|=
C|= (9)
D|=|

All the other A_n, B_n, C_n and D_n should be zero.

Therefore, once the constants $\varepsilon_1, \mu_1, \sigma_1$; $\varepsilon_2, \mu_2, \sigma_2$; $\varepsilon_3, \mu_3, \sigma_3$ are given, the field outside and inside the cell can be found.

3. Electromagnetic scattering wave by a biological cell

Assume that a cell is under radiation of a plane electromagnetic wave:

$$\vec{E} = \vec{E}_0 e^{j(\omega t - kz)} = \vec{E}_0 e^{j\omega t - jkr\cos\theta} \tag{10}$$

For a plane wave, we have:

$$\begin{cases} \vec{E} = E_0 \vec{e}_x e^{j\omega t - jkr\cos\theta} \\ \vec{H} = \dfrac{k}{\omega\mu} E_0 \vec{e}_y e^{j\omega t - jkr\cos\theta} \end{cases} \tag{11}$$

The plane wave should be expanded into two parts with different polarization in r direction: TM and TE waves. In spherical coordinate system, the TM and TE waves can be written as follows:

$$r\hat{V}_0 = \frac{E_0 \cos\varphi}{\omega\mu} \sum_{n=1}^{\infty} \frac{j^{-n}(2n+1)}{n(n+1)} \hat{J}_n(kr) P_n^1(\cos\theta) \tag{12}$$

for TM wave; and

$$r\hat{U}_0 = \frac{E_0 \sin\varphi}{k} \sum_{n=1}^{\infty} \frac{j^{-n}(2n+1)}{n(n+1)} \hat{J}_n(kr) P_n^1(\cos\theta) \tag{13}$$

for TE wave.

Here $r\hat{V}_0$ and $r\hat{U}_0$ are Debye potentials for TM and TE, respectively. $\hat{J}_n(kr)$ is spherical Bessel function, and $P_n^1(\cos\theta)$ is associated Legendre function.

Outside the cell there exists scattering wave, the Debye potentials in this region can be written as:

$$\begin{cases} r\hat{V}_3 = \cos\varphi \sum_{n=1}^{\infty} \left[\alpha_n \hat{J}_n(k_3 r) + A_n \hat{H}_n^{(2)}(k_3 r)\right] P_n^1(\cos\theta) & (TM) \\ r\hat{U}_3 = \sin\varphi \sum_{n=1}^{\infty} \left[\beta_n \hat{J}_n(k_3 r) + \overline{A}_n \hat{H}_n^{(2)}(k_3 r)\right] P_n^1(\cos\theta) & (TE) \end{cases} \tag{14}$$

In region II, we have:

$$\begin{cases} r\hat{V}_2 = \cos\varphi \sum_{n=1}^{\infty} \left[C_n \hat{J}_n(k_2 r) + D_n \hat{H}_n^{(2)}(k_2 r)\right] P_n^1(\cos\theta) & (TM) \\ r\hat{U}_2 = \sin\varphi \sum_{n=1}^{\infty} \left[\overline{C}_n \hat{J}_n(k_2 r) + \overline{D}_n \hat{H}_n^{(2)}(k_2 r)\right] P_n^1(\cos\theta) & (TE) \end{cases} \tag{15}$$

And in region III, the kern of the cell, the Debye potentials can be written as:

$$\begin{cases} r\hat{V}_1 = \cos\varphi \sum_{n=1}^{\infty} F_n \hat{J}_n(k_1 r) P_n^1(\cos\theta) & (TM) \\ r\hat{U}_1 = \sin\varphi \sum_{n=1}^{\infty} \overline{F}_n \hat{J}_n(k_1 r) P_n^1(\cos\theta) & (TE) \end{cases} \tag{16}$$

where:

$$\begin{cases} \alpha_n = \dfrac{E_0}{\omega\mu} \dfrac{j^{-n}(2n+1)}{n(n+1)} \\ \beta_n = \dfrac{E_0}{k} \dfrac{j^{-n}(2n+1)}{n(n+1)} \end{cases} \tag{17}$$

$\hat{H}_2^{(2)}(k_i r)$ is spherical Hankel functon of second kind, it represents an outgoing wave. $k_i^2 = \omega^2 \varepsilon_i \mu_i - j\omega\mu_i\sigma_i$, here $i = 1,2,3$.

It can be proved that, except the nonlinear media, there is no coupling between TM and TE waves, so we can deal with them separately.

By using the boundary conditions of fields, we can derive the expressions of the field coefficient as follows:

$$A_n = \frac{\Delta_v^{(1)}}{\Delta_v},\ C_n = \frac{\Delta_v^{(2)}}{\Delta_v}, D_n = \frac{\Delta_v^{(3)}}{\Delta_v}, F_n = \frac{\Delta_v^{(4)}}{\Delta_v} \tag{18}$$

$$B_n = \frac{\Delta_u^{(1)}}{\Delta_u}, \; \overline{C}_n = \frac{\Delta_u^{(2)}}{\Delta_u}, \overline{D}_n = \frac{\Delta_u^{(3)}}{\Delta_u}, \overline{F}_n = \frac{\Delta_u^{(4)}}{\Delta_u} \tag{19}$$

where:

$$\begin{aligned}
\Delta_v = {} & \hat{H}_n^{(2)}(k_2a).\hat{J}_n(k_1b)\left[\hat{J}_n(k_2a)\hat{H}_n^{(2)'}(k_2b) - \hat{H}_n^{(2)}(k_2a).\hat{J}'_n(k_2b)\right] + \\
& + \tau_2.\hat{J}'_n(k_1b)\left[\hat{H}_n^{(2)}(k_2a).\hat{J}_n(k_2b) - \hat{J}_n(k_2a)\hat{H}_n^{(2)}(k_2b)\right] + \\
& \tau_3.\hat{J}'_n(k_2a)\hat{H}_n^{(2)}(k_3a)\left[\tau_2.\hat{J}'_n(k_1b)\hat{H}_n(k_2b) - \hat{J}_n(k_2b)\hat{H}_n^{(2)'}(k_2b)\right] + \\
& + \tau_3 H_n^{(2)'}(k_2a)\hat{H}_n^{(2)}(k_3a)\left[\tau_2.\hat{J}'_n(k_1b).\hat{J}_n(k_2b) - \hat{J}_n(k_1b).J'_n(k_2b)\right]
\end{aligned} \tag{20}$$

$$\begin{aligned}
\Delta_v^{(1)} = {} & \alpha_n \hat{J}'_n(k_3a)\left[\hat{H}_n^{(2)}(k_2a)\hat{J}'_n(k_2b).\hat{J}_n(k_1b) + \tau_2.\hat{J}'_n(k_1b)\hat{H}_n^{(2)}(k_2b).\hat{J}_n(k_2b) - \right. \\
& \left. - \hat{J}_n(k_2a)\hat{H}_n^{(2)'}(k_2b).\hat{J}_n(k_1b) - \tau_2.\hat{J}'_n(k_1b)\hat{H}_n^{(2)}(k_2a).\hat{J}_n(k_2a)\right] + \\
& + \alpha_n \tau_3.\hat{J}_n(k_3a).\hat{J}_n(k_2a)\left[\hat{J}_n(k_1b)\hat{H}_n^{(2)'}(k_2b) - \hat{J}'_n(k_1b)\hat{H}_n^{(2)}(k_2b)\right] + \\
& + \alpha_n \tau_3.\hat{J}_n(k_3a)\left[\hat{J}'_n(k_2b).\hat{J}_n(k_1b) - \hat{J}'_n(k_1b).J_n(k_2b)\right]
\end{aligned} \tag{21}$$

$$\begin{aligned}
\Delta_v^{(2)} = {} & \alpha_n.\hat{J}_n(k_3a)\hat{H}_n^{(2)'}(k_3a)\left[\hat{H}_n^{(2)}(k_2b).\hat{J}_n(k_1b) - \tau_2.\hat{J}'_n(k_1b)\hat{H}_n^{(2)}(k_2b)\right] \\
& + \alpha_n.\hat{J}'_n(k_3a)\hat{H}_n^{(2)}(k_3a)\left[\tau_2.\hat{J}'_n(k_2b)\hat{H}_n^{(2)}(k_2b) - \hat{J}'_n(k_1b)\hat{H}_n^{(2)'}(k_2b)\right]
\end{aligned} \tag{22}$$

$$\Delta_v^{(3)} = -j\alpha_n\left[\tau_2.\hat{J}'_n(k_1b).J_n(k_2b) - J'_n(k_2b).\hat{J}_n(k_1b)\right] \tag{23}$$

$$\Delta_v^{(4)} = -\alpha_n^2 \tag{24}$$

in which:

$$\begin{cases} \tau_2 = \dfrac{k_1(\sigma_2 + j\omega\varepsilon_2)}{k_2(\sigma_1 + j\omega\varepsilon_1)} \\ \tau_3 = \dfrac{k_2(\sigma_3 + j\omega\varepsilon_3)}{k_3(\sigma_2 + j\omega\varepsilon_2)} \end{cases} \tag{25}$$

Let $\tau'_2 = k_1/k_2, \tau'_3 = k_2/k_3$ replace τ_2 and τ_3, then the expressions of $\Delta_u^{(1)}$, $\Delta_u^{(2)}$, $\Delta_u^{(3)}$, $\Delta_u^{(4)}$ and Δ_u are given by those of $\Delta_v^{(1)}, \Delta_v^{(2)}, \Delta_v^{(3)}, \Delta_v^{(4)}$ and Δ_v, respectively.

4 A cell as an electromagnetic cavity

An individual biological cell can be considered as a spherical cavity, a special kind of EM cavity. Based on the theory obtained in last section we can derive some electromagnetic parameters of the special resonance cavity. One of the most difficulties to build up a theory for the cell cavity is that what are the criteria of the "resonance" for a cell. There are three approaches ~~need~~ for a dielectric cavity: (1) magnetic wall approximation (magnetic wall open); (2) cut-off waveguide; (3) cut-off radial transmission line approximation. For a biological cell, the first and second conditions seem not to be acceptable. The third one depends on the dielectric constant ε and permeability μ of the cell, in particular, the ε_2 of the cell's membrane. The approximation is very good, provided that ε_2 is large, say 50~100. But if ε_2 is ~~too~~ small, the accuracy will be poor, because this leads to that more E.M. energy spreads outside the cell, and the resonance curve will not appear like a peak. Here we suppose that the dielectric constant of the cell membrane is large enough. The fields outside and inside the cell can be expressed as:

$$\begin{cases} r\hat{V}_3 = \sum_{n=0}^{\infty}\sum_{m=0}^{n} A_{mn}\hat{H}_n^{(2)}(k_3 r)P_n^m(\cos\theta)e^{jm\varphi} \\ r\hat{U}_3 = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \bar{A}_{mn}\hat{H}_n^{(2)}(k_3 r)P_n^m(\cos\theta)e^{jm\varphi} \end{cases} \qquad (r \geq a) \qquad (26)$$

$$\begin{cases} r\hat{V}_2 = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\left[C_{mn}\hat{J}_n(k_2 r) + D_{mn}\hat{H}_n^{(2)}(k_2 r)\right]P_n^m(\cos\theta)e^{jm\varphi} \\ r\hat{U}_2 = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\left[\bar{C}_{mn}\hat{J}_n(k_2 r) + \bar{D}_{mn}\hat{H}_n^{(2)}(k_2 r)\right]P_n^m(\cos\theta)e^{jm\varphi} \end{cases} \qquad (b \leq r \leq a) \qquad (27)$$

$$\begin{cases} r\hat{V}_1 = \sum_{n=0}^{\infty}\sum_{m=0}^{n} B_{mn}\hat{J}_n(k_1 r)P_n^m(\cos\theta)e^{jm\varphi} \\ r\hat{U}_1 = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \bar{B}_{mn}\hat{J}_n(k_2 r)P_n^m(\cos\theta)e^{jm\varphi} \end{cases} \qquad (0 \leq r \leq b) \qquad (28)$$

The boundary conditions are:

$$\begin{cases} E_\theta^{III} = E_\theta^{II} \\ E_\varphi^{III} = E_\varphi^{II} \\ H_\theta^{III} = H_\theta^{II} \\ H_\varphi^{III} = H_\varphi^{II} \end{cases} \qquad (r = a) \qquad (29)$$

$$\begin{cases} E_\theta^{II} = E_\theta^{I} \\ E_\varphi^{II} = E_\varphi^{I} \\ H_\theta^{II} = H_\theta^{I} \\ H_\varphi^{II} = H_\varphi^{I} \end{cases} \qquad (r = b) \qquad (30)$$

By using the above boundary conditions we can obtain the field expansion coefficients, i.e. A_n, $\bar{A}_n$, B_n, $\bar{B}_n$, C_n, $\bar{C}_n$, D_n and $\bar{D}_n$.

As we described above, when almost all the energy is kept within a cell, the cell acts like a resonator. So the resonant condition should be

$$E_{\varphi}^{II} H_{\theta}^{II} - E_{\theta}^{II} H_{\varphi}^{II} = 0 \qquad (r = a) \tag{31}$$

5 possible applications of the theory

The theory given in the paper is an attempt to using the well-developed electromagnetic theory to study the electric or electromagnetic characteristics of biological cell/cells. The theory may have some attractive applications in the biological field. For example, the equivalent circuit in [1] may be derived from the theory as follows:

Suppose a cell is under the extra field $E_0 \vec{e}_z$, the field components in the cell membrane and in the inner part have been given in section 2. The energy stored in the cell membrane is:

$$\zeta_m = \frac{1}{2}\varepsilon_0\varepsilon_2 \int (E_r^2 + E_{\varphi}^2 + E_{\theta}^2) dV \tag{32}$$

The integration is along the volume of membrane. According to the stored energy expression of a capacitor, we can express the equivalent capacitance of the cell membrane as:

$$C_m = \frac{\varepsilon_0\varepsilon_2 \int (E_r^2 + E_{\varphi}^2 + E_{\theta}^2) dV}{\langle U \rangle^2} \tag{33}$$

$\langle \cdots \rangle$ denotes taking average.

The current density within the membrane and in the cell's kern is:

$$J_i = \sigma_i (E_{ri} \cos\theta + E_{\theta i} \sin\theta) \tag{34}$$

Then the average resistance of the cell is:

$$R_1 = \frac{\langle U_1 \rangle}{\langle I_1 \rangle} \tag{35}$$

The conductivity of cell membrane is:

$$G_m = \frac{\langle I_1 \rangle}{\langle U_1 \rangle} \tag{36}$$

6 conclusion

Theoretical study on electromagnetic characteristics of an individual biological cell has been worked out. A cell in electrostatic field is studied. The results obtained can be used to find the parameters of the equivalent circuit of the cell. The scattering wave of a cell to a plane wave has been obtained. It may be used to study the behavior of a cell under the irradiation of electromagnetic wave. For example, by measuring the scattering wave we can find the parameters of the cell $(\mu, \varepsilon, \sigma)$. A cell can be considered as a special cavity when its radiation loss is very small. The results of the paper, therefore, are valuable for understanding the electrical characteristics of the cell and provide a theoretical base for further study on biological cell.

References:

1. K.H. Schoenbach, IEEE Trans. PS-25, 2,284, 1997.
2. G.A. Hoffman and G.A. Evans, IEEE Eg. Med. Biol. Mag. P.6, Dec. 1986.
3. Ulrich Zimmermann; G.A. Neil: "Electromanipulation of Cell", CRC Press, 1996.
4. Proceeding of the First International Symposium on Medical/Biological Treatments Using Electromagnetic Fields and Ionized Gases, (ElectroMed'99), Norfolk, Apr.12-14, 1999.
5. Paul Lorrain and Dale R. Corson: "Electromagnetic Field and Waves", W. H. Freeman and company, San Francisco, 1970.

6. **Shenggang Liu, Robert J. Barker, Yan Yang, and Zhu Dajun, "A New Type of Waves in Magnetized Plasma Waveguide".**

A New type of Waves in Magnetized Plasma Waveguide*

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Abstract A new type of wave containing a quasi-static part has been found. the features of this new type of waves are examined and studied in this paper. This type of waves has not been reported in published papers to date. Characteristics of these waves are analyzed. The theoretical analysis is found to be consistent with numerical calculations.

Key Words Magnetized Plasma Waveguide Quasi-Static New type of Wave

Summary

Wave propagation in a waveguide filled with plasma immersed in a magnetic field has been an important subject to study in plasma physics and microwave electronics [1]-[7]. It was found that the efficiency of high power microwave devices can be increased greatly when filled with plasma. Research interest in this subject continues to grow. There are three kinds of waves in a magnetized plasma waveguide: (1)Low frequency plasma wave, $\omega < \min(\omega_p, \omega_c)$. (2)Electron cyclotron wave, $\omega_p < \omega < \omega_c$. (3)Waveguide wave, $\omega > \omega_h$, where $\omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$ is electron plasma frequency, $\omega_c = \frac{eB}{m}$ is electron cyclotron frequency, n is electron density, e and m are electron charge and mass, e_0 is permitty in vacuum, $\omega_h = \sqrt{\omega_p^2 + \omega_c^2}$ is the upper hybrid frequency.

Plasma has great influence on the dispersion characteristics of electromagnetic waves in a waveguide. There are no stand-alone TE or TM wave unless the magnetic field is zero or infinite, according to waveguide theory. The electromagnetic wave in a empty waveguide satisfies Helomhtz's equation rather than Laplace's equation, However, we find a new type of wave in a waveguide filled with magnetized plasma which exists at discrete frequencies and with quasi-static characteristics satisfying Laplace's equation.

The theory given in this paper shows there exists a new kind of waves in a magnetized plasma waveguide, the new kind of wave contains a quasi-static part of field components satisfying

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Laplace's equation, and these waves can only exist at a series of discrete frequencies. The numerical calculations verify the existence of this new kind of wave.

Physically, this shows that in a waveguide without plasma, the quasi-static field (such as a TEM wave) cannot exist and only fast waveguide modes can propagate. However,When the waveguide is plasma filled, there are a large number of charged particles in the space inside the waveguide. So the quasi-static field can exist and it is closely related to the plasma. The importance of this new wave is that it can propagate at the discrete frequencies satisfying $\nabla_{\perp}^2 E_z = 0$ and keeps the continuity of the dispersion curve.

Mathematically, it indicate that D=0 is not a singularity, the solutions of wave equation exists and there is no break of wave propagation, but the wave mode is changed significantly at the discrete frequencies.

We can see that wave propagation along a magnetized plasma waveguide has quite unusual dispersion characteristics.

References

1 Liu Shenggang, Lee J K ,Zhu Dajun. Theory of Wave propagation along waveguide filled with plasma. Science in china E,1996,26(4):348□363

2 Ivanov S T, Alexov E G. Electromagnetic waves in a plasma waveguide. J. of Plasma Physics, 1990, 43(1): 51□66

3 Ivanov S T ,Alexov E G. Electromagnetic waves in fully-filled plasma waveguide. J of Plasma (in Russian), 1989,15:1131□1142

4 Uhm H S. Wave dispersion theory in a plasma column bounded by a cylindrical waveguide. J Appl Phys. 1988, 64(3):1108□1115

5 Margot J,Moisan M. Electromagnetic surface waves for a new approach to the investigation of plasma produced at electron cyclotron resonance(ECR). J Physics D, Applied Physics,1991,24:1765□1788

6 Liu Shenggang, Lee J K *et al.* Theory of wave propagation along corrugated waveguide filled with plasmas immersed in an axial magnetic field. IEEE trans on plasma Science, 1996.24(3):918□923

7 Liu Shenggang, Zhu Dajun. An alternative analysis for wave propagation along plasma waveguide in finite magnetic field. Chinese Journal of Electronics,1996,5(2):1□7

7. Shenggang Liu, Robert J. Barker, Gao Hong, and Yan Yang, "Electromagnetic Wave Pumped Ion-Channel Free-Electron Laser".

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Electromagnetic Wave Pumped Ion-Channel Free-Electron Laser ***

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Summary

Ion-channel guiding of an electron beam in a free-electron laser can eliminates the need for conventional focusing magnets, thereby reducing the capital and running cost.[1] Moreover, the presence of the ion-channel allows beam currents higher than the vacuum limit and also helps radiation guiding.[2] So there should be higher efficiencies and lower wavelengths for ion-channel guiding FEL. The linear theory of a wiggler FEL with ion-channel guiding has been presented by P. Jha *et al.*[3] They have shown that the steady-state transverse electron motion generates a similar set of stable and unstable electron orbits, and a substantial enhancement in peak growth rate could be obtained when the ion-channel frequency approaches the wiggler frequency. Comparing with wiggler FEL, there is a shorter period in an electromagnetic wave pumped FEL. So under the same injection energy of electrons, one can get a more shorter laser wavelength output. Motivated by the benefits, the study on electromagnetic wave pumped ion-channel free-electron laser (EPIC-FEL) will be more useful. Radiation generation in an ion-channel was first proposed by Whittum and Sessler in 1990.[4] Then Whittum studied the electromagnetic wave instability of the ion-focused regime in detail.[5] The Ion-ripple Laser was proposed by Chen and Dawson in 1992.[6,7] Tang *et al.*[8] studied electromagnetic wave instability in an ion-channel electron cyclotron maser (ECM) and proposed the concept of the ICECM. Recently, The ion-channel hybrid instability was proposed by Liu et al.[9] It was indicated by K.R. Chen[10] that two bunching mechanisms exist for wave amplification in the ion-channel laser, cyclotron maser and FEL. There should be a competition between axial bunching and azimuthal bunching, however, axial bunching is mainly considered in FEL, including the discussion in [3].

In this paper, theoretical study on electromagnetic wave pumped ion-channel free-electron laser (EPIC-FEL) is presented. The physical mechanism responsible for the generation of coherent radiation in the EPIC-FEL is described and the fundamental role of the ponderomotive wave in bunching and trapping the beam is emphasized. The dispersion relation of the EPIC-FEL has been obtained and growth rates are calculated for different parameters. It shows that EPIC-FEL has very bright future.

The EPIC-FEL dispersion relation can be obtained as:

$$(\omega^2 - c^2k^2 - \frac{\omega_b^2}{\gamma_0\Omega_{i1}})\cdot\{[(\omega-\omega_0)-(k+k_0)v_{z0}]^2 - \frac{\omega_b^2}{\gamma_0\gamma_{z0}^2}\}$$
$$= \frac{\omega_b^2}{\gamma_0}\frac{V_0^2}{\Omega_{i0}}(k+k_0)[\frac{k_0}{\Omega_{i1}} + \frac{k}{\Omega_{i0}} - \frac{\omega\beta_{z0}}{c\Omega_{i0}} + \frac{\omega_0\beta_{z0}}{c\Omega_{i1}} - \frac{\omega_i^2}{\Omega_{i1}\Omega_{i0}\Omega_0}(\frac{\Omega_0}{\Omega_1}-1)] \quad (1)$$

*** presented at 24th Int. Conf. on IRMMW. Montery. CA. USA.

where $V_0 = \frac{|e| A_0}{m\gamma_0 c}$ is EM wave wiggle velocity. When $\omega_i = 0$, we may get the vacuum case. In this case, $\Omega_{i0} = \Omega_{i1} = 1$, the above equation reduces to the conventional EM wave pumped FEL dispersion relation[11]. Under the conditions of resonation for backward wave pumped, i.e. $k = \frac{(1+\beta_{z0})^2}{(1-\beta_{z0}^2)} k_0$, we can get $\Omega_0 = \Omega_1$ and $\Omega_{i0} = \Omega_{i1}$. Then the eq. (1) is reduced to

$$(\omega^2 - c^2k^2 - \frac{\omega_b^2}{\gamma_0 \Omega_{i0}}) \cdot \{[(\omega - \omega_0) - (k + k_0)v_{z0}]^2 - \frac{\omega_b^2}{\gamma_0 \gamma_{z0}^2}\} = \frac{\omega_b^2}{\gamma_0} \frac{V_0^2}{\Omega_{i0}^2} 4kk_0 \qquad (2)$$

We have calculated the above equation, the results show that the presence of ion-channel may lead to substantial enhancements in growth rate as $\omega_i \approx \Omega_0$. These agreement with results obtained in Ref.[3], however the values of the peak growth rates and resonate frequencies are larger than that of in the former case when $A_w = A_0$. For all kinds of FEL, we can derive $\lambda \approx \lambda_0 / 2\gamma_z^2$, where λ and λ_0 are the radiation and pump EM wave wavelengthes in the kind of EM pumped FEL case or radiation wave wavelength and wiggler period in the kind of wiggler FEL case, respectively. In order to operate FEL at low wavelength, two methods have been used. First, reducing the wiggler period; We know it is very difficult to obtain for conventional wiggler FEL case. However it has no problem for conventional EM wave pumped FEL. Second, increasing the beam energy. But it may cause lower efficiencies. However, in ion-channel kind of FELs, the ion-channel can aid the electron beam focusing and can compensate the efficiency lose by adjusting the plasma frequency (eq.(2)), so under high beam energy case, this kind of FEL can be operation at very low radiation wavelength without lose many efficiencies; meanwhile, the decreasing of pump wavelength is very easy to accomplish for EPIC-FEL. So with such benefits, we believe that EPIC-FEL will has a very bright future.

References

1 K. Takayama and S. Hiramatsu, Phys. Rev. A 37, 173(1988)

2 Y. Seo, V.K. Tripathi, and C.S. Liu. Phys. Fluids B1, 221(1989)

3 P. Jha and P. Kumar. Phys. Rev. E 57, 2256(1998)

4 D.H. Whittum and A.M. Sessler, *Phys. Rev. Lett.* 64, 2511(1990)

5 K.R. Chen, T.C. Katsouleas, J.M. Dawson, *IEEE Trans. Plasma Sci.* 18, 837(1990)

6 D.H. Whittum. *Phys. Fluids B* 4, 730(1992)

7 K.R. Chen and J.M. Dawson, *Phys. Rev. Lett.* 68, 29(1992)

8 C.J. Tang, P.K. Liu, S.G. Liu, *J. Phys. D:Appl. Phys.* 29, 90(1996)

9 S.G. Liu and R.J. Barker, 23rd Int. Conf. on IR/MM waves, Essex Univ. Sept. 7-11, 1998

10 K.R. Chen, J.M. Dawson, A.T. Lin, T. Katsouler, *Phys. Fluids. B* 3, 1270(1991)

11 V.L. Braman, G.G. Denisov, N.S. Ginzburg, *Int. J. Electron.* 59, 247(1985)

10

Electromagnetic Wave Pumped Ion-Channel Free-Electron Laser

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Abstract

Theoretical study on electromagnetic wave pumped ion-channel free-electron laser (EPIC-FEL) is presented. The physical mechanism responsible for the generation of coherent radiation in the EPIC-FEL is described and the fundamental role of the ponderomotive wave in bunching and trapping the beam is emphasized. The dispersion relation of the EPIC-FEL has been obtained and growth rates are calculated for different parameters. It shows that EPIC-FEL has very bright future.

I. Introduction

Ion-channel guiding of an electron beam in a free-electron laser can eliminates the need for conventional focusing magnets, thereby reducing the capital and running cost.[1] Moreover, the presence of the ion-channel allows beam currents higher than the vacuum limit and also helps radiation guiding.[2] So there should be higher efficiencies and lower wavelengths for ion-channel guiding FEL. The linear theory of a wiggler FEL with ion-channel guiding has been presented by P. Jha *et al.*[3] They have shown that the steady-state transverse electron motion generates a similar set of stable and unstable electron orbits, and a substantial enhancement in peak growth rate could be obtained when the ion-channel frequency approaches the wiggler frequency. Comparing with wiggler FEL, there is a shorter period in an electromagnetic wave pumped FEL. So under the same injection energy of electrons, one can get a more shorter laser wavelength output. Motivated by the benefits, the study on electromagnetic wave pumped ion-channel free-electron laser (EPIC-FEL) will be more useful.

Radiation generation in an ion-channel was first proposed by Whittum and Sessler in 1990.[4] Then Whittum studied the electromagnetic wave instability of the ion-focused regime in detail.[5] The Ion-ripple Laser was proposed by Chen and Dawson in 1992.[6,7] Tang *et al.*[8] studied electromagnetic wave instability in an ion-channel electron cyclotron maser (ECM) and proposed the concept of the ICECM. Recently, The ion-channel hybrid instability was proposed by Liu et al.[9] It was indicated by K.R. Chen[10] that two bunching mechanisms exist for wave amplification in the ion-channel laser, cyclotron maser and FEL. There should be a competition between axial bunching and azimuthal bunching, however, axial bunching is mainly considered in FEL, including the discussion in [3].

In this paper, we presented the linear theory of EPIC-FEL. In the following discussions we also mainly consider the axial bunching for simplifier. Section II shows that there has an unstable electron orbits regime when $\Omega_0 = \omega_i$, while there has a ion-channel instability when $\Omega_1 = \omega_i$. Electron axial bunching is introduced in section III and the dispersion relation is obtained in section IV. Section V is the discussions and calculation results. Conclusion is given in section VI.

II. Electron transverse motion

Consider a relativistic electron (charge -|e|, rest mass m, relativistic factor γ_0) moving along the z axis. A backward scattering electromagnetic (EM) wave as a pump wiggler field is described by

$$A_\theta = A_0[e^{i(\omega_0 t+k_0 z)}e_+ + e^{-i(\omega_0 t+k_0 z)}e_-] \tag{1}$$

where A_0 is the amplitude of EM wave field, $k_0 = 2\pi/\lambda_0$, λ_0 is the EM wave wavelength, $e_\pm = (e_x \pm ie_y)/2$. The transverse static field caused by the ion-channel has a form of

$$E_i = A_i(x, y, 0) \tag{2}$$

where $A_i = 4\pi|e|n_i$, n_i is the plasma density. In a form similar to representation of the wiggler field (1) we represent the forward stimulated radiation by

$$A_R(z,t) = A_R[e^{i(kz-\omega t)}e_- + e^{-i(kz-\omega t)}e_+] \tag{3}$$

where A_R is the amplitude, k is the complex wavenumber, and ω is the real frequency. So, the total electric field and magnetic field are

$$E(z,t) = -\frac{\partial\Phi}{\partial z}e_z - \frac{1}{c}\frac{\partial(A_\theta + A_R)}{\partial t} + A_i x \tag{4a}$$

$$B(z,t) = -\frac{\partial}{\partial z}[e_z \times (A_\theta + A_R)] \tag{4b}$$

where x represents the transverse displacement, Φ is space-charge potential with the perturbed density δn, and is given by

$$\frac{\partial^2 \Phi}{\partial z^2} = 4\pi|e|\delta n \tag{4c}$$

The transverse velocity can be determined by the motion equation

$$\frac{dP}{dt} = -|e|[E + \frac{1}{c}(v \times B)] \tag{5}$$

where $P = m\gamma V$, γ is the total relativistic factor. Substituting eqns. (4a) and (4b) into (5) yields

$$\frac{dP_\perp}{dt} = \frac{|e|}{c} \cdot \frac{d(A_0 + A_R)}{dt} - eA_i x_\perp \tag{6}$$

In above equation the subscript $\perp$ represents the transverse component. For the electron stable motion, assuming $x_{\perp 0} \propto e^{i\Omega_0 t} = e^{i(\omega_0 + k_0 v_{z0})t}$, we have

$$\begin{cases} x_{\perp 0} = -\dfrac{i}{\Omega_0} V_{\perp 0} \\ V_{\perp 0} = \dfrac{|e|}{m\gamma_0 c} \cdot \dfrac{A_0}{\Omega_{i0}} \end{cases} \tag{7}$$

where $\Omega_{i0} = 1 - \frac{\omega_i^2}{\Omega_0^2}$, $\omega_i = (4\pi|e|^2 n_i / m\gamma_0)^{1/2}$ is plasma frequency. We can see that for the electron steady-state motion, there is an unstable value when $\Omega_0 = \omega_i$. The electron motion orbits was similar with Ref. [3], i.e., also have two stable regimes.

For the perturbed motion of electron, assuming $x_{\perp 1} \propto e^{-i\Omega_1 t} = e^{-i(\omega - kv_{z0})t}$, then

$$\begin{cases} x_{\perp 1} = \dfrac{i}{\Omega_1} V_{\perp 1} \\ V_{\perp 1} = \dfrac{|e|}{m\gamma_0 c} \cdot \dfrac{A_R}{\Omega_{i1}} \end{cases} \tag{8}$$

where $\Omega_{i1} = 1 - \frac{\omega_i^2}{\Omega_1^2}$. In above derivation we used the approximate of γ_0 instead of γ. That means we mainly consider the axial bunching. From eq. (8), the ion-channel instability will happen when $\Omega_1 = \omega_i$.

III. Electron axial motion and bunching

From charge conservation, the perturbed beam density is given by

$$|e| \frac{\partial \delta n}{\partial t} = \frac{\partial \delta J_z}{\partial z} \tag{9}$$

where δJ_z is the perturbed axial beam current given by

$$\delta J_z(z,t) = -|e|(n_0 \delta v_z + \delta n v_{z0}) \tag{10}$$

In eq. (10) δv_z and v_{z0} are the perturbed and unperturbed axial electron velocities. Combining (9) and (10) yields the following expression for the perturbed density:

$$\frac{d\delta n}{dt} = -n_0 \frac{\partial \delta v_z}{\partial z} \qquad (11)$$

Taking the axial component of the motion equation, eq.(5), and using the relation $d\gamma / dt = -|e|(v \cdot E)/mc^2$, we find that

$$\frac{dv_z}{dt} = -\frac{|e|}{m\gamma_0}[-\frac{\partial \Phi}{\partial z} + \frac{1}{c}(v \times B)\cdot e_z - \frac{v_z}{c^2}(v \cdot E)] \qquad (12)$$

Linearizing (12) by keeping terms to first order in the radiation field yields

$$\frac{d\delta v_z}{dt} = \frac{|e|}{m\gamma_0}\{-\gamma_{z0}^{-2}\frac{\partial \Phi}{\partial z} + i[(\frac{k_0}{\Omega_{i1}} + \frac{k}{\Omega_{i0}}) - \frac{\omega\beta_{z0}}{c\Omega_{i0}} + \frac{\omega_0\beta_{z0}}{c\Omega_{i1}} - \frac{\omega_i^2}{\Omega_{i1}\Omega_{i0}\Omega_0}(\frac{\Omega_0}{\Omega_1} - 1)]\Phi_p(z,t)\} \qquad (13)$$

where $$\Phi_p(z,t) = \frac{-|e|}{m\gamma_0 c^2} A_0 A_R e^{i[(k+k_w)z-(\omega-\omega_0)t]} \qquad (14)$$

is the ponderomotive potential and $\gamma_{z0} = (1-\beta_{z0}{}^2)^{-1/2}$, $\beta_{z0} = v_{z0}/c$. Taking the convective time derivative of both sides of (11) and employing (13) and (4c) yields

$$(\frac{d^2}{dt^2} + \frac{\omega_b^2}{\gamma_0\gamma_{z0}^2})\delta n = -\frac{in_0|e|}{m\gamma_0}[(\frac{k_0}{\Omega_{i0}} + \frac{k}{\Omega_{i0}} - \frac{\omega\beta_{z0}}{c\Omega_{i0}} + \frac{\omega_0\beta_{z0}}{c\Omega_{i1}} - \frac{\omega_i^2}{\Omega_{i1}\Omega_{i0}\Omega_0}(\frac{\Omega_0}{\Omega_1} - 1)]\frac{\partial \Phi_p(z,t)}{\partial z} \qquad (15)$$

where $\omega_b = (4\pi|e|^2 n_0/m)^{1/2}$ is beam electron frequency, n_0 is beam electron density. Equation (15) shows that the perturbed charge density is driven by the ponderomotive potential wave.

IV. Dispersion relation

The wave equation of radiation field is

$$(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})A_R = -\frac{4\pi}{c}J_\perp \qquad (16)$$

where $J_\perp = -|e|(n_0 V_{\perp I} + \delta n V_{\perp\theta})$, substituting eqns. (7) and (8) into eq. (16) yields

$$(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\omega_b^2}{\gamma_0\Omega_{i1}})A_R = \frac{4\pi|e|^2}{m\gamma_0}\frac{A_0}{\Omega_{i0}}\delta n \qquad (17)$$

Since the phase of the ponderomotive wave is $[(k+k_0)-(\omega-\omega_0)t]$, we see from eq. (15) that the perturbed density should have a similar dependence in the time asymptotic limit, hence we write

$$\delta n(z,t) = \delta\tilde{n}\, e^{i[(k+k_w)z-(\omega-\omega_0)t]} \tag{18}$$

Using (18) together with (14) and (15), by eliminating $\delta\tilde{n}$ and A_R, eq. (17) becomes

$$\begin{aligned}(\omega^2 - c^2k^2 - \frac{\omega_b^2}{\gamma_0\Omega_{i1}})\cdot\{[(\omega-\omega_0)-(k+k_0)v_{z0}]^2 - \frac{\omega_b^2}{\gamma_0\gamma_{z0}^2}\}\\ = \frac{\omega_b^2}{\gamma_0}\frac{V_0^2}{\Omega_{i0}}(k+k_0)[\frac{k_0}{\Omega_{i1}} + \frac{k}{\Omega_{i0}} - \frac{\omega\beta_{z0}}{c\Omega_{i0}} + \frac{\omega_0\beta_{z0}}{c\Omega_{i1}} - \frac{\omega_i^2}{\Omega_{i1}\Omega_{i0}\Omega_0}(\frac{\Omega_0}{\Omega_1}-1)]\end{aligned} \tag{19}$$

where $V_0 = \frac{|e|A_0}{m\gamma_0 c}$ is EM wave wiggle velocity. The above equation is the EPIC-FEL dispersion relation. When $\omega_i = 0$, we may get the vacuum case. In this case, $\Omega_{i0} = \Omega_{i1} = 1$, the eq.(19) reduces to the conventional EM wave pumped FEL dispersion relation[11].

Comparing the dispertion relation with that of the wiggler ion-channel FEL (WIC-FEL) case, there are some differences. First, there is additional effect coming from ω_0 in EPIC-FEL; second, A_w which is the amplitude of the wiggler field is replacde by A_0; third, there is a product that 1/2 multiple the left of the dispertion equation in the wiggler FEL case. So, they may bring some difference for two cases when we analyze the relation properties. For example, considering the case when $A_w \approx A_0$, we can easy conclude the growth rate in EPIC-FEL will double higher than that in WIC-FEL.

V. Numerical Calculations

Note that the phase velocity of the wave should be synchronize with electron beam, i.e.: $v_{ph} \approx v_{z0}$. The axial phase velocity is

$$v_{ph} = \frac{\omega-\omega_0}{k+k_0} \tag{20}$$

So we can get $k = \frac{(1+\beta_{z0})^2}{(1-\beta_{z0}^2)}k_0$ and $\omega = \frac{(1+\beta_{z0})^2}{(1-\beta_{z0}^2)}\omega_0$. But for a forward EM wave pump, we get the ponderomotive potential as

$$\Phi_p \sim e^{i[(\omega-\omega_0)t-(k-k_0)z]} \tag{21}$$

A_0

$$\Phi_p \approx e^{i[(\omega-\omega_0)t-(k-k_0)z]} \tag{21}$$

Therefore $v_{ph} = \dfrac{\omega - \omega_0}{k - k_0}$. It required that $\beta_{z0} = 1$, we can see it is difficult to satisfy thus condition for forward wave to amplifier the wave.

Under the conditions of resonation for backward wave pumped, i.e. $k = \dfrac{(1+\beta_{z0})^2}{(1-\beta_{z0}^2)} k_0$, we can get $\Omega_0 = \Omega_1$ and $\Omega_{i0} = \Omega_{i1}$. Then the eq. (19) is reduced to

$$(\omega^2 - c^2k^2 - \frac{\omega_b^2}{\gamma_0\Omega_{i0}})\cdot\{[(\omega-\omega_0)-(k+k_0)v_{z0}]^2 - \frac{\omega_b^2}{\gamma_0\gamma_{z0}^2}\} = \frac{\omega_b^2}{\gamma_0}\frac{V_0^2}{\Omega_{i0}^2}4kk_0 \tag{22}$$

In obtaining eq. (22) the use of the approximations, $\omega \approx ck$, $\omega_0 \approx ck_0$, $\beta_{z0} \approx 1$ and $k >> k_0$ has been made. Above equation has been solved on a computer with $\gamma_0 = 6.0$, $k_0 = 2\pi/3cm^{-1}$, $\beta_0 = V_0/c = 0.2/\gamma_0$, $\beta_{z0} = 0.98$ and $\omega_0 = ck_0$. The growth rates (Im k) are plotted versus frequency for appropriate values of x (ω_i/Ω_0) for two stable regimes in Fig.1 and 2, respectively.[3] The parameters used here are list all in the figure captions. In order to satisfy the Budker condition for propagation of an electron beam in the ion-focused regime, the beam electron frequencies are different for regimes I and II in Figs.1 and 2. A plot of peak growth rate with x for regimes I and II (for the same parameters as in Figs. 2 and 3) is shown in Fig. 3. In regime I, with and increase in x the peak growth rate increases monotonically up to the singularity at the orbital stability boundary ($x \approx 1$). In regime II, the peak growth rate decreases slowly as ω_i becomes large. The results of the calculations are very interesting. Our results show that the values of the growth rates are lager than that of the magnetic wiggler case and the operating frequencies are much higher than that in magnetic wiggler case. From $\omega = \dfrac{(1+\beta_{z0})^2}{(1-\beta_{z0}^2)}\omega_0 = \gamma_z^2(1+\beta_{z0}^2)\omega_0$, we can see that these can be explained by providing the pumping frequency and the beam energy are high enough. The growth rate in this case enhances as $\omega_i \approx \Omega_0$.

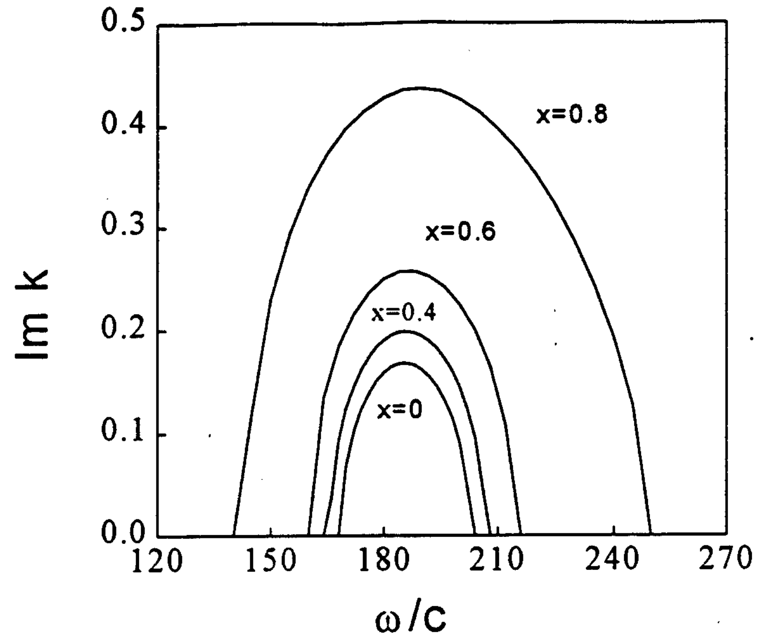

Fig. 1 Spatial growth rate Im k(cm^{-1}) vs ω/c (cm^{-1}) for regime I orbit, $\omega_b = 1.8\text{x}10^{11}\,s^{-1}$.

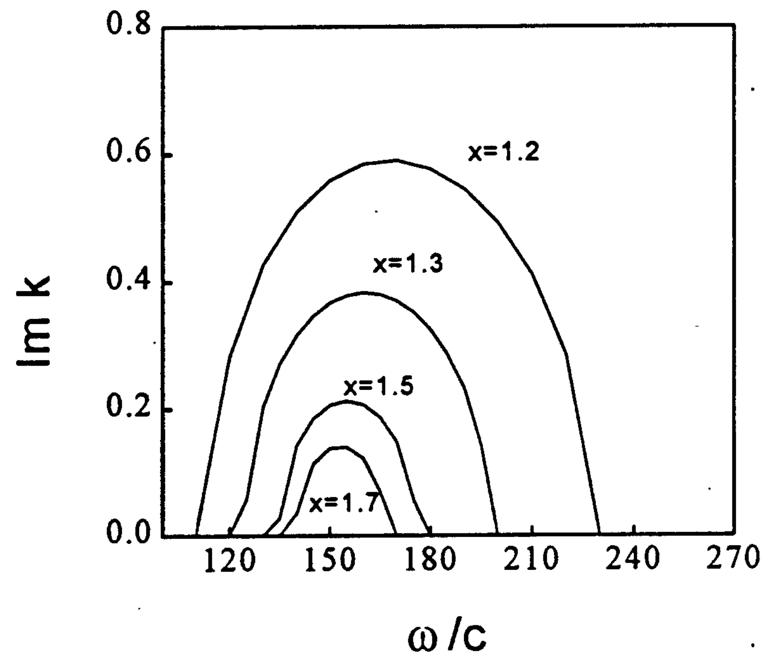

Fig. 2 Spatial growth rate Im k(cm^{-1}) vs ω/c (cm^{-1}) for regime II orbit, $\omega_b = 4.4\text{x}10^{11}\,s^{-1}$.

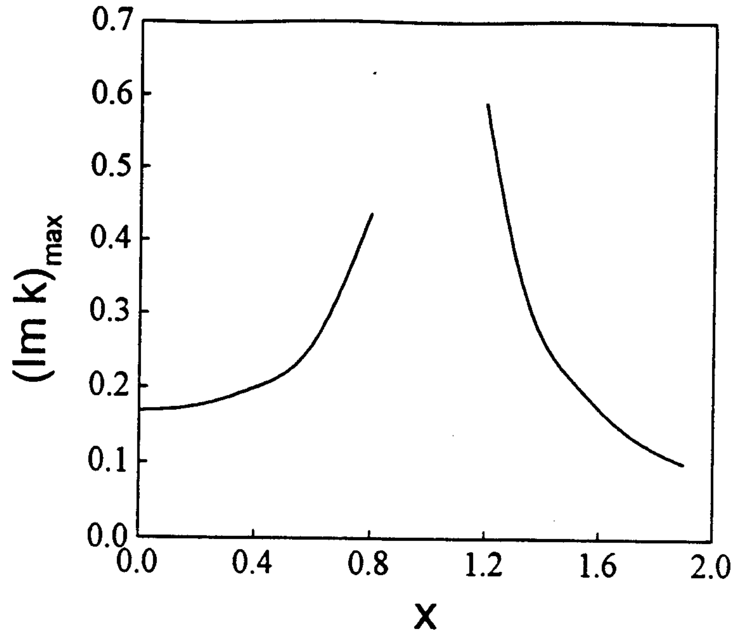

Fig. 3 Peak growth rate $(\text{Im } k)_{max}$ (cm^{-1}) as a function of x for regimes I and II.

VI. Conclusions

In conclusion, we have analyzed the wave generation in an ion-channel EM wave pumped FEL, and derived the dispersion relation. The presence of ion-channel may lead to substantial enhancements in growth rate as $\omega_i \approx \Omega_0$. These agree~~ment~~ with results also that obtained in Ref.[3]. But the values of the peak growth rates and the resonate frequencies are larger ~~than that~~ ~~of~~ in ~~the former~~ our case. For all kinds of FEL, we can ~~derive~~ get $\lambda \approx \lambda_0 / 2\gamma_z^2$, where λ and λ_0 are the radiation and pump EM wave wavelengthes in the kind of EM pumped FEL case or radiation wave wavelength and wiggler period in the kind of wiggler FEL case, respectively. In order to operate FEL at low wavelength, two methods have been used. First, reducing the wiggler period, we know it is very difficult ~~to obtain~~ for conventional wiggler FEL case. However it has no problem for ~~conventional~~ EM wave pumped FEL. Second, increasing the beam energy. But it may cause lower efficiencies. However, in ion-channel kind of FELs, the ion-channel can aid the electron beam focusing and can compensate the efficiency lose by adjusting the plasma frequency (eq.(22)), so under high beam energy case, this kind of FEL can be operation at very low radiation wavelength without lose many efficiencies~~, meanwhile~~, and the decreasing of pump wavelength is very easy to accomplish for EPIC-FEL. So with such benefits, we believe that EPIC-FEL will has a very bright future.

References

1 K. Takayama and S. Hiramatsu, Phys. Rev. A 37, 173(1988)

2 Y. Seo, V.K. Tripathi, and C.S. Liu, Phys. Fluids B1, 221(1989)

3 P. Jha and P. Kumar, Phys. Rev. E 57, 2256(1998)

4 D.H. Whittum and A.M. Sessler, *Phys. Rev. Lett.* 64, 2511(1990)

5 K.R. Chen, T.C. Katsouleas, J.M. Dawson, *IEEE Trans. Plasma Sci.* 18, 837(1990)

6 D.H. Whittum, *Phys. Fluids B* 4, 730(1992)

7 K.R. Chen and J.M. Dawson, *Phys. Rev. Lett.* 68, 29(1992)

8 C.J. Tang, P.K. Liu, S.G. Liu, *J. Phys. D:Appl. Phys.* 29, 90(1996)

9 S.G. Liu and R.J. Barker, 23rd Int. Conf. on IR/MM waves, Essex Univ. Sept. 7-11, 1998

10 K.R. Chen, J.M. Dawson, A.T. Lin, T. Katsouler, *Phys. Fluids. B* 3, 1270(1991)

11 V.L. Braman, G.G. Denisov, N.S. Ginzburg, *Int. J. Electron.* 59, 247(1985)

8. Shenggang Liu, Zhu Dajun, Yan Yang, and Robert J. Barker, "Dispersion Characters of Wave Propagation Along a Plasma Waveguide in a Finite Magnetic Field".

Dispersion characters of wave propagation along a plasma waveguide in a finite magnetic field*

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Abstract Further theoretical study and computer calculations on wave propagation along plasma waveguide in finite magnetic field are given in the paper. It gives accurate dispersion equations and shows that dispersion characters of wave propagation in partially plasma filled waveguide are quite different with that in completely plasma filled waveguide, and the dispersion curves of all waves can not pass across the critical frequency line: $\omega = \omega_h = \left(\omega_{ce}^2 + \omega_{pe}^2\right)^{\frac{1}{2}}$. It has been found that some calculations given in published papers should be checked and corrected. It shows that the $p_2 = 0$ curve in the p_1^2, p_2^2 diagram. for example, has only one line rather than two lines. as it was given in [5] and [8]. and there is no so called anisotropic waves dispersion curve HE_1 as it was shown in [5]. A new kind of waves which is unknown in published papers has been found , the unusual dispersion characters of this kind of waves are discussed, and hence there are two kinds of waves, electromagnetic waves and waves containing quasi-static parts: The paper also calls in question the concept of "coupling of dispersion curves between modes" and presents an alternative physical explanation on the unusual curvature of dispersion curves of some modes.

I. Introduction

The wave propagation along a waveguide filled with plasmas immersed in a magnetic field has been well studied[1]-[13]. References [1-2] are pioneer

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works. In [4]-[6] , completely plasma filled waveguide was studied, and in [8], dispersion equations of partially plasma filled waveguide was given, but only for symmetric modes. In [7], theory of partially plasma filled waveguide was presented, but there was no calculation, and the boundary conditions used was not standard. Theoretical study both for partially and completely plasma filled waveguide has been given in [3], based on this, detailed analysis and complete numerical calculations are given in this paper. The paper shows:

1. The accurate dispersion equations should be $\frac{F(p_1,p_2,k)}{D}=0$. Only when $D\neq 0$, we have $F(p_1,p_2,k)=0$, as it was given in previous published papers, and we can prove that when $D\to 0$, the limit of $\lim\limits_{D\to 0}\frac{F(p_1,p_2,k)}{D}$ exists and continues. The dispersion curves, therefore, remain unchanged.

2. The cut-off frequencies and the dispersion characters are quite different in partially plasma filled waveguide with that in completely plasma filled waveguide.

3. Numerical calculations show that the dispersion curves can not go across the critical frequency line $\omega=\omega_h=\left(\omega_{ce}^2+\omega_{pe}^2\right)^{\frac{1}{2}}$ for all modes, and that also gives strong influences on wave propagation along plasma filled waveguide in finite magnetic field.

4. It has also been found that some calculations given in published papers should be checked and corrected. The $p_2^2=0$ curve in p_1, p_2 diagram, for example, only has one line rather than two lines as it was shown in [4]-[6], [8]. And also, the existence of "anisotropic waves" HE_1 in [5] was incorrect..

5. A new kind of waves which is unknown in published papers has been found and hence there are two kinds of waves that can propagate along plasma filled waveguide in finite magnetic field: they are electromagnetic

waves and waves containing quasi-static parts. The unusual dispersion characters of this kind of waves are discussed in the paper.

6. It calls in question the concept of "coupling of dispersion curves between modes" presented in [5] and [6], and an alternative physical explanation on the unusual curvature of dispersion curves of some modes.

The paper is organized as following: Section I is introduction. Dispersion equations for both partially and completely plasma filled waveguide are discussed and analyzed in section II. The calculations of cut-off frequencies are given in section III, and the numerical calculations of dispersion equations both for partially and completely plasma filled waveguide are given in section V. A new kind of wave that contains quasi-static parts and did not appeared in published papers are analyzed in section IV and it shows that there are two kinds of waves in plasma filled waveguide. Section VI calls in question the concept on "coupling between dispersion curves of modes" and an alternative physical explanation for the unusual curvature of dispersion curves of some modes was presented. Section VII is the conclusion. The derivations of the dispersion equations of the waves containing quasi-static parts and the corresponding expressions of field components are given in Appendix.

II. Dispersion Equations

It has been shown that for plasma filled waveguide in finite magnetic field, we have two coupled wave equations for E_z and H_z:

$$\nabla_\perp^2 E_z + aE_z = bH_z \tag{1}$$

$$\nabla_\perp^2 H_z + cH_z = dE_z \tag{2}$$

Decoupling eq. (1) and (2), we get:

$$\left[\nabla_\perp^4 + (a+c)\nabla_\perp^2 + (ac-bd)\right]E_z = 0 \tag{3}$$

$$\left[\nabla_\perp^4 + (a+c)\nabla_\perp^2 + (ac-bd)\right]H_z = 0 \tag{4}$$

Factoring eq. (3), we get:

$$\left(\nabla_{\perp}^{2}+p_{1}^{2}\right)E_{z1}=0 \tag{5}$$

$$\left(\nabla_{\perp}^{2}+p_{2}^{2}\right)E_{z2}=0 \tag{6}$$

Where:

$$E_{z}=E_{z1}+E_{z2} \tag{7}$$

Eigen values p_1^2 and p_2^2 are given by:

$$p_{1,2}^{2}=\frac{1}{2}\left\{(a+c)\pm\left[(a+c)^{2}-4(ac-bd)\right]^{\frac{1}{2}}\right\} \tag{8}$$

or:

$$p_{1.2}^{2}=\frac{1}{2\varepsilon_{1}}\left[\gamma^{2}\left(\varepsilon_{1}+\varepsilon_{3}\right)+k^{2}\left(\varepsilon_{1}\varepsilon_{3}+\varepsilon_{1}^{2}+\varepsilon_{2}^{2}\right)\right]\pm \\ \frac{1}{2\varepsilon_{1}}\left\{\left[\gamma^{2}\left(\varepsilon_{3}-\varepsilon_{1}\right)+k^{2}\left(\varepsilon_{1}\varepsilon_{3}-\varepsilon_{1}^{2}-\varepsilon_{2}^{2}\right)\right]^{2}+4k^{2}\gamma^{2}\varepsilon_{2}^{2}\varepsilon_{3}\right\}^{\frac{1}{2}} \tag{9}$$

The same equations may be obtained for H_Z[3]-[5].

If p_1^2 and p_2^2 are assumed:

$$p_{1}^{2}\neq 0, \qquad p_{2}^{2}\neq 0 \tag{10}$$

we may find the corresponding solutions (eigen functions), for example, for cylindrical waveguide, we have:

$$E_{z}=E_{z1}+E_{z2} \tag{11}$$

$$\begin{cases} E_{z1}=A_{1}J_{m}\left(p_{1}r\right) \\ E_{z2}=A_{2}J_{m}\left(p_{2}r\right) \end{cases} \tag{12}$$

Then we get:

$$H_{z}=A_{1}h_{1}J_{m}\left(p_{1}r\right)+A_{2}h_{2}J_{m}\left(p_{2}r\right) \tag{13}$$

where:

$$h_{1,2}=\frac{\left(\gamma^{2}+k^{2}\varepsilon_{1}\right)\frac{\varepsilon_{3}}{\varepsilon_{1}}-p_{1.2}^{2}}{j\omega\mu_{0}\gamma\frac{\varepsilon_{2}}{\varepsilon_{1}}} \tag{14}$$

The transverse field components may be expressed by means of E_Z and H_Z[3].

The boundary conditions are:

$r = R_c$:

$$E_z = E_\theta = 0 \tag{15}$$

$r = R_p$:

$$\left.\begin{aligned} E_z^{I} &= E_z^{II} \\ E_\theta^{I} &= E_\theta^{II} \\ H_z^{I} &= H_z^{II} \\ H_\theta^{I} &= H_\theta^{II} \end{aligned}\right\} \tag{16}$$

Then, the dispersion equations may be found:

$$\frac{1}{p^2 D} C_1 + \frac{1}{D^2} C_2 + \frac{1}{p^4} C_3 + \left(\frac{m}{R_p}\right)\left(\frac{1}{p^2 D} C_4 + \frac{1}{D^2} C_5\right) + \left(\frac{m}{R_p}\right)^2 C_6 = 0 \tag{17}$$

where

$$\begin{aligned} C_1 = {} & \gamma^2 p k_g^4 \left[p_1 J_m'(p_1 R_p) J_m(p_2 R_p) - p_2 J_m'(p_2 R_p) J_m(p_1 R_p)\right] \cdot \Bigg[\varepsilon_D Q + \\ & \frac{(\varepsilon_3 K^2 - \varepsilon_1 p_1^2)(\varepsilon_3 K^2 - \varepsilon_1 p_2^2)}{\varepsilon_g \gamma^2 k_g^2} F\Bigg] + k^2 p \Big\{\varepsilon_D K^2 Q\big[(\varepsilon_3 K^2 - \varepsilon_1 p_2^2) p_2 J_m'(p_2 R_p) J_m(p_1 R_p) \\ & -(\varepsilon_3 K^2 - \varepsilon_1 p_1^2) p_1 J_m'(p_1 R_p) J_m(p_2 R_p)\big] + (\varepsilon_1 K^2 - \varepsilon_g k_g^2) F \\ & \big[(\varepsilon_3 K^2 - \varepsilon_1 p_2^2) p_1 J_m'(p_1 R_p) J_m(p_2 R_p) - (\varepsilon_3 K^2 - \varepsilon_1 p_1^2) p_2 J_m'(p_2 R_p) J_m(p_1 R_p)\big]\Big\} \end{aligned} \tag{18}$$

$$C_2 = \varepsilon_1 p_1 p_2 J_m'(p_1 R_p) J_m'(p_2 R_p)(p_2^2 - p_1^2)\left[k_g^4(\gamma^2 - K^2) + \varepsilon_1 k^2 K^4\right] \tag{19}$$

$$C_3 = k^2 \varepsilon_D p^2 \varepsilon_1 (p_2^2 - p_1^2) Q F J_m(p_1 R_p) J_m(p_2 R_p) \tag{20}$$

$$C_4 = \varepsilon_1 k^2 p\left(p_2^2 - p_1^2\right) J_m\left(p_1 R_p\right) J_m\left(p_2 R_p\right)\left(\varepsilon_g \gamma^2 F - \varepsilon_D k_g^2 Q\right) +$$
$$\frac{p_2}{\varepsilon_g} J_m\left(p_1 R_p\right) J_m^{'}\left(p_2 R_p\right)\left[\gamma^2 k_g^2 \varepsilon_1 \varepsilon_g\left(p_1^2 - p_2^2\right) + \varepsilon_g \gamma^2 k_g^2\left(\varepsilon_1 K^2 - \varepsilon_g k_g^2\right) +\right.$$
$$\left. K^2\left(\varepsilon_3 K^2 - \varepsilon_1 p_1^2\right)\left(\varepsilon_3 K^2 - \varepsilon_1 p_2^2\right)\right] - \frac{p_1}{\varepsilon_g} J_m^{'}\left(p_1 R_p\right) J_m\left(p_2 R_p\right) \cdot$$
$$\left[\gamma^2 k_g^2 \varepsilon_1 \varepsilon_g\left(p_2^2 - p_1^2\right) + \varepsilon_g \gamma^2 k_g^2\left(\varepsilon_1 K^2 - \varepsilon_g k_g^2\right) + K^2\left(\varepsilon_3 K^2 - \varepsilon_1 p_1^2\right)\left(\varepsilon_3 K^2 - \varepsilon_1 p_2^2\right)\right]$$

(21)

$$C_5 = \frac{p_2}{\varepsilon_g} J_m\left(p_1 R_p\right) J_m^{'}\left(p_2 R_p\right)\left[\gamma^2 k_g^2 \varepsilon_1 \varepsilon_g K^2\left(p_2^2 - p_1^2\right) - \varepsilon_g^2 \gamma^2 k_g^4\left(\gamma^2 - K^2\right) -\right.$$
$$\gamma^2 k_g^2 \varepsilon_1 \varepsilon_g K^4 + \left(k_g^4 - K^4\right)\left(\varepsilon_3 K^2 - \varepsilon_1 p_1^2\right)\left(\varepsilon_3 K^2 - \varepsilon_1 p_2^2\right) + k_g^4\left(\varepsilon_1 K^2 - \varepsilon_g k_g^2\right) \cdot$$
$$\left.\left(\varepsilon_3 K^2 - \varepsilon_1 p_1^2\right) + \varepsilon_g \gamma^2 k_g^2 K^2\left(\varepsilon_3 K^2 - \varepsilon_1 p_2^2\right)\right] - \frac{p_1}{\varepsilon_g} J_m^{'}\left(p_1 R_p\right) J_m\left(p_2 R_p\right) \cdot$$
$$\left[\gamma^2 k_g^2 \varepsilon_1 \varepsilon_g K^2\left(p_1^2 - p_2^2\right) - \varepsilon_g^2 \gamma^2 k_g^4\left(\gamma^2 - K^2\right) - \gamma^2 k_g^2 \varepsilon_1 \varepsilon_g K^4 + \left(k_g^4 - K^4\right) \cdot\right.$$
$$\left(\varepsilon_3 K^2 - \varepsilon_1 p_1^2\right)\left(\varepsilon_3 K^2 - \varepsilon_1 p_2^2\right) + k_g^4\left(\varepsilon_1 K^2 - \varepsilon_g k_g^2\right)\left(\varepsilon_3 K^2 - \varepsilon_1 p_2^2\right) +$$
$$\left.\varepsilon_g \gamma^2 k_g^2 K^2\left(\varepsilon_3 K^2 - \varepsilon_1 p_1^2\right)\right]$$

(22)

$$C_6 = \varepsilon_1 \gamma^2 J_m\left(p_1 R_p\right) J_m\left(p_2 R_p\right)\left(p_1^2 - p_2^2\right)\left(\frac{2K^2}{p^2 D} - \frac{1}{p^4} - \frac{1}{D}\right) \quad (23)$$

$$p^2 = \gamma^2 + k^2 \varepsilon_D \quad (24)$$

$$D = K^4 - k_g^4 \quad (25)$$

$$K^2 = \gamma^2 + k^2 \varepsilon_1 \quad (26)$$

$$k_g^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_g = k^2 \varepsilon_g \quad (27)$$

$$\varepsilon_g = \left|\varepsilon_2\right|, \qquad \varepsilon_2 = -j\varepsilon_g$$

and:

$$Q = \frac{N_m(pR_c)J'_m(pR_p) - J_m(pR_c)N'_m(pR_p)}{J_m(pR_p)N_m(pR_c) - J_m(pR_c)N_m(pR_p)} \tag{28}$$

$$F = \frac{N'_m(pR_c)J'_m(pR_p) - J'_m(pR_c)N'_m(pR_p)}{J_m(pR_p)N'_m(pR_c) - J'_m(pR_c)N_m(pR_p)} \tag{29}$$

Eq. (17) is a general dispersion equation for magnetized plasma waveguide. Now, based on this dispersion equation, we can discuss the following special cases:

1. The general dispersion equation (17) shows that it contains $\left(\frac{m}{R_p}\right)^2$, $\left(\frac{m}{R_p}\right)$ and symmetrical (m=0) terms. For m=0, the symmetric mode, we can get the following dispersion equation:

$$\frac{1}{p^2 D}C_1 + \frac{1}{D^2}C_2 + \frac{1}{p^4}C_3 = 0 \tag{30}$$

2. When $R_r = R_c$, i. e. the waveguide is fully filled with plasma, the dispersion equation (17) reduces to the following equation:

$$p_1 J'_m(p_1R_c)J_m(p_2R_c)\left[\gamma k_g^2 - \frac{K^2}{\gamma\varepsilon_g}(\varepsilon_3 K^2 - \varepsilon_1 p_1^2)\right] - p_2 J'_m(p_2R_c)J_m(p_1R_c)\cdot$$
$$\left[\gamma k_g^2 - \frac{K^2}{\gamma\varepsilon_g}(\varepsilon_3 K^2 - \varepsilon_1 p_2^2)\right] + \left(\frac{m}{R_c}\right)\frac{\varepsilon_1 k_g^2}{\gamma\varepsilon_g}(p_1^2 - p_2^2)J_m(p_1R_c)J_m(p_2R_c) = 0$$

(31)

Actually and more accurately, the dispersion equations should be written as:

$$\frac{1}{D}F(p_1, p_2, k) = 0 \tag{32}$$

Where D is given in [25] and $F(p_1, p_2, k)$ is the rest part function of the dispersion equations (17) or (31).

Only under the condition D=0, we can get $F(p_1,p_2,k)=0$ as usual.

We can prove that the limit $\lim\limits_{D\to 0}\dfrac{F(p_1,p_2,k)}{D}$ exists and continues.

It has been shown that no matter eigen values p_1^2 and p_2^2 are real, imaginary or complex, there are waves that can propagate along plasma filled waveguide in finite magnetic field, the classification of waves will be discussed later.

It has been shown also that there is a critical frequency[3]:

$$\omega=\omega_h=\left(\omega_{ce}^2+\omega_{pe}^2\right)^{\frac{1}{2}} \tag{33}$$

If this resonance occurs, there is disruption of the wave propagation, this means, the dispersion curves for all wave can not go across the critical frequency line.

Since the Bessel function has the property of $J_m(x)=J_{-m}(x)$, and the sign of ε_g depends on the wave propagation direction along or against the direction of magnetic field $\vec{B}_0=B_0\vec{e}_z$. Eq. (17) and (31) show that there is some kind of nonreciprocal property for the waves propagating along plasma waveguide in finite magnetic field, the problem about the nonreciprocal property will be discussed in detail in authors' another paper.

III. Cut-off frequency

Let $\gamma=0$, from eq. (17) we get the following equation for cut-off frequencies:

$$k\left[p_1\frac{J_m'(p_1R_p)}{J_m(p_1R_p)}-pQ\right]\left[\frac{\varepsilon_1^2+\varepsilon_2^2}{\varepsilon_D}pF-\varepsilon_1 p_2\frac{J_m'(p_2R_p)}{J_m(p_2R_p)}-\frac{m}{R_p}\varepsilon_g\right]=0 \tag{34}$$

$$(p_1^2=\left(p_1^2\right)_c,\qquad p_2^2=\left(p_2^2\right)_c)$$

Where at $\gamma=0$, we have $p_c^2=k^2\varepsilon_D$, $\left(p_1^2\right)_c=k^2\varepsilon_3$, $\left(p_2^2\right)_c=k^2\dfrac{\varepsilon_1^2+\varepsilon_2^2}{\varepsilon_1}$.

In the case of $R_p=R_c$, the completely plasma filled case, $Q\to\infty$, $F\to 0$, from equation (34), we get:

$$kJ_m(p_1R_p)\left[\frac{J'_m(p_2R_p)}{J_m(p_2R_p)}+\frac{m\varepsilon_g}{R_p\varepsilon_1 p_2}\right]=0 \qquad (35)$$

From eq. (34), we can get three equations for cut-off frequencies of the waves:

$$k=0 \qquad (36)$$

$$p_1\frac{J'_m(p_1R_p)}{J_m(p_1R_p)}-pQ=0, \quad p_1^2=(p_1^2)_c \qquad (37)$$

$$\frac{\varepsilon_1^2+\varepsilon_2^2}{\varepsilon_D}pF-\varepsilon_1 p_2\frac{J'_m(p_2R_p)}{J_m(p_2R_p)}-\frac{m}{R_p}\varepsilon_g=0, \qquad p_2^2=(p_2^2)_c \qquad (38)$$

We can get analytical calculation expressions by using eq. (35), (37), (38).

So we can classify the electromagnetic waves using the cut-off frequencies. The waves their cut-off frequency equation is (36) are plasma waves. Eq. (37) represents EH waves, the cut-off frequencies of which are only the function of ω_p and have not any relation with ω_c. Equation (38) are the cut-off frequency equation of HE waves, which are not only the function of ω_p, but also of ω_c.

The numerical calculation results of equation (37) are shown in figure 1. It gives the relation of cut-off frequency of EH waves with ω_p. The results of equation (38) are shown in figure 2 and figure 3. They give the relations of cut-off frequency of HE waves with ω_p and ω_c. In these figures, (a) is for completely plasma filled case ($R_p/R_c=1$), and (b) is for partially plasma filled case ($R_p/R_c=0.7$). In figure 2 and 3, there are two special curves of ω_1 and ω_2. Where $\omega_1=\dfrac{\sqrt{\omega_c^2+4\omega_p^2}+\omega_c}{2}$ is the solution of equation $K^2=-k_g^2$ (D=0) under the condition of $\beta=0$. $\omega_2=\dfrac{\sqrt{\omega_c^2+4\omega_p^2}-\omega_c}{2}$ is the solution of equation $K^2 = k_g^2$ (D=0) under the condition of $\beta = 0$. Curves ω_1 and ω_2 will be discussed later.

From the calculation results of cut-off frequency, as shown in Fig.1, Fig. 2, and Fig. 3, we have the following important points:

(1). For EH modes: the cut-off frequencies are all higher than ω_p in completely plasma-filled case, but can be lower than ω_p in partially plasma-filled case. (2). For HE modes: the cut-off frequencies are all higher than ω_2 in completely plasma-filled case, but can be lower than ω_2 in partially plasma-filled case. Also, with the increasing of ω_c, more modes are condensed up near ω_h.

(3). For each mode, no matter EH wave or HE wave, the cut-off frequency is increasing with the increasing of ω_p. But this increasing rate becomes slow when $\omega_{cut} > \omega_p$ in partially plasma-filled case.

(4). There are more condensed modes in partially plasma filled case compared with completely plasma filled case. Also, for HE modes, there are many modes nearly below ω_h.

(5). For HE modes, if the cut-off frequencies are below ω_h, they are called cyclotron modes. If the cut-off frequencies are higher than ω_h, they are called waveguide modes. The curve of mode HE_{01} is usually coincided with the line ω_h.

IV. Numerical calculations of dispersion equation

Now, we give the numerical calculations of dispersion equation (17) and (31). and make analysis of the calculations. We have found that some calculations given in [4] and [5] should be checked and corrected.

At first, we have calculated the eigen value diagram, as shown in Fig. 4, it can be seen that the curve **a** of $p_2 = 0$ is one line going across ω_h, not two lines which was given in Fig. 1 of [5] (We present it as a little figure on Fig. 4 in this paper). Both analytical and numerical study show that on the line of

$\omega = \omega_p$, we also have $p_2 = 0$. But the curve **a** of $p_2 = 0$ is the line of equation $\beta^2 = k^2 \dfrac{\omega^2 + \omega\omega_c - \omega_p^2}{\omega(\omega + \omega_c)}$ (see section V and Appendix), it is one continuos line, not two lines as it was shown in [4]-[6], [8] (for example, Fig. 1 in [5]).

Fig. 5 shows rather complete dispersion curves. Dashed lines show three special critical lines: D=0 and $p_2 = 0$. Here again there is difference between our calculations with that given in Fig. 2 of [5]. Mainly, the cut-off frequency of HE_{11} mode is below ω_1, so the mode dispersion curve will go across the line of $p_2 = 0$. We have also found that there is no HE_1 mode, and the curve for HE_1 mode given in Fig.2 of [5] is actually the line of $p_2 = 0$.

From Fig. 5, we can see that there are mainly three kind of waves: the first is plasma modes their cut-off frequencies are zero. The second is the modes their cut-off frequencies are lower than ω_h, these modes, include EH modes and HE modes, are cyclotron wave modes. The dispersion curves of these cyclotron waves will be condensed between ω_h and $\mathrm{Max}(\omega_c, \omega_p)$. All these dispersion curves can not go through ω_h line. The third is the modes their cut-off frequencies are higher than ω_h. They are all waveguide modes. Fig. 6(a) and 6(b) show some waveguide waves, there is also difference with that given in Fig. 3 of [5], the mode HE_{01} should be below mode EH_{02}.

Fig. 7 and Fig. 8 show some waveguide waves the cut-off frequencies of which are close to ω_h. Again there are differences between our calculations with that given in Fig. 4(a) and Fig. 5 of [5]. It seems that in [5] Fig. 4(a) the curve EH_{11} was confused with the line of D=0, $p_2 = 0$, and the curve HE_1 is really the dispersion curve of HE_{11}.Fig. 8 shows that many EH modes are cyclotron modes.

It should be mentioned that in order to easily make comparison, all parameters used in our calculations are the same as that used in [5].

One of the important differences between us and [5] which should be noted is that in [5] it was presented that there are so called anisotropic waves for which one of the eigen value is negative ($p_1^2 < 0$ or $p_2^2 < 0$). Our opinion is that there is no such kind of mode. The reasons are:

1. A number of waves their dispersion curves can go through one zone ($p_1^2 > 0$ or $p_2^2 > 0$) to another zone ($p_1^2 < 0$ or $p_2^2 < 0$), that means one of the eigen values (p_1^2 or p_2^2) changes sign naturally, it does not cause to appear a new kind of waves.
2. The dispersion curves of anisotropic waves HE_1 given in [5] is actually nothing but the curve of D=0, $p_2 = 0$. It has no mean with "anisotropic waves", the waves associates with D=0, $p_2 = 0$, are discussed in section V and Appendix.

For the partially plasma filled case, the dispersion characters are different. Fig. 9(a), 9(b) show the dispersion curves of the partially plasma filled case. They are the dispersion curves in a partially plasma-filled waveguide with $\frac{R_p}{R_c} = 0.714$. The dielectric is vacuum. Compared with the completely plasma filled case, we can see that for these modes of $\omega_{cut} > \omega_h$, the dispersion curves have a slower increasing rate of frequency with βR_c, because the curves can go across the second line of D=0, $p_2 = 0$. So by changing the thickness of dielectric the phase velocity of each mode can be changed. Also, there are more modes in partially plasma-filled case. In partially plasma filled case, there are also a line of p=0, this line will go through the dispersion curves of cyclotron modes. As in the condition of D=0, the dispersion character of waves in this case is also different. The discussion in this condition is just as in the case of D=0, which will be discussed in the follow section.

V. A new kind of wave

Mathematically, if one of eigen values vanish, for example, $p_2 = 0$, (i. e. $K^2 = \pm k_g^2$) then the corresponding wave equation becomes Laplace equation:

$$\nabla_\perp^2 E_{z2} = 0 \qquad (39)$$

The solution is:

$$E_{z2} = A_2 r^m \qquad (40)$$

$$H_{z2} = A_2 h_2 r^m \qquad (41)$$

Physically, this belongs to quasi-static waves. It is obvious that this part of waves is closely connected with plasma.

We obtain, therefore:

$$E_z = A_1 J_m(p_1 r) + A_2 r^m \qquad (42)$$

$$H_z = A_1 h_1 J_m(p_1 r) + A_2 h_2 r^m \qquad (43)$$

As Appendix shows, the transverse components of fields present two kinds of wave polarization, the left hand polarized wave (LHP) which corresponding to $K^2 = k_g^2$ and right hand polarized wave (RHP) which corresponding to $K^2 = -k_g^2$. By using the boundary conditions, we can obtain the dispersion equations at $R_p = R_c$:

For LHP, the dispersion equation is:

$$h_1\left[\frac{m}{2K^2 R_c} J_m(p_1 R_c) - \frac{J_{m+1}(p_1 R_c)}{p_1}\right] - h_2 J_m(p_1 R_c)\left[\frac{m}{2K^2 R_c} - \frac{R_c}{2m+2}\right] = 0$$

(44)

For RHP, the dispersion equation is:

$$h_1\left[\frac{m}{2K^2 R_c} J_m(p_1 R_c) + \frac{J_{m+1}(p_1 R_c)}{p_1}\right] - h_2 J_m(p_1 R_c)\left[\frac{m}{2K^2 R_c} + \frac{R_c}{2m+2}\right] = 0$$

(45)

The detailed derivation of eq. (44) and (45) is given in Appendix I.

modes"

In [2], [5] and [6], a concept on the "coupling between dispersion curves of modes" has been presented to explain the unusual shape curvature of the dispersion curves of some modes, as shown in Fig. 3 and Fig. 5 in [5]. In this section we deal with the concept. The reason for presenting such concept is stated in [5] and [2]: "there exists a value of magnetic field B_0 such that the cut-off frequency of the lowest waveguide mode (EH_{01}) is less than the upper-hybrid frequency. The dispersion curve of EH_{01} mode is then incorporated with the family of cyclotron waves and coupled with one of them. When the cut-off frequency of waveguide mode (EH_{01}) is less than the cut-off frequency of the lowest cyclotron mode HE^c_{01}, the coupling occurs between dispersion curves of these two modes".

In fact, there is no solid ground to support this concept. Since each mode is independent, the dispersion equation, which is valid and independent for each mode, is derived by using the smooth wall boundary conditions and the field components expressions that are also independent for each mode. Therefore, during the entail physical-mathematical process of the problem concerning the dispersion characters of the wave propagation along magnetized plasma waveguide, except the coupling between TE_{mn} and TM_{mn} which causes that the waves are always hybrid, there is no any other coupling mechanism at all. It is hard to believe that there exists any coupling between dispersion curves of modes. This paper, therefore, calls in question this coupling concept.

The numerical calculations show that the unusual curvature sharp of the dispersion curves of some modes really exists. The problem is how to make a proper explanation about this phenomena. In this paper, an alternative explanation for this phenomena is presented.

Looking at the figures given in [2]-[5], the Fig. 8 in this paper, for example, it is easy to find a fact that all these "couplings" occur in the vicinity of the line

In addition, there is one more equation:

$$p_{1,2}^2 = a + c = \frac{1}{\varepsilon_1}\left[\gamma^2(\varepsilon_1 + \varepsilon_3) + k^2(\varepsilon_1\varepsilon_3 + \varepsilon_1^2 + \varepsilon_2^2)\right] \qquad (46)$$

So together with the equations of $p_2 = 0$, the dispersion characters of the new waves can be determined.

The numerical calculation results show the solutions of eq. (44) are some spots on the curve of $K^2 = k_g^2$, that is:

$$\beta^2 = k^2 \frac{\omega^2 + \omega\omega_c - \omega_p^2}{\omega(\omega + \omega_c)} \qquad (47)$$

where ($\gamma = j\beta$), and that for eq. (45) are some spots on the curve of $K^2 = -k_g^2$, that is:

$$\beta^2 = k^2 \frac{\omega^2 - \omega\omega_c - \omega_p^2}{\omega(\omega - \omega_c)} \qquad (48)$$

Carefully theoretical study and numerical calculations show that we have three kinds of curves on dispersion diagram: dispersion curves (eq. (31) or (17) for example), dispersion curves (eq. (44), (45)), and D=0 curve. These three kinds of curves may go across each other on some common discrete points, as shown in Fig. 10. At these points the waves consist of two parts, electromagnetic wave parts and quasi-static parts.

We, therefore, have found a new kind of waves that contain quasi-static field components, and there are two kinds of waves: electromagnetic waves, for which $p_1^2 \neq 0$, $p_2^2 \neq 0$, and waves containing quasi-static parts, for which $p_2^2 = 0$ (or $p_1^2 = 0$). The electromagnetic waves have been well studied, but the new kind of waves that contains quasi-static parts is unknown in published papers.

Thus we have found that the wave propagation along waveguide filled with plasma in finite magnetic field may have quite unusual dispersion characters.

VI. The question of "the coupling between dispersion curves of

Thus, we call in question of "the coupling between dispersion curves of mode". We think these curvatures represent their own dispersion characters of each mode, coupling does not exist.

VII. Conclusion

This paper gives a completely study of the wave propagation along waveguide both partially and completely filled with plasma immersed in finite magnetic field. The following main contributions have been obtained:

1. It shows that there exist quite big difference of cut-off frequencies and dispersion waves between partially and completely plasma filled cases. These differences have been discussed in sections III and IV. It shows, for example, that by means of changing the thickness of the dielectric tube, one can tun the dispersion curves of the propagating waves in partially plasma filled waveguide, and there are more condensed modes of waves near $\omega = \omega_h$ in partially plasma filled case.

2. It has been found that the calculations given in [5], [6], [8] should be checked or corrected. For example, the curve of $p_2^2 = 0$ in the eigen values diagram shown in [4]-[6], [8] has two lines seems incorrect, it only has one line as shown in Fig. 4 in this paper. It shows that the so called anisotropic wave HE_1 presented in [5] seems can not exist, and it also shows that the dispersion curves of HE_1 given in [5] is really nothing but the curve of $p_2^2 = 0$.

3. A new kind of waves has been found . These waves contain quasi-static field components and are associated with D=0, $p_2^2 = 0$. The dispersion characters are quite unusual: the dispersion characteristics depend on the discrete points, rather than curves, which are the common cross points of the curves $p_2^2 = 0$, the usual dispersion curves of wave propagation (eq. (31) or eq. (17), and the dispersion curves given in section V and Appendix of this paper.

4. The paper calls in question the concept of "coupling of dispersion curves between modes" presented in [2] and [5], and presented an alternative

$\omega = \omega_h$, where $\omega_h = \left(\omega_c^2 + \omega_p^2\right)^{\frac{1}{2}}$, and are happened among the dispersion curves of the modes the cut-off frequencies of which are close or even less than $\omega = \omega_h$. It reminds us that, at $\omega = \omega_h$, all modes can not propagate, there is disruption of wave propagation at $\omega = \omega_h$ for all modes and waves. It will be shown that by using this fact, the unusual curvature sharp of the dispersion curves of some related modes may be explained successfully.

As shown in Fig. 3, with the increasing of ω_c, more modes are condensed up near ω_h. Their cut-off frequencies should all be a little bit higher than ω_h, the dispersion curves of these modes must be unusually curvature in the frequency region close to the line of $\omega = \omega_h$. Because the dispersion curves can not go across the line $\omega = \omega_h$ and the factor $1/\left(\omega - \omega_h\right)$, is involved in the dispersion equations through ε_1, ε_2, ε_3 and p_1^2, p_2^2, and becomes very large in the vicinity of $\omega = \omega_h$, it is obvious that the influence of this factor on the dispersion curves is very strong. Therefore, the dispersion curves of these modes (the cut-off frequencies are less than ω_h or close to ω_h) must have unusual curvature. These facts are enough to explain the unusual curvature sharp of the dispersion curves of the related modes.

We have the following points: at first, for the modes their cut-off frequencies are a little bit higher than ω_h, there are condensed dispersion curves of modes near the line $\omega = \omega_h$. Secondly, as a results of these condensed dispersion curves, these curves must have unusually curvature. Thirdly, in addition to these two points, for those modes, the dispersion curves of which can be close to the line of $\omega = \omega_h$, the factor of $1/\left(\omega - \omega_h\right)$ becomes very large in the vicinity of $\omega = \omega_h$ line, and this factor is involved in the dispersion equation, therefore, its influence on the dispersion curves is strong. And finally. the lines of D=0 and p=0 in the partially plasma filled case also give some influences on the dispersion curves.

12. Song Liqun and Liu Shenggang, Acta Electronics, (1995)

13. Liu Shenggang and Zhu Dajun. An alternative analysis for wave propagation along plasma waveguide in finite magnetic field. Submitted for publication and presented at the first academic meeting of the national key project "The Fundamental Research on Microwave Plasma and its Applications". Jan. 4-6, 1996, Kunming, China.

physical explanation on the unusual sharp of the dispersion curves of some modes.

Thus, the paper provides a good theoretical ground for better understanding in physics and further study of the problems of wave propagation along waveguide filled with plasma in finite magnetic field, that is very important in a variety of areas in science and engineering, such as high power microwave generation and microwave excited plasmas.

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References

1. A. W. Trivelpiece and R. W. Gould, J. Appl. Phys. 30, 1784(1959).
2. V. Bevc and T. E. Everhart, J. of Electron. Control. 13, 185(1962).
3. Liu Shenggang, Wave propagation along waveguide filled with plasma immersed in finite magnetic field. ICPST'94, C1, 34(1994).
4. M. Moison and J. Pelletier, Microwave Excited Plasma (Elsevier 1992), Chap. 4-6.
5. S. T. Ivanov and E. G. Alexov, J. of Plasma Physics, 43, 1, 51(1990).
6. S. T. Ivanov and E. G. Alexov, J. of Plasma (in Russian). 15, 1131(1989).
7. H. S. Uhm, et al. J. Appl. Phys. 64, 1108(1988).
8. S. T. Ivanov, E. G. Alexov and P. N. Malinov, Plasma Physics and Controlled Fusion, 31, 6, 941-953(1989).
9. J. Margot and M. Moisan, J. Physics D: Applied Physics. 24, 1765(1991).
10. M. V. Kugelev, et al. Soviet J. Plasma Physics. 2, 244 (1982, JETP, 4, 1358).
11. J. A. Kong, Electromagnetic Wave Theory, (John Willey & Sons, Inc., 1985)

. Equation (A8) is taken into consideration first, which is corresponding to LHP waves. When $K^2 = -k_g^2$, the equation (A8) becomes:

$$jE_r + E_\theta = \frac{j}{2K^2}\left[-\gamma\left(\frac{\partial E_z}{\partial r} + \frac{m}{r}E_z\right) + \omega\mu_0\left(\frac{\partial H_z}{\partial r} + \frac{m}{r}H_z\right)\right] \qquad \text{(A10)}$$

From the Maxwell's equations:

$$\nabla \times \bar{E} = -j\omega\mu_0\bar{H} \qquad \text{(A11)}$$

So we get:

$$\frac{\partial E_\theta}{\partial r} + \frac{E_\theta}{r} - j\frac{m}{r}E_r = -j\omega\mu_0 H_z \qquad \text{(A12)}$$

The equation for E_θ can be derived from (A10) and (A12):

$$r\frac{\partial E_\theta}{\partial r} + (m+1)E_\theta = -j\omega\mu_0 r H_z + \frac{jm}{2K^2}\left[-\gamma\left(\frac{\partial E_z}{\partial r} + \frac{m}{r}E_z\right) + \omega\mu_0\left(\frac{\partial H_z}{\partial r} + \frac{m}{r}H_z\right)\right] \qquad \text{(A13)}$$

The equation (A13) can be transformed into following:

$$r^{-m}\left[\frac{\partial}{\partial r}\left(r^{m+1}E_\theta\right)\right] = -j\omega\mu_0 r H_z + \frac{jm}{2K^2}r^{-m}\left\{-\gamma\left[\frac{\partial}{\partial r}\left(r^m E_z\right)\right] + \omega\mu_0\frac{\partial}{\partial r}\left(r^m H_z\right)\right\}. \qquad \text{(A14)}$$

The E_θ can be solved from (A14), and the field components can be obtained:

$$E_\theta = -j\omega\mu_0\left[A_1 h_1\frac{J_{m+1}(p_1 r)}{p_1} + A_2 h_2\frac{r^{m+1}}{2m+2}\right] + \frac{jm}{2K^2 r}\left(-\gamma E_z + \omega\mu_0 H_z\right) \qquad \text{(A15)}$$

$$E_r = \omega\mu_0\left[A_1 h_1\frac{J_{m+1}(p_1 r)}{p_1} + A_2 h_2\frac{r^{m+1}}{2m+2}\right] + \frac{1}{2K^2}\left(-\gamma\frac{\partial E_z}{\partial r} + \omega\mu_0\frac{\partial H_z}{\partial r}\right) \qquad \text{(A16)}$$

$$H_r = \frac{j}{\omega\mu_0}\left(\gamma E_\theta + \frac{jm}{r}E_z\right) \qquad \text{(A17)}$$

Appendix Field distribution under D=0

According to the definition of a, b, c, d:

$$a = \left(\gamma^2 + k^2\varepsilon_1\right)\varepsilon_3 / \varepsilon_1 \tag{A1}$$

$$b = j\omega\mu_0\gamma\varepsilon_2 / \varepsilon_1 \tag{A2}$$

$$c = \gamma^2 + k^2\left(\varepsilon_1^2 + \varepsilon_2^2\right) / \varepsilon_1 \tag{A3}$$

$$d = -j\omega\varepsilon_0\gamma\varepsilon_2\varepsilon_3 / \varepsilon_1 \tag{A4}$$

Under the condition of D=0, where $D = K^4 - k_g^4$, we can get:

$$p_1 = a + c, \quad p_2 = 0$$

$$h_1 = -\frac{c}{b} \qquad h_2 = \frac{a}{b} \tag{A5}$$

From equations (3)-(7), under $p_2 = 0$, we can get the expressions of E_Z and H_Z:

$$E_z = A_1 J_m(p_1 r) + A_2 r^m \tag{A6}$$

$$H_z = A_1 h_1 J_m(p_1 r) + A_2 h_2 r^m \tag{A7}$$

Since D=0, the other field components can not be derived directly from the E_Z and H_Z under the condition of D=0, and a different method is employed to get them.

The E_r and E_θ satisfy following equations:

$$jE_r + E_\theta = \frac{j}{K^2 - k_g^2}\left[-\gamma\left(\frac{\partial E_z}{\partial r} + \frac{m}{r}E_z\right) + \omega\mu_0\left(\frac{\partial H_z}{\partial r} + \frac{m}{r}H_z\right)\right] \tag{A8}$$

$$jE_r - E_\theta = \frac{j}{K^2 + k_g^2}\left[\gamma\left(-\frac{\partial E_z}{\partial r} + \frac{m}{r}E_z\right) - \omega\mu_0\left(\frac{\partial H_z}{\partial r} - \frac{m}{r}H_z\right)\right] \tag{A9}$$

D=0 means $K^2 = k_g^2$ or $K^2 = -k_g^2$. The equation (A8) is employed to derive fields for $K^2 = -k_g^2$ and equation (A9) for $K^2 = k_g^2$.

(A25)

Actually the dispersion equations in section II, equations (17), (30) and (31), should be written more accurately:

$$\frac{F(k,\gamma,\varepsilon_1,\varepsilon_g,\varepsilon_3)}{D}=0 \qquad \text{(A26)}$$

Where $$D=K^4-k_g^4 \qquad \text{(A27)}$$

and $F(k,\gamma,\varepsilon_1,\varepsilon_g,\varepsilon_3)=F(p_1,p_2,p)$ is the rest parts of the dispersion equation. The dispersion equation, for example, eq. (31) and eq. (17) are valid only when

$$D\neq 0 \qquad \text{(A28)}$$

If eq. (A28)is not fulfilled, we get $p_2=0$, then we have the case discussed in this appendix. It is very important to note that we can prove that when $D\rightarrow 0$, the limit of all field components are exist and approach to the results given in this appendix, and the dispersion equation (A26) reduces to eq. (A24) or (A25).

For $p_1^2=0$, we can get similar results.

$$H_\theta = -\frac{j}{\omega\mu_0}\left(\gamma E_r + \frac{\partial E_z}{\partial r}\right) \tag{A18}$$

The transverse field components of RHP waves, which corresponding to $K^2 = k_g^2$, can be obtained in a similar way:

$$E_\theta = -j\omega\mu_0\left[A_1 h_1 \frac{J_{m+1}(p_1 r)}{p_1} + A_2 h_2 \frac{r^{m+1}}{2m+2}\right] + \frac{jm}{2K^2 r}(\gamma E_z + \omega\mu_0 H_z)$$

(A19)

$$E_r = -\omega\mu_0\left[A_1 h_1 \frac{J_{m+1}(p_1 r)}{p_1} + A_2 h_2 \frac{r^{m+1}}{2m+2}\right] - \frac{1}{2K^2}\left(\gamma\frac{\partial E_z}{\partial r} + \omega\mu_0\frac{\partial H_z}{\partial r}\right)$$

(A20)

$$H_r = \frac{j}{\omega\mu_0}\left(\gamma E_\theta - \frac{jm}{r}E_z\right) \tag{A21}$$

$$H_\theta = -\frac{j}{\omega\mu_0}\left(\gamma E_r + \frac{\partial E_z}{\partial r}\right) \tag{A22}$$

In completely plasma filled case, the dispersion equation can be derived by the boundary condition:

$$E_z\big|_{r=R_c} = E_\theta\big|_{r=R_c} = 0 \tag{A23}$$

So the dispersion equation for LHP wave is:

$$h_1\left[\frac{m}{2K^2R_c}J_m(p_1R_c) - \frac{J_{m+1}(p_1R_c)}{p_1}\right] - h_2 J_m(p_1R_c)\left[\frac{m}{2K^2R_c} - \frac{R_c}{2m+2}\right] = 0$$

(A24)

And the dispersion equation for RHP is:

$$h_1\left[\frac{m}{2K^2R_c}J_m(p_1R_c) + \frac{J_{m+1}(p_1R_c)}{p_1}\right] - h_2 J_m(p_1R_c)\left[\frac{m}{2K^2R_c} + \frac{R_c}{2m+2}\right] = 0$$

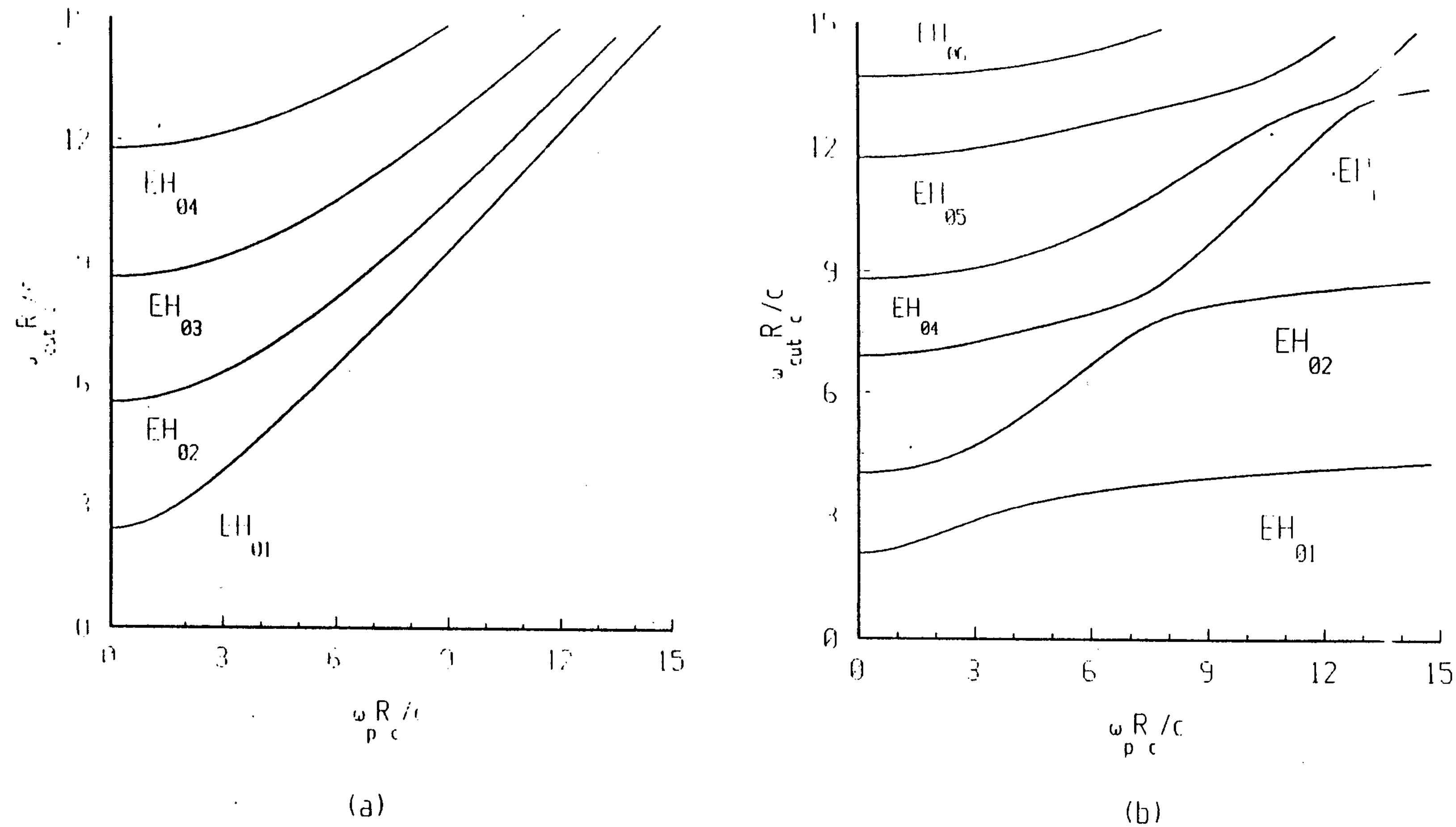

Fig. 1 Cut-off frequencies of EH waves versus plasma frequency ω_p

(a) Completely plasma filled case. (b) Partially plasma filled case (R_p/R_c=0.7, ε_D=4)

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(a) Completely plasma filled case.

(b) Partially plasma filled case (R_p/R_c=0.7, ε_D=4)

Fig. 2 Cut-off frequencies of HE waves vs. plasma frequency ω_p ($\omega_c R_c/c$=5)

(a) Completely plasma filled case.

(b) Partially plasma filled case (R_p/R_c=0.7, ε_D=4)

Fig. 3 Cut-off frequencies of HE waves vs. cyclotron frequency ω_c

(ω_p=1.85 × $10^{10}s^{-1}$, R=1.5cm). (a) Completely plasma filled case.

(b) Partially plasma filled case (R_p/R_c=0.7, ε_D=4)

Fig. 4 Areas which determine the signs of eigen values p_1^2 and p_2^2

, $p_1^2 > 0$; , $p_2^2 > 0$; , p_1^2 and p_2^2 are complex; , $p_1^2 < 0$, $p_2^2 < 0$

Fig. 5 Dispersion curves of the lowest modes of each type in a completely plasma filled waveguide for B_0=0.175T (ω_c/ω_p=1.86),

ω_p=1.85 × $10^{10}s^{-1}$, R=1.5cm.

Fig. 6 Dispersion curves of symmetrical waveguide modes in a completely plasma filled waveguide for ω_p=1.85 × $10^{10}s^{-1}$, R=1.5cm:

(a) ω_c/ω_p=5.76, (b) ω_c/ω_p=5.86

Fig. 7 Dispersion curves of waveguide modes in a completely plasma filled waveguide for ω_p=1.85 × $10^{10}s^{-1}$, R=1.5cm and ω_c/ω_p=4.07.

Fig. 8 Dispersion curves of the non-symmetrical waveguide modes in a completely plasma filled waveguide for ω_p=1.85 × $10^{10}s^{-1}$, R=1.5cm

and (a) ω_c=8.8 × $10^{10}s^{-1}$, (b) ω_c=14.14 × $10^{10}s^{-1}$.

Fig. 9 Dispersion curves of symmetrical waveguide modes in a partially plasma filled waveguide for ω_p=1.85 × $10^{10}s^{-1}$, R_p/R_c=0.714,

ε_D=1, R_c=2.95cm, and (a) ω_c/ω_p=0.75, (b) ω_c/ω_p=4.14.

Fig. 10 Three kinds of curves on dispersion diagram (m=0, $\omega_c R/c$=5,

$\omega_p R/c$=6). (The solid lines from eq. (31), the dashed lines

from eq. (45), the dotted line for D=0)

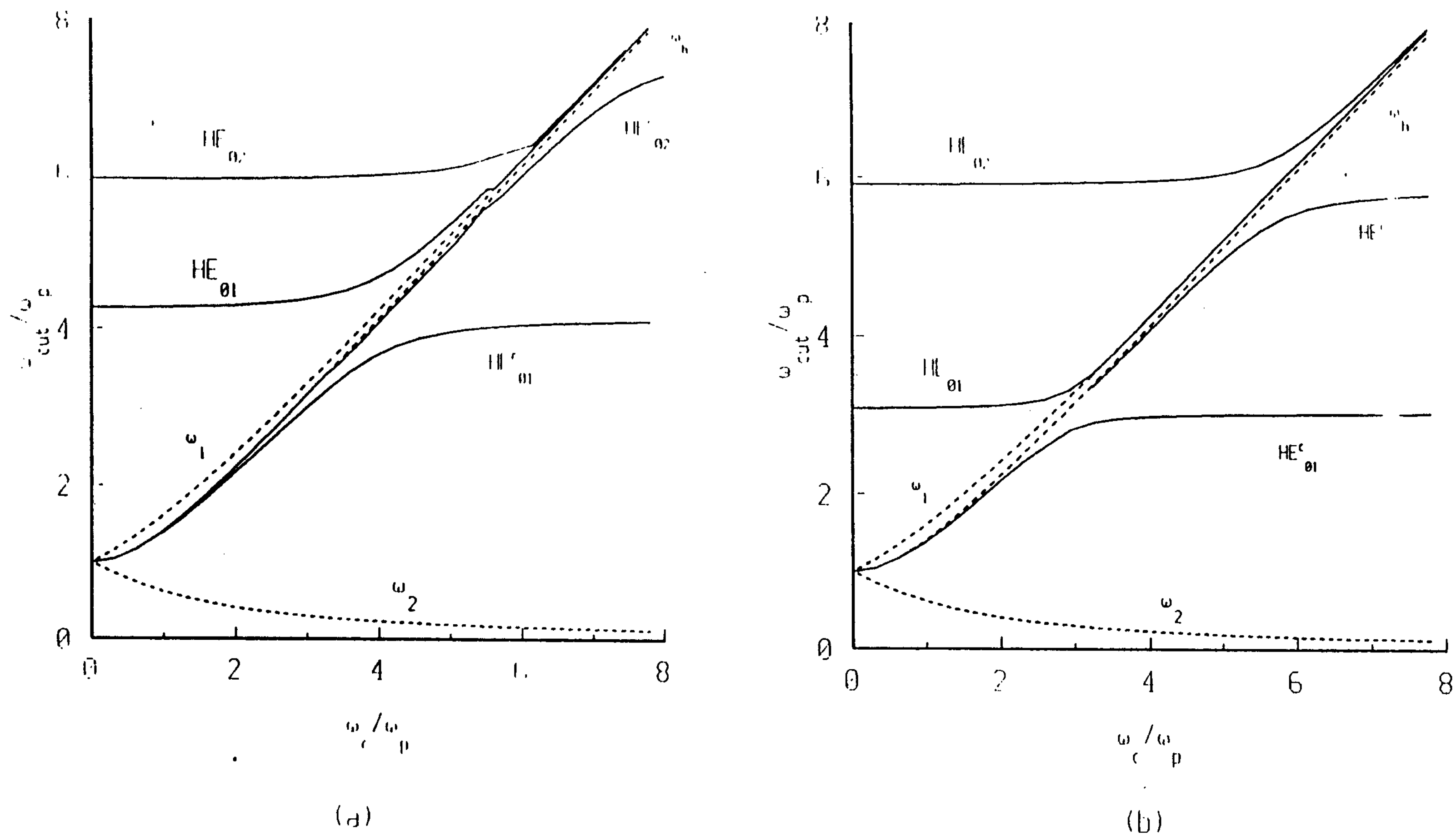


Fig. 3 Cut-off frequencies of HE waves vs. cyclotron frequency ω_c (ω_p=1.85 × $10^{10}s^{-1}$, R=1.5cm). (a) Completely plasma filled case. (b) Partially plasma filled case (R_p/R_c=0.7, ε_D=4)

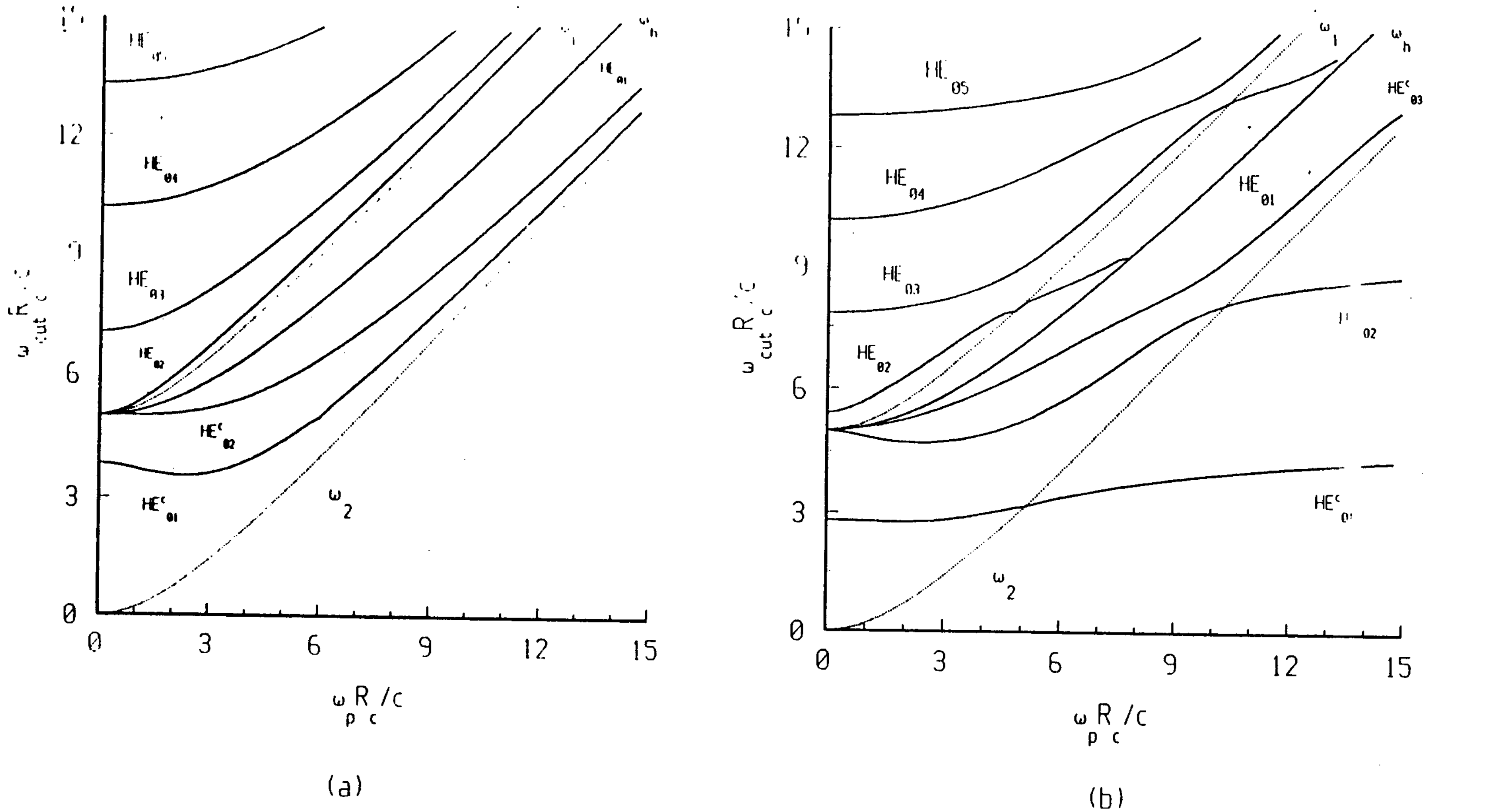
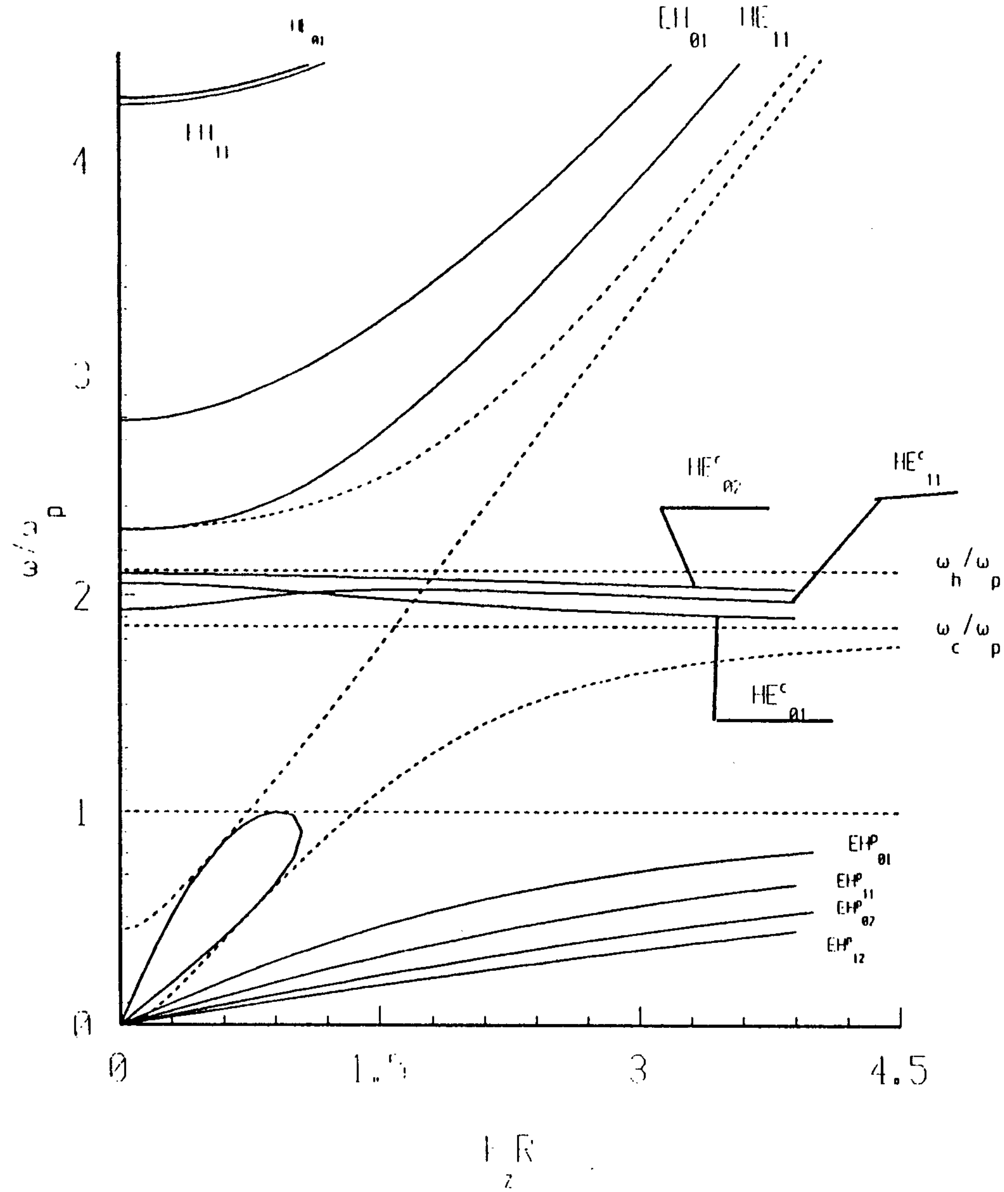


Fig. 2 Cut-off frequencies of HE waves vs. plasma frequency ω_p ($\omega_c R_c/c=5$)
(a) Completely plasma filled case. (b) Partially plasma filled case ($R_p/R_c=0.7$, $\varepsilon_D=4$)



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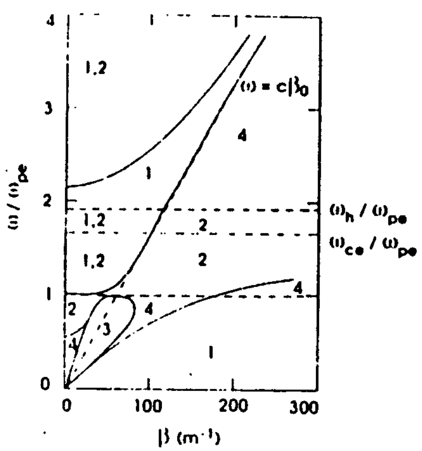
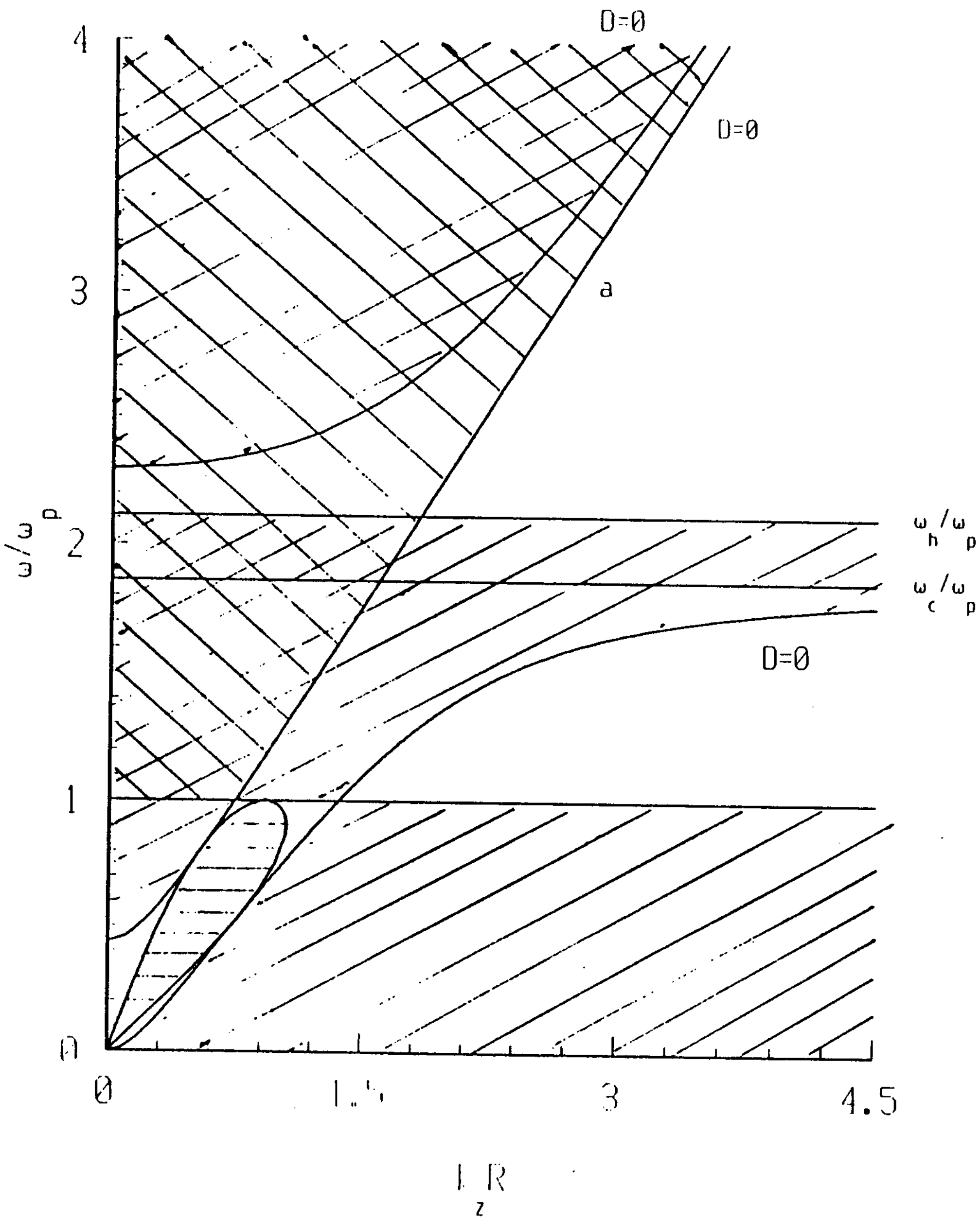


Fig. 1 in reference [5]



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▨, $p_1^2 > 0$; ▨, $p_2^2 > 0$; ▭, p_1^2 and p_2^2 are complex; ▭, $p_1^2 < 0$, $p_2^2 < 0$

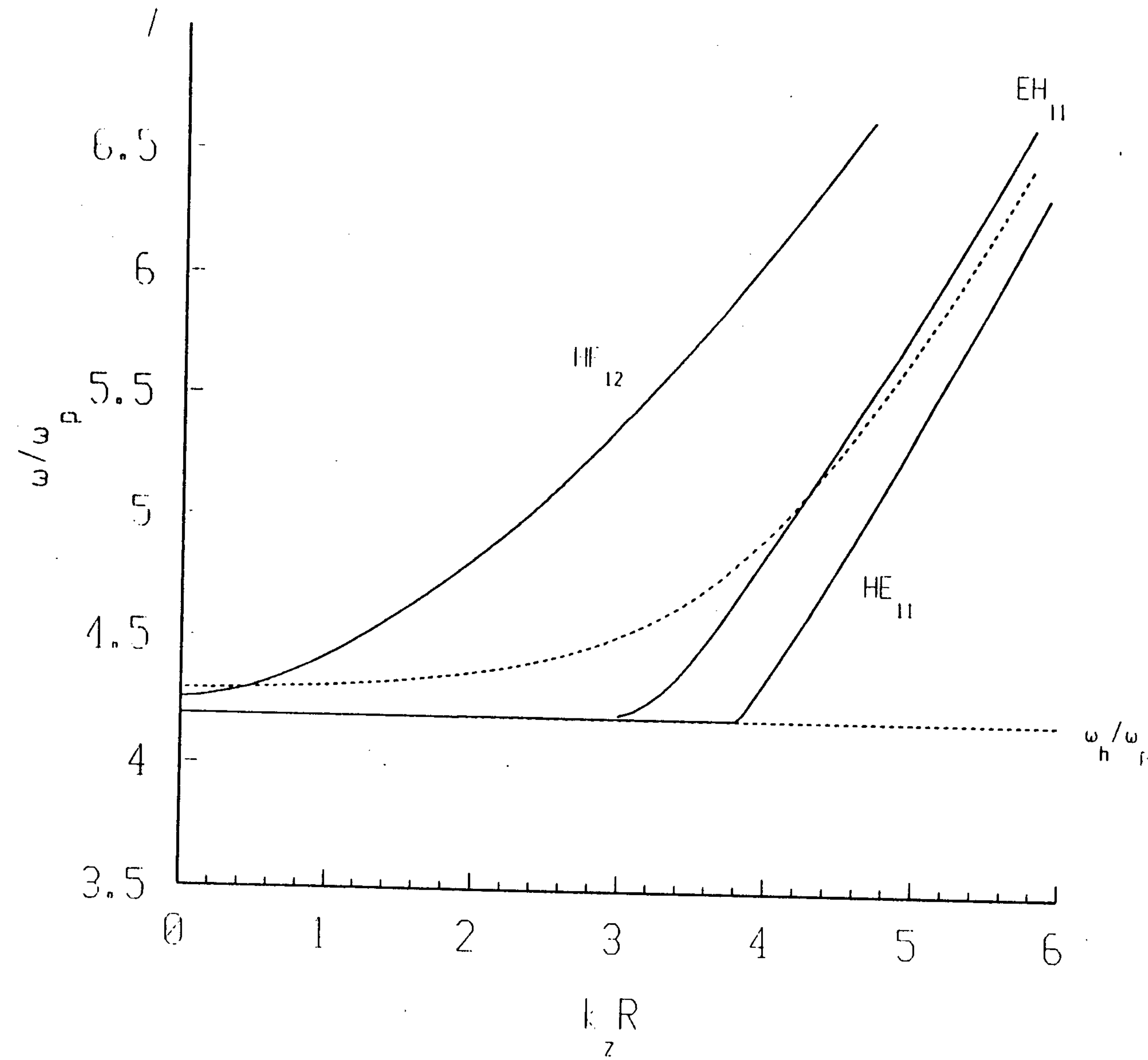

Fig. 7 Dispersion curves of waveguide modes in a completely plasma filled waveguide for $\omega_p = 1.85 \times 10^{10} s^{-1}$, R=1.5cm and $\omega_c/\omega_p = 4.07$.

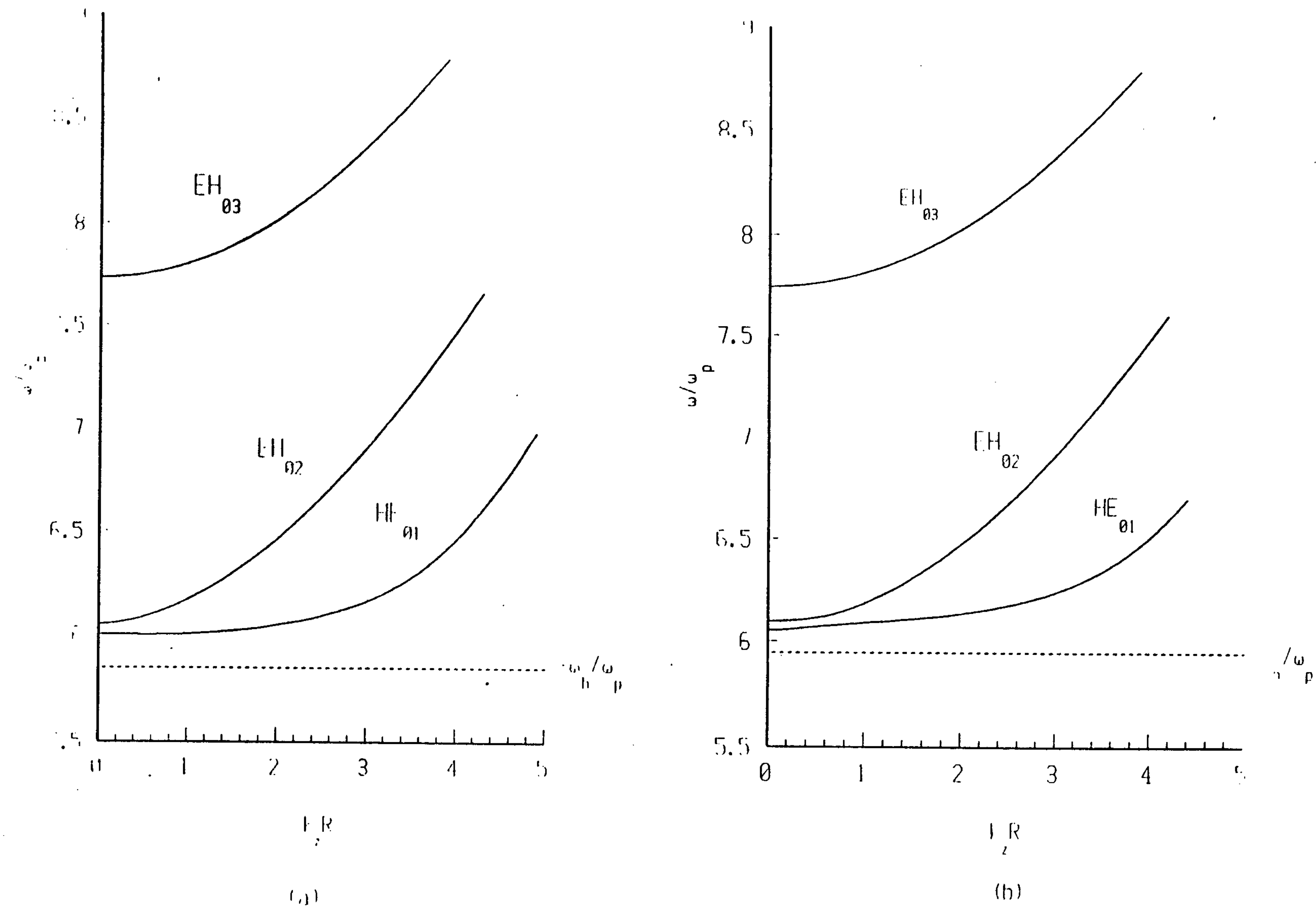

Fig. 6 Dispersion curves of symmetrical waveguide modes in a completely plasma filled waveguide for $\omega_p = 1.85 \times 10^{10} s^{-1}$, R=1.5cm: (a) $\omega_c/\omega_p = 5.76$, (b) $\omega_c/\omega_p = 5.86$

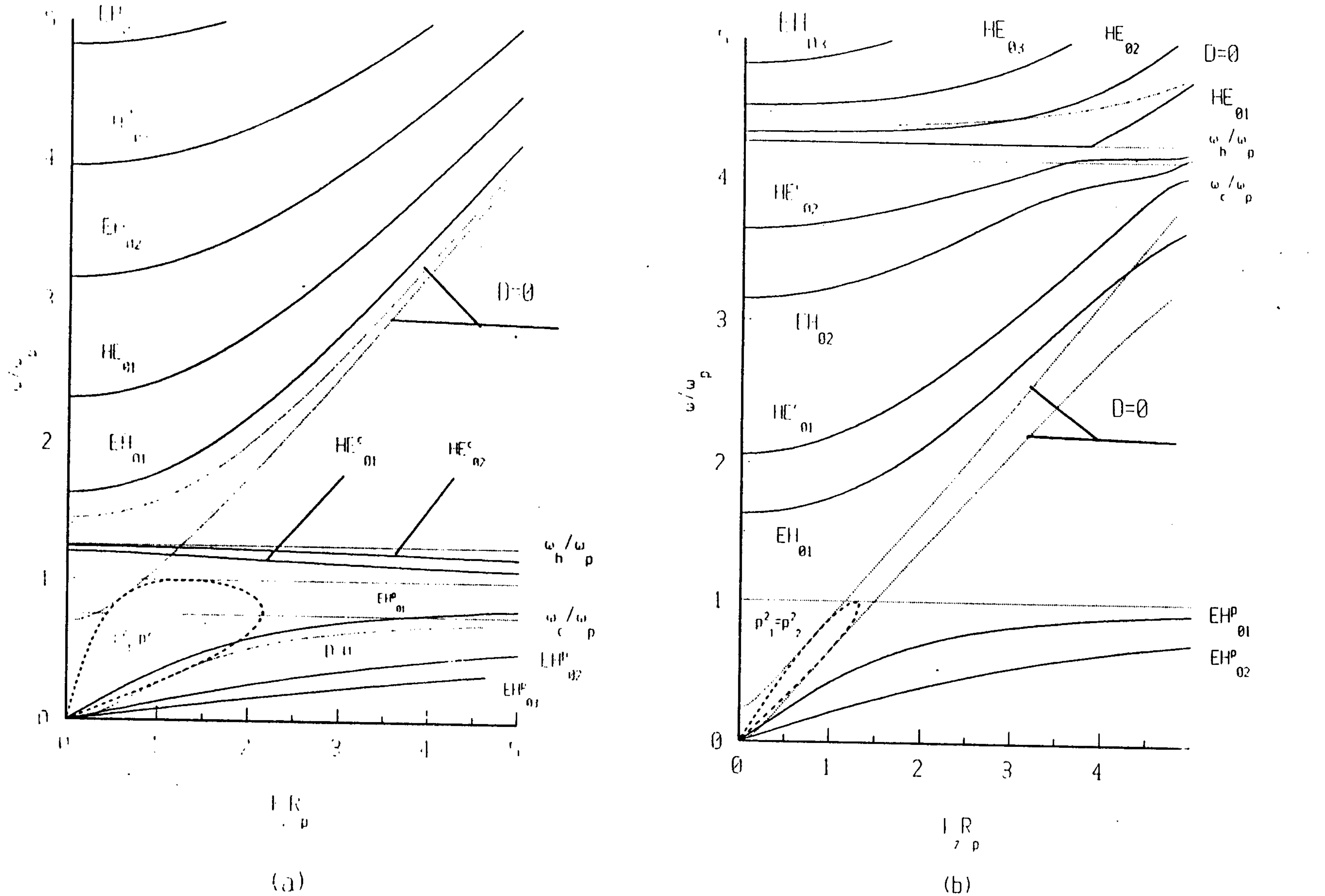

Fig. 9 Dispersion curves of symmetrical waveguide modes in a partially plasma filled waveguide for $\omega_p=1.85\times10^{10}s^{-1}$, $R_p/R_c=0.714$, $\varepsilon_D=1$, $R_c=2.95$cm, and (a) $\omega_c/\omega_p=0.75$, (b) $\omega_c/\omega_p=4.14$.

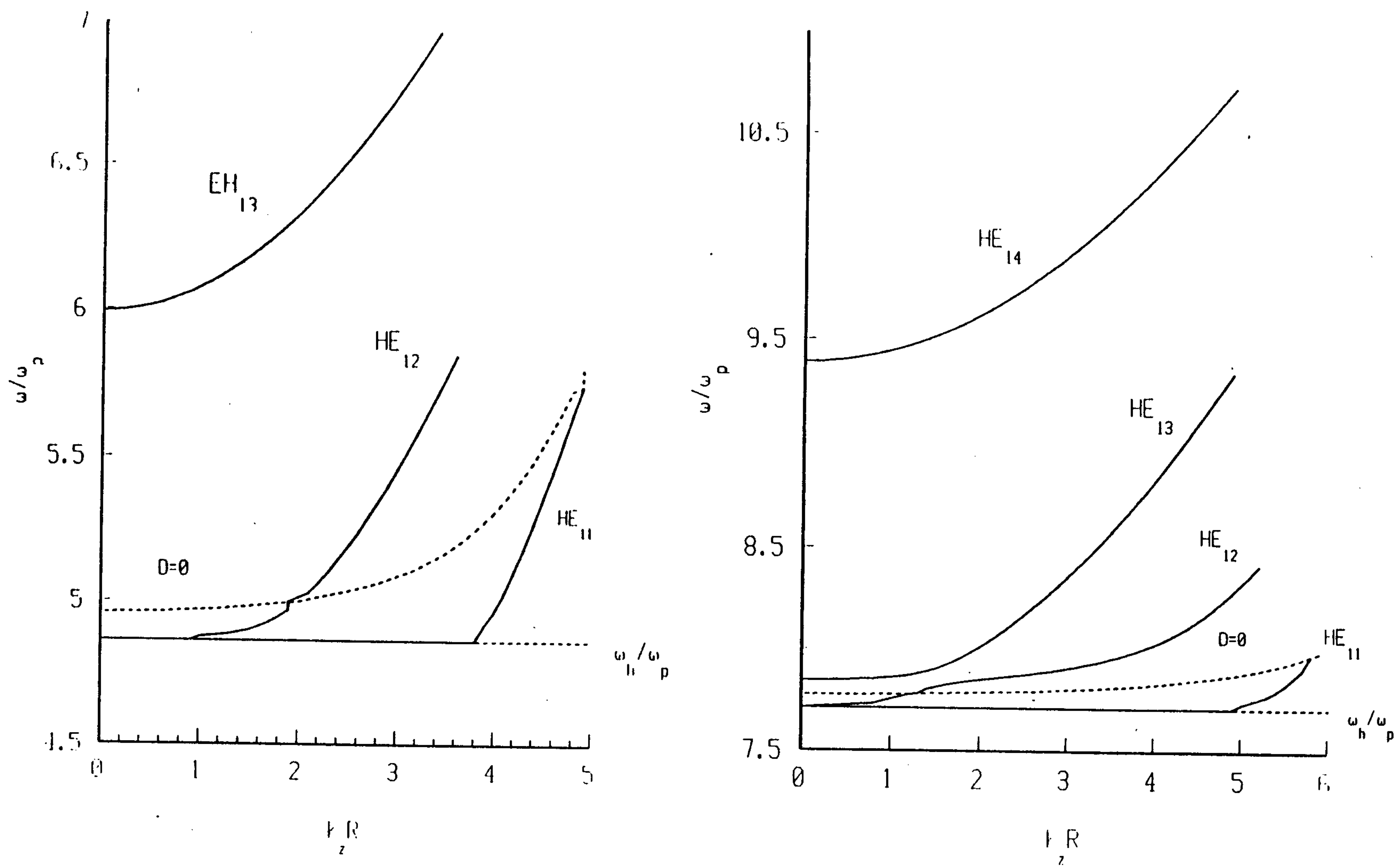

Fig. 8 Dispersion curves of the non-symmetrical waveguide modes in a completely plasma filled waveguide for $\omega_p=1.85\times10^{10}s^{-1}$, R=1.5cm and (a) $\omega_c=8.8\times10^{10}s^{-1}$, (b) $\omega_c=14.14\times10^{10}s^{-1}$.

9. Robert J. Barker and Shenggang Liu, "An Examination of Plasma Microwave Electronics".

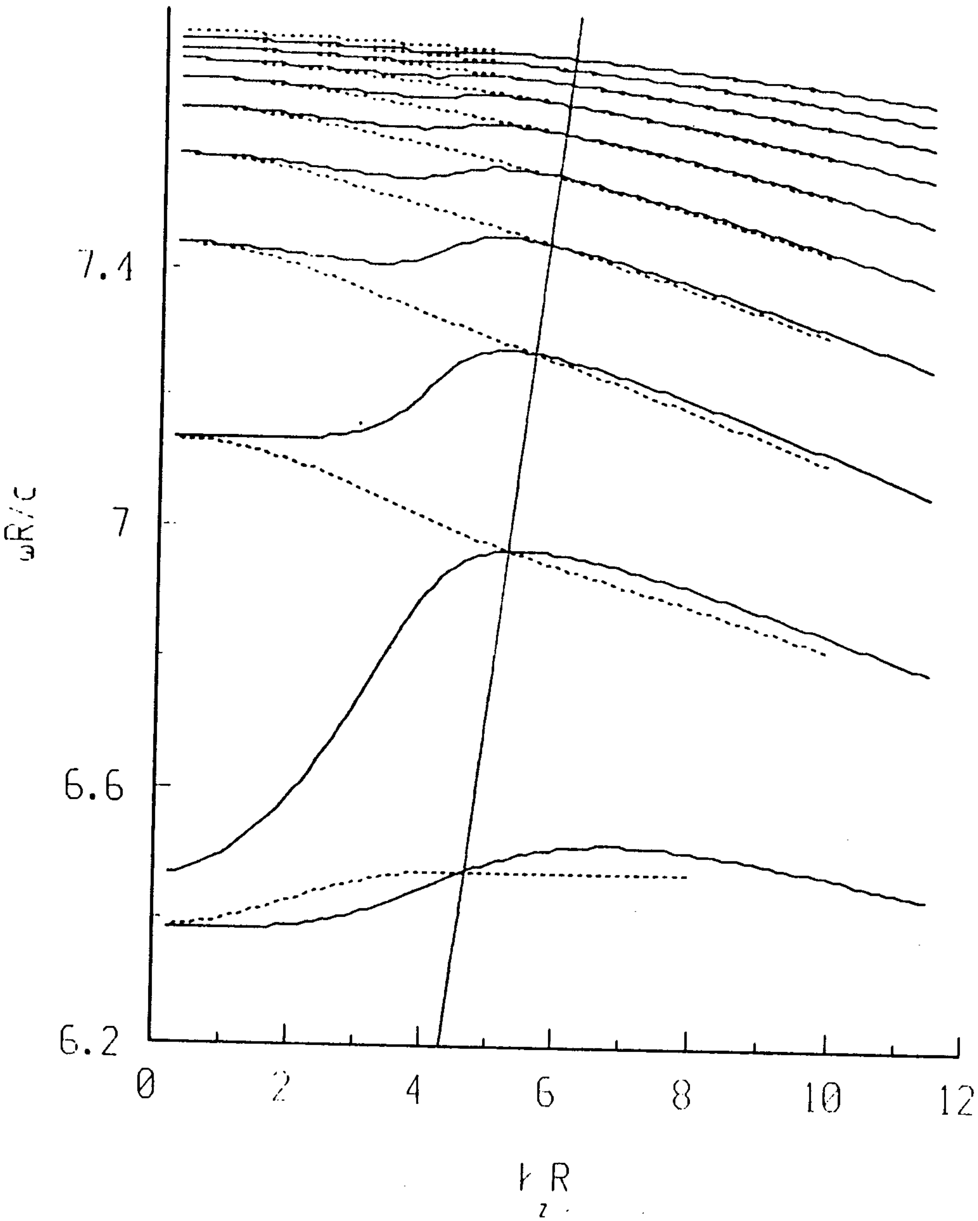

Fig. 10 Three kinds of curves on dispersion diagram (m=0, $\omega_c R/c=5$, $\omega_p R/c=6$)
(The solid lines from eq. (31), the dashed lines from eq. (45), the dotted line for D=0)

2

An Examination of Plasma Microwave Electronics *

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Drafted by
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Abstract - *Gas in microwave tubes is an anathema to the traditional microwave vacuum electronics (MVE) community. The surge in high power microwave (HPM) R&D that began in the late 1970s signaled a departure from that engineering mindset. The plasma that poisoned the thermionic cathode and destroyed signal purity in MVE devices became a shining hope of HPM researchers. Only cathode plasmas could supply the kiloamperes per square centimeter necessary to drive the proposed HPM tube concepts. A plasma fill provided an ion background that helped neutralize electron-beam space charge, thereby permitting greatly elevated beam currents. Unfortunately, the cathode plasma that supplied prodigious numbers of electrons to feed the drive beam also expanded to short out anode-cathode gaps. The highly nonuniform cathode plasma surface is unable to produce the uniform high-quality e-beam necessary for high efficiency and reproducible output performance. This paper discusses these and other blessings and curses that are held by plasma for the overall microwave community. A detailed analytical formulation is proposed to facilitate studies in this challenging field.*

The microwave tube engineers who are in the business of producing rugged, reliable microwave sources take great pains to eliminate every trace of residual gas from their devices. To achieve vacuums better than 10(-9) Torr, every internal component is meticulously cleaned. No plastics or liquids are allowed inside the vacuum envelope. The entire device is baked for days in order to facilitate the rigorous pump-out. Afterward, cathodes are fired repeatedly thousands and even millions of times to "condition" them, boiling off impurities and increasing beam quality.

These precautions are meant to protect the active thermionic cathode surface from being poisoned and reducing beam current. They also minimize the presence of ions in the tube that could be accelerated to cause impact damage to various components. In addition, they reduce chances for surface flashover of contaminants that short out segments of the microwave configuration. It is not surprising that gas (and resultant plasma) is the enemy of traditional microwave tube engineers.

The late 1970s witnessed the birth of a new field of microwave source research that sought extremely high power microwave (HPM) devices. Significantly, the overwhelming majority of the researchers who flocked to this new field emerged from the plasma physics rather than from the electrical engineering community. The reasons for this staffing anomaly are debatable but the consequences are clear. HPM researchers view plasma sympathetically, without the gas-abhorrence traditional to vacuum electronics. For them, a vacuum of 10(-6) Torr could be quite sufficient. Furthermore, the basic research required by their new crop of HPM source concepts mandated the ability to readily take apart and reassemble their microwave devices in order to continuously make adjustments and/or replace components that had succumbed to the omnipresent intense fields and beam. In short, the HPM community worked with "gassy" tubes (by the standards of traditional vacuum electronics) right from the start. It is only natural that this field gave birth to the Plasma Microwave Electronics (PME) area that is the subject of this paper.

The initial motivation for injecting plasma background fills into microwave tubes was to boost beam current. HPM demands high power electron drive beams. Safety considerations limit the allowed beam voltage, thus encouraging higher beam currents. Unfortunately, space-charge blow-up restricts the

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Sept. 3-9, 1999

11. Conclusion

It is the tradition that Chinese people always pay much attention to education, in particular, the higher education. There is an old proverb in China, it says, To plant a tree, it takes 10 years, but to plant a person, it requires 100 years. Education has been existing in China more than 4000 years. To further improve the education, my idea is: keep our own feature and learn from outside—do our best for the international exchange and cooperation.

I have been working in universities more than 40 years. My experience is that the most important goal of university is to train students to have some good customs rather than to teach them some new knowledge. Having a good custom, students can keep up study through their life, maximum beam current in vacuum devices. The ions in a background plasma, however, help to neutralize that negative space-charge and thereby allow greater beam current densities and powers. [1]

HPM cathodes also demand the presence of a plasma. To produce the prodigious electron-beams necessary for these tubes, cathodes were required to emit kiloamperes per sq.cm. This was far beyond the capabilities of the best thermionic cathodes. Only explosive electron emission (EEE) in which an electron-emitting surface plasma was produced by flashover at the cathode, could fulfill this need.

More recent studies have found other benefits to be reaped from plasmas in microwave devices. The background plasma has been shown to provide *frequency tunability* to a microwave source. By adjusting the plasma density (and/or the axial magnetic field strength) one could change the operating frequency of a given device by 10% or more while leaving the geometry and e-beam parameters fixed. [2] Furthermore, the frequency upshifting realizable in such devices could somewhat relax the engineering costs to produce higher frequency tubes.

It was also found that for certain parameter regimes, a plasma fill could reduce or even eliminate the need for the heavy, expensive axial guide-field magnets. In effect, the plasma not only negated the beam-electrons' electrostatic self-repulsion, but also allowed their beam current attractive forces to dominate, resulting in self-pinching of the beam. [3]

Even more interesting from a scientific standpoint, the plasma background could serve as an *active* component of the microwave source. The dynamics of e-beam/plasma interactions result in plasma waves that participate in the microwave generation process. Given the correct choice of parameters, these waves could replace traditional metal slow-wave-structures. The richness of new instabilities has opened the door to entirely new device concepts. [4]

Finally, it must be admitted that *if* one could accept 10(-6) Torr vacuums in microwave tubes, the fabrication costs would be dramatically reduced.

Conventional vacuum electronics condemns gas (plasma) fills for the reasons already stated. The HPM community has slowly grown to concede that even in their devices, although devoid of thermionics and "more relaxed' regarding signal purity, plasma can be an enemy. It is now generally accepted that internal plasmas are the single greatest cause for the "pulse shortening" phenomenon that has always plagued that field. Plasma in the "wrong" places can short-out critical structural components and vastly complicate the problem of microwave extraction from an HPM device. It is clear that the plasma blessings have come with a price.

This paper seeks to avoid the *emotional* prejudices of both the vacuum electronics and HPM communities and to present the benefits and the costs of plasma microwave electronics from a strictly technical perspective. A fact that emerges is that *both* communities continue to benefit from the scientific dialogue that has emerged from this debate.

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References

[1] R.J. Barker, "Advanced Research in Millimeter-Wave Vacuum Electronics," *Int. Journal of IR&MM-Waves*, Vol. 10, No.10, pp. 1213-1222, 1989.

[2] G.S. Nusinovich, Y. Carmel, T.M. Antonsen, Jr., D.M. Goebel, and J. Santoru, "Recent Progress in the Development of Plasma-Filled Traveling-Wave Tubes and Backward-Wave Oscillators," *IEEE Trans. on Plasma Science*, Vol. 26, No. 3, pp. 628-645, June 1998.

[3] D.M. Goebel, J.M. Butler, R.W. Schumacher, J. Santoru, and R.L. Eisenhart, "High-Power Microwave Source Based on an Unmagnetized Backward-Wave Oscillator," *IEEE Trans. on Plasma Science*, Vol. 22, No. 5, pp. 547-553, Oct 1994.

[4] Liu Shenggang, R.J. Barker, et al., "Basic Theoretical Formulations of Plasma Microwave Electronics – Parts I & II," submitted to *IEEE Trans. on Plasma Science*, May 1999.

10. Karl H. Schoenbach, Robert J. Barker, and Shenggang Liu, "Nonthermal Medical/Biological Treatments Using Electromagnetic Fields and Ionized Gases".

8. Development and Enterprises.

Some universities have their own R$D centers and enterprises.

9. Budget

Central Government gives Budget to the universities and colleges affiliated by national minstries and local governments give budgets to universities and colleges affiliated by local governments each year. It is always far from enough.

10. Reform

Different reform approaches are carried on in institutions of higher learning in order to improve the efficiency and to upgrade the academic level.

6

NONTHERMAL MEDICAL/BIOLOGICAL TREATMENTS USING ELECTROMAGNETIC FIELDS AND IONIZED GASES

Karl H. Schoenbach (Old Dominion University), Robert J. Barker (U.S. Air Force Office of Scientific Research), and Shenggang Liu (University of Electronic Science and Technology of China)

Abstract

The application of intense electrical pulses to biological cells causes changes in the permeability of the cell membranes. This effect, electroporation, is the basis of numerous medical, industrial and environmental applications. The development of nanosecond and even subnanosecond high power pulse generators and pulsed microwave sources promises to expand this range of nonthermal electric field – cell interactions from the cellular to the molecular level, with the potential for novel medical treatments. A research field which is mainly established in the FSU countries and in China deals with the biological effects of microwaves and millimeter waves. Extended scientific studies in this field may lead to novel therapeutic, health and safety applications. A second technological area, which has developed rapidly, is the generation of nonthermal, atmospheric pressure plasmas, with application in bacterial and chemical decontamination. In order to provide a forum for exchange of information and ideas the "First International Symposium on Nonthermal Medical/Biological Treatments Using Electromagnetic Fields and Ionized Gases" was held in April of 1999 in Norfolk, VA. The paper provides an overview of the state of these emerging research and technology fields based on the conference presentations, and conclusions and recommendations for research and development, developed in discussion groups at the symposium.

INTRODUCTION

There is generally a negative connotation with the use of electricity on biological systems. Electric effects on biological cells are often related to electric shocks, electric burns, or to electrocution. There is fear that exposure to electromagnetic fields may cause cancer, the opinion that electromagnetic radiation is kind of a pollutant. Much of the research on biological/medical effects of electricity has therefore concentrated on potential hazards of electromagnetic radiation, on a better understanding of damage mechanisms and remedies against them. The positive effects in bioelectrics have received much less attention, although there are spectacular research results. In the medical field, electric fields have been shown to accelerate wound healing, they are used for transdermal drug delivery, in gene therapy, and even in treatment of cancer. There are indirect effects of electric field/cell interactions, which have a positive effect on our well being. Pulsed electric fields have been shown to be useful in bacterial decontamination, an effect used for example to make our food and drinking water safer. Electric field techniques promise to be useful in bacterial decontamination of air, an important research area considering the threat of biological warfare.

In all these positive effects of electric field-cell interaction are nonthermal, that means that heating of cell suspensions or tissues can be neglected. The reason is that for many of these applications the electric fields are either very small, or the duration for which the electric field is applied is too short to cause heating. The latter is the case in the applications listed above. The spectacular results in these nonthermal techniques have been achieved with a technology which is based to a large extend on pulse power technology of the sixties and seventies. More recent advances in the generation of ultrashort electrical power pulses (pulse power technology), allow us now to extend the field of bioelectrics into a range where promising new effects of field-cell interaction can be expected. By reducing the pulse duration into the nanosecond or even shorter regime, the external electric fields in the cell become comparable and even exceed the internal electric fields, allowing us to affect processes in the cell interior without heating the cell.

The effect of ionized gases on biological cells, in physics terminology, "plasmas", was the second topic in the symposium. In nature they appear as lightning, in the laboratories and manufacturing plants as arcs and sparks. These "conventional plasmas" are violently destructive when applied to biological cells. Researchers have in the past years, however, developed "cold" high pressure plasmas, plasmas with gas temperatures close to room temperature. When applied to surfaces and gas volumes for seconds or minutes, they kill bacteria, and offer a chance to be used as sterilizing agents much simpler than conventional methods and without harming the environment.

In order to provide a forum for the exchange of information and ideas in these emerging research and technology fields the "First International Symposium on Nonthermal Medical/Biological Treatments Using electromagnetic Fields and Ionized Gases" has been held on April 12 - 14, in Norfolk, VA. The symposium was sponsored by the U.S. Air Force Office of Scientific Research, the National Science Foundation, the IEEE Nuclear and Plasma Sciences Society, the Biolectromagnetics Society, Old Dominion University, the College of William and Mary, and the Eastern Virginia Medical School. One hundred and thirty five scientists from 12 countries were attending. Eighteen Invited Talks were given covering a range from basic microbiology to pulse power technology. Approximately 60 contributed papers were presented in a poster session. The last day of the symposium was devoted to discussion on the status of the research field and strategies to expand it. The meeting was concluded with presentations on funding opportunities and application procedures. Presenters were P. Dunne for U.S. Army RD&E. R. Ellis for USDA. L. Goldberg for NSF. E. Postow for NIH. D. Quass for EPRI. and G. Roy for ONR. The session was chaired by R. Barker (AFOSR).

STATUS OF RESEARCH AND DEVELOPMENT

The interaction of electromagnetic fields with biological cells has been a topic of many studies since Galvani. at the end of the 18th century. explored the muscular contraction of frog legs under the influence of electric fields. With the expansion on the amplitude and frequency range of electromagnetic fields. which are now accessible and controllable by us. the possible dangers and opportunities have multiplied. There are several professional societies devoted to this topic. the most important one being the Bioelectromagnetics Society.

Although the topic of the symposium covers only a small part of bioelectromagnetic effects and treatments. only such that are based on nonthermal processes, there is still a large spectrum of research directions in this area. In order to introduce the audience. which consisted of engineers. physicists. biologists and clinicians. to these research areas we had invited prominent scientists to give tutorials and reviews on the important research areas in nonthermal treatments and on supporting pulse power and microwave technology. The sessions began with a historic overview and an introduction into fundamental concepts, presented by C. Polk. University of Rhode Island. Particularly emphasis was given to the definition of nonthermal processes at high electric fields. the topic of this conference. A classic. nonthermal medical application of pulsed electric field effects is the treatment of ventricular fibrillation by means of strong electrical shocks. A report on the latest research on ventricular fibrillation and defibrillation was presented by J. Leon. University of Montreal.

One of the most important nonthermal processes in bioelectrics is electroporation. the reversible or irreversible changes in the permeability of cell membranes due to the application of high electric fields. Basic principles of this effect were discussed by J. Weaver. MIT/Harvard. with applications given. The speaker concentrated particularly on the use of this effect for transdermal drug delivery. This topic was expanded by U. Zimmermann. University of Wuerzburg. Germany. who discussed the application of electroporation and electrofusion as a means to generate antibodies for the treatment of certain types of cancer. G. Hofmann. Genetronics Inc. carried this topic into an even more applied area. the therapeutic use of electroporation for transdermal delivery of large molecules. for gene therapy. electroporation mediated therapy of cancer. and electroporation generated delivery of drugs and genes through vessel walls for treatment of cardiovascular diseases. Following this presentation. W.R. Panje. Rush-Presbyterian-St. Luke's Medical Center. Chicago. reported about clinical trials using electroporation mediated therapy on head and neck cancer. and concluded that this method offers promising possibilities in the treatment of these cancers. Whereas controlled electroporation. the topic of these review talks has its place in therapeutic applications. uncontrolled electroporation was found to be the important mechanism in electric force injury. rather than thermal (burn) mechanisms. Research results on electric tissue injuries and consequences for their treatment were discussed by R. Lee. University of Chicago.

Theoretical considerations to the coupling of electric fields to cells were presented by K. Foster. University of Pennsylvania. His talk was particularly devoted to the effect of electrical pulses with high frequency content (ultrawideband pulses) on cell membranes and cell nuclei. Experimental results with short. high intensity electric field pulses on multicellular organisms and cells in vitro were presented by K. Schoenbach. Old Dominion University. Results of laboratory and field experiments with microsecond pulses for biofouling prevention were presented. and the potential of electric field interaction with cell nuclei for pulses in the submicrosecond range was discussed. Another application of high electric fields is the so-called Pulsed Electric Field (PEF) method. where high electric fields applied to liquid food serves to decontaminate the food. A report on the status of this rather mature technology. where industrial interest and support is in place. was given by P. Dunne. US Army Natick R&D Center.

Electrical field interaction with DNA. through coupling of the field to electron and ion transfer reactions. was discussed by M. Blank. Columbia University. He based most of the discussions on this effect on the stress protein inducing effect of low frequency magnetic fields. Many other low intensity. electric and magnetic field effects are not as well studied as the effect described by M. Blank. however are already widely used in therapeutic applications. Millimeter wave therapy e.g. is widely considered as therapeutic modality. particularly in the former Soviet Union. A review on the treatments based on low intensity millimeter wave irradiation was given by A. Pakhomov. Brooks AFB. Although seemingly effective. millimeter wave therapy is not well understood. and the research in field suffers from lack of reliable studies. The effect of intense pulses of microwave radiation on bacteria. spores and mammalian cells was the topic of a presentation by J. Kiel. AFRL. Brooks AFB. Pulsed microwave radiation in the 1.25 to 9.35 GHz range was found to affect the growth of bacteria in the presence of certain chemicals. Preliminary results suggest that pulsed microwave radiation could be directed toward pathological targets and organs while sparing normal tissue.

The presentations on electric (and magnetic) field effects on biological cells were complemented by two tutorial talks on the state of pulse power devices and high power microwave and millimeter-wave sources for possible applications in medical/biological research. The first topic. on pulsed electric power systems. was presented by M. Kristiansen, Texas Tech University. He concentrated on nanosecond and subnanosecond. pulse generators. where research institutes in Russia seem to have a leading role in expanding the source parameters to ever-shorter pulses and higher power. The status of high power microwave and millimeter wave sources and the role of pulse power technology in the development of high intensity generators were presented by E. Schamiloglu, University of New Mexico. Again, as in pulse power in general. most of dramatic increases in power are relatively recent. Only in the 1970s pulse power technology begin to emerge as an independent research field.

Whereas electrical interaction with biological cells has a long history. the use of nonthermal plasmas in atmospheric pressure air for medical/biological applications has been only recently recognized. An introduction into the physics of nonthermal plasmas. and the various types of atmospheric pressure plasmas and their features was given by E. Kunhardt, Stevens Institute of Technology. It was followed by a talk by T. C. Montie. University of Tennessee. on the application of a special type of atmospheric pressure plasma, a radio frequency driven glow discharge. for sterilization of surfaces and materials. Similar discharges. barrier discharges. have also been used for sterilization. The presentation on results of this discharge type was given by J. Birmingham, MesoSystems Technology. Inc.. For both types of plasmas the sterilization rate was reported as faster than by heat alone. The session was concluded by a talk of P. Netzer. National Naval Medical Center. on the need of new sterilization processes in healthcare, and the role of plasma methods in such an environment.

Contributed papers were presented in a poster session. The 76 accepted poster papers were placed into seven categories:

Basic Phenomena (15 contributions);
Pulsed Electric Fields (11 contributions);
Microwaves (5 contributions);
Ultraviolet Radiation (3 contributions);
Electron and Ion Beams (7 contributions);
Ionized Gases (16 contributions). and
Advanced Pulsed Power and Plasma Generators (19 contributions).

The largest research and development area represented in the poster session was bacterial decontamination, using pulsed electric fields, UV radiation, and nonthermal plasmas: the second largest area dealt with basic studies and medical applications of pulsed electric fields. The relatively large number of papers on pulsed field generators, mainly presented by scientists from the former Soviet Union, provided the audience with a good overview of leading edge pulse power systems.

Abstracts of invited and contributed papers have been published in Proceedings of this Symposium.

CONCLUSIONS AND RECOMMENDATIONS

Following the presentations. the status of research and development of nonthermal medical/biological treatments was discussed in four discussion sessions. Discussions concentrated on the following topics

1. Pulsed Electric Field Effects: Basic Research and Applications
2. Microwave and Millimeter Waves Interaction with Biological Cells
3. Medical Applications of Pulsed and cw Electric Field Technology
4. Ionized Gases for Biological Decontamination

The discussion sessions were chaired by J. Dunn (ALP. Chicago), J. Kiel (Brooks AFB), R. Lee (University of Chicago). and I. Alexeff (University of Tennessee).

A. Pulsed Electric Field Effects: Basic Research and Applications (J. Dunn)

The effect of pulsed electric fields on biological cells depends on pulse duration. pulse shape. and amplitude. Three pulse domains were identified, dependent on major applications and/or physical mechanisms:

1. "Traditional Electroporation" :
   Ten's to hundred's of microsecond duration, several kV/cm electric fields
2. "Traditional" Biological Decontamination
   Less than 10 microsecond duration. greater 16 kV/cm electric fields
3. "Cell Modifications" Targets: cell substructures. molecules/bonds
   Submicrosecond (nanosecond, picosecond) duration; time domain is accessible with modern pulse power technology. however field-cell interaction mechanisms are not explored

The mechanism, which leads to cell death through electric field application, is not well understood. Although it is accepted that in the "traditional debacterialization" range poration of the outer membrane. is the ultimate mechanism, the pore formation process itself is controversial. An approach which considers cells as perturbations in homogeneous fluid was presented by J. Dunn. It was hypothesized that electrical double layers at the cell surface cause localized heating of the cell membrane, and consequent membrane breakdown. Independent on the breakdown mechanism it was agreed that for "traditional biological decontamination", particularly in the electric field range of less than 30 kV/cm, at pulse duration of less than 2.5 microseconds, electrochemical effects can be neglected.

One of the more mature applications of PEF technique is bacterial decontamination of food, although there is still much room for better engineering of systems. Electrical requirements in food treatment depend on the type of the food. For low acid food, e.g., with a pH value of greater than 4.5, the conditions for the electrical pulses are more stringent, than for food of high acidity (pH < 4.5), where spores are absent. The dominant problem in food preservation using PEF is the high energy requirement, 100 – 400 J/ml. Improving this efficiency requires either to consider combination processing (thermal + PEF) or to search for electric field processes which are based on a mechanisms different from electroporation, such as subcellular processes or resonant molecular processes.

Collaboration among scientists in engineering biology and medicine is a key in successful research and development of PEF bacterial decontamination methods. This becomes obvious, when the results of pulsed electric field experiments on gram-positive and gram-negative bacteria, and on spores, are considered. In order to develop PEF systems for bacterial decontamination, engineers need to communicate more with biologists and physicians.

B. Microwave and Millimeter-Wave Effects (J. Kiel)

Methods using extremely short electromagnetic pulses and pulsed microwave are related. One of the main differences, which is purely technical, is seen in the coupling mechanism: direct coupling through electrodes inserted in tissue or suspensions, for high electric field pulses, and remote coupling, for microwave and millimeter wave interaction. Medical effects of microwaves and millimeter waves include post-operative septic wound healing, pain relief, treatment of hypertension, and promoting the recovery after heart attacks. Hypothermia for cancer treatment is another, however thermal, application of high power microwaves therapy. It reduces the risk for collateral (cardiovascular) damage compared to surgery and radiation treatment. Other biological effects of major interest are bacterial decontamination.

Strong fundamental research on medical and biological treatments exists in Russia and other FSU countries, and in China. Particularly, the FSU have developed a large spectrum of marketable sources for these applications. Custom made high power devices in the US range from 200 k$ to one million $. The high cost might be one of the reasons that research and development in the USA has more focused on large scale environmental (clean up of hazardous material) and industrial use (material processing, such as sintering of ceramics) by high power micro- and millimeter-wave sources. In medical research, however, the emphasis in the USA has concentrated more on the potential negative effects of micro and millimeter waves. An example is the ongoing discussion on the health and safety issues in wireless communication.

Recent developments in millimeter wave technology have provided researchers with new opportunities for work in this field. Proper scientific studies have the potential of uncovering significant benefits for all mankind. Therapeutic, health and safety applications appear to be feasible and within our reach. Preliminary reports from efforts in China, Russia, and several other countries have already produced encouraging results. Despite leading in the technological development of sources, the US is lagging behind many parts of the world in the understanding of the interaction of biological systems and microwaves. This lack of understanding is resulting in the potential loss of medical therapies (such as enhanced septic wound treating) as well as new potential health and safety issues for personal working with intense electromagnetic fields.

C. Medical Applications (R. Lee)

Medical applications in therapy and diagnostics were discussed with respect to the coupling modes of electric fields with biological systems.

1. ***Cellular Coupling*** is achieved by using relatively long pulses (> 1 ms). Applications include:

    Electrochemotherapy or Electroporation Therapy
    Electro-Transdermal Delivery
    Rhythm Disorders (defibrillation)

Cellular coupling (electroporation) is also related to electrical injury.

Corresponding to long pulses are low-frequency electric (and magnetic) fields.

Applications of low frequency, cw fields are:

    Tissue ablation
    Electrical Injury (therapy)
    Electrical Stimulation

2. ***Molecular Coupling*** is considered to be the dominant mechanism when short pulses (10^{-6} to 10^{-9} s) are applied. Potential applications of such nonthermal methods are treatment of birth defects and cancer, through triggering of apoptosis.

cw treatments, using radiofrequency, microwave and optical radiation are thermal methods which have applications in:

    Musculoskeletal Heating
    Hypothermia: Treatment of Cancer
    Treatment of Burns

3. ***Atomic Coupling*** occurs for ultrashort pulses (e.g. in photolysis), and in the case of ionizing radiation, used in cancer therapy.

Diagnostic applications are ***cell sensing***:

Tissue damage and tumor detection by means of impedance spectroscopy,

Cancer cell and viral particulates by dielectrophoresis, and ***molecular sensing***,

used in tumor detection and localization by means of conformal radiofrequency imaging.

D. Ionized Gases for Biological Decontamination (I. Alexeff)

The use of ionized gases has been demonstrated as being very effective for biological decontamination. A major US company has already a commercial product on sale. An in-depth analysis of promising decontamination technologies sponsored by the U.S. Army's Edgewood Chem-Bio Center concluded that plasmas have great potential, particularly in the areas of sensitive equipment and vehicle/shelter decontamination, but that significant development will be required before this potential can be realized.

As major applications for plasma decontamination three areas were identified:

a) bacterial decontamination in bacterial warfare,
b) sterilization of food, and
c) sterilization of medical instruments in hospitals.

The development of appropriate diagnostics for nonthermal, high-pressure air plasmas was considered as most important. As an example, there is still controversy on the role of atomic oxygen in the discharge with respect to decontamination. Whereas some claim that atomic oxygen in the high-pressure glow is extremely effective in biological contamination, others state that atomic oxygen disappears in microseconds, and so is of no effect whatsoever. However, everybody seemed to agree that identifying the killing mechanisms for bacteria, spores, and viruses is of very high priority. It is not yet clear what the most active killing species (ions, radicals, active molecules, or UV light) is.

The question of research funding was discussed. Research on these nonthermal plasmas is both basic and applied; also, since the interdisciplinary nature of the research requires teams of scientists, rather than single investigators, relatively large grant are needed. Due to the new and unique requirements for this area of research, there was no clear picture where to obtain funds. It was therefore recommended to set up a task force to develop funding.

SUMMARY

The consensus of the participants was that the research field offered exciting possibilities to expand available technologies into new areas of research, and consequently has a strong potential for breakthrough results. The research discussed at the meeting carries the promise of commercial and medical applications. It is a research field to which the general public can easily relate. This was demonstrated by strong newspaper and television coverage. It seems also to have the support of our legislators. In the introductory address at the symposium, the Honorable O. Picket, ranking member in the House Subcommittee on military research promised his support for increased research spending. Members of the Virginia legislature voiced similar support.

The question of funding was addressed in the final session, where representatives of funding agencies talked about opportunities and procedures, but it was also brought up in any of the discussion sessions. Financial research support seems to be sporadic and limited. There is no interagency research program, something, which would be strongly benefit this interdisciplinary research. The lack of support is rather surprising considering the importance this research field is given in other countries, the spectacular results in clinical applications, and its commercial potential.

To explore the full potential of nonthermal treatments using electromagnetic fields and ionized gases, the topic of our symposium, the interaction and the cooperation of engineers and physicists on one side and biologists and clinicians on the other side is needed. The symposium seems to have served as a catalyst to stimulate discussions between scientists with various backgrounds but the same interest. It is hoped that these conversations lead to collaborations. The response from the participants was overwhelmingly positive, and the need for an ongoing conference was voiced. The program committee decided to hold the "Second International Symposium on Nonthermal Medical/Biological Treatments Using Electromagnetic Fields and Ionized Gases" again in Norfolk, VA, in spring of 2001, with S. Beebe, Eastern Virginia Medical School, as chair. A Special Issue of the IEEE Transactions on Plasma Science with over forty contributions on the topic of the symposium will appear in February 2,000.

ACKNOWLEDGEMENTS

This symposium was sponsored by the U.S. Air Force Office of Scientific Research (AFOSR), National Science Foundation (NSF), IEEE Nuclear and Plasma Science Society (NPSS), Bioelectromagnetics Society (BEMS), Old Dominion University (ODU), the College of William and Mary (CWM), and Eastern Virginia Medical School (EVMS).

$$\varepsilon_1 = \frac{1-\xi^2-\delta^2}{(1+\delta^2)}; \qquad \varepsilon_i = -\frac{(2-\xi^2)}{(1+\delta^2)} \tag{22}$$

$$\xi^2 = \frac{\omega_{pe}^2}{\omega^2}; \qquad \delta = \frac{\nu_{eff}}{\omega} \tag{23}$$

ω_{pe} and ν_{eff} is the plasma frequency and the effective collision frequency, respectively.

Then, we can calculate the Q factor of the cavity:

$$Q = \omega_0 \frac{W_T}{P_T} \tag{24}$$

$$P_T = P_L + P_{col} \tag{25}$$

Actually, knowing P_T, we can also calculate the

$$P_L^c = \frac{k_z^2 \varepsilon_i^2}{\varepsilon_0^2(\varepsilon_1^2+\varepsilon_i^2)} p_p^2 D_2^2 \frac{R_0}{2}[J_1^2(p_p R_0) - J_0(p_p R_0) J_2(p_p R_0)] + \frac{p_p^2 \varepsilon_i^2 (p_p R_0)^2}{\varepsilon_0^2(\varepsilon_1^2+\varepsilon_i^2)} D_2^2 [J_0^2(p_p R_0) + J_1^2(p_p R_0)] \tag{26}$$

Then, we can calculate the Q factor of the cavity:

$$Q = \omega_0 \frac{W}{P_L} \tag{27}$$

$$W = W^v + W^p \tag{28}$$

$$P_L = P_L^W + P_c^p \tag{29}$$

we can also calculate the attenuation constant of the wave propagation: $e^{j\omega t - jk_z z - \alpha z}$.

$$\alpha = \frac{P_L}{2P} \tag{30}$$

where P is the propagating power: $P = P_1 + P_2$.

$$P_1 = \frac{1}{2}\oiint (\bar{E}\times\bar{H}^*)ds = \frac{1}{2}\int_0^{R_0}\int_0^{2\pi}(\bar{E}\times\bar{H}^*)rdrd\varphi \tag{31}$$

and

$$P_2 = \frac{1}{2}\int_0^a dx \int_0^b dy(\bar{E}\times\bar{H}^*) \tag{32}$$

P_1 and P_2 is the propagating power in the vacuum area and in the plasma column, respectively.

$$P_2 = \frac{ab}{2}\frac{k_z^2\omega}{\varepsilon_0}\{((\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 - \frac{k_z\omega}{\varepsilon_p\varepsilon_1}\pi D_2^2 \frac{(\bar{p}_p R_0)^2}{2} \times [J_1^2(\bar{p}_p R_0) - J_0(\bar{p}_p R_0) J_2(\bar{p}_p R_0)]\} \tag{33}$$

$$P_1 = \pi \frac{k_z\omega}{\varepsilon_p\varepsilon_0}\frac{(p_p R_0)^2}{2} D_2 [J_1^2(p_p R_0) - J_0(p_p R_0) J_2(p_p R_0)] \tag{34}$$

Substituting eq.(31)-(34) and (16), (17) and (20) into eq.(30), the attenuation constant α can be obtained.

Therefore, we obtained all formulas that we need for calculating and design the resonant cavity for microwave excited excimer laser.

III. Discussion and analysis

To simply our formulas obtained in previous section: let $D_2 = 0$, we get from eq.(16):

Theory of Resonant Cavity for Microwave Excited Excimer Laser
(Draft)

Abstract: Theory of Resonant cavity for Microwave Excited Excimer Lasers presented in the paper. The cavity is a set of Rectangular waveguide with a cylindrical plasma column on the center. The formulas of the resonant spectrum, the stored energy, the lose on the wall and due to the collision on the plasma and the Q factor of the cavity have been worked out.

I. Introduction

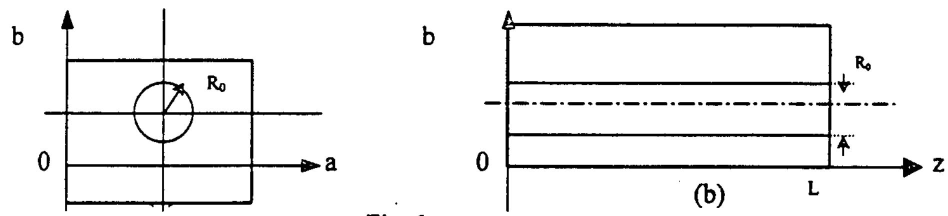

Fig. 1

A most convenient and useful structure of the Microwave plasma Excited excimer laser is a Rectangular Waveguide cavity with a cylindrical plasma column in the center, as shown in Fig.1.
It seems that there is no theoretical study for this kind of resonant cavity appeared in the published papers. It makes inconvenient for study and design this kind of Excimer Laser. Based on the paper of "Theory of microwave rectangular waveguide for microwave plasma excited excimer laser", the theory of this kind of cavity have been worked out. The formulas of the resonant spectrum, the stored energy, the loses on the cavity wall and in the plasma column due to the collision have been obtained. These formulas provide the basic theory for computer calculations and the basis of the understanding and the design of this kind of microwave plasma excited excimer laser.

The paper is organized as follow: section I is the introduction. Theory of the cavity is given in section II. Discussion and analysis are given in section III. Section IV deals with computer calculations (to be carried on), and section V is the conclusion.

II. Theory of Resonant Cavity of a Rectangular waveguide with a Cylindrical Plasma Column in the Center of the Waveguide

Based on the theory given in the paper "Theory of waveguide system for microwave plasma excited excimer laser" making use of the boundary conditions:

$$z = 0, \qquad z = L$$
$$E_x = E_y = 0; \qquad E_r = 0 \tag{1}$$

The field components outside and inside of the plasma can be obtained: outside the plasma, we have:

$$E_z = 2\frac{\pi^2}{\varepsilon_0}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos(k_z z) \tag{2}$$

$$E_x = -2\frac{jk_z}{\varepsilon_0}\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin(k_z z) \tag{3}$$

$$E_y = -2\frac{jk_z}{\varepsilon_0}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(k_z z) \tag{4}$$

$$H_x = j\pi\omega\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos(k_z z) \tag{5}$$

$$H_y = -j\pi\omega\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos(k_z z) \tag{6}$$

$$H_z = 0 \tag{7}$$

for TM mode.

Inside the plasma column:

encourage themselves constantly and gain new knowledge by themselves, while any new knowledge may out of date sooner or later, as science and technology are developing so rapidly.

To conclude my talk, I would say that education is a lifelong issue.

$$E_z = D_2 \cdot 2 \frac{p_p^2}{\varepsilon_0 \varepsilon_p} J_0(p_p r) \cos k_z z \tag{8}$$

$$E_r = -j2D_2 \frac{k_z^2}{\varepsilon_0 \varepsilon_p} p_p J_0'(p_p r) \sin k_z z \tag{9}$$

$$H_\varphi = -j2D_2 \omega p_p J_0'(p_p r) \cos k_z z \tag{10}$$

The TE modes can be studied by using the same approach.

Thus, the resonance frequency can be found by:

$$\sin k_z L = 0 \tag{11}$$

If gives:

$$k_z L = 2\pi l, \qquad k_z = 2\pi \frac{l}{L} \tag{12}$$

where k_z should be found by using the dispersion equation given in eq. (21) (or in eq.(1), in complement) of the paper [1].

Now we can calculate the stored energy.

$$W_T = W_v + W_p \tag{13}$$

where W_v and W_p is the stored energy in the vacuum space and in the plasma column, respectively.

$$W_v = \int_0^a dx \int_0^b dy \int_0^L dz [E_v]_v^2 - \int_0^L dz \int_0^{R_0} r dr \int_0^{2\pi} | E_v |^2 \, d\varphi \tag{14}$$

$$W_p = \int \int_0^{R_0} \int_0^{2\pi} | E_p |^2 \, r dr d\varphi dz \tag{15}$$

Substituting eq. (4)-(7) and (8)-(10) into eq. (14) and (15), we can obtain:

$$W_v = \frac{abL}{\varepsilon_0^2} \{ \frac{1}{2} k_z [(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2] + \frac{1}{2} (\frac{m\pi}{a})^4 + \frac{1}{2} (\frac{n\pi}{b})^4 + \pi^2 \cdot [(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2] \} - \pi \{ \frac{k_z^2}{\varepsilon_0^2 \bar{\varepsilon}_p^2} \tilde{D}_2^2 [J_1^2(\bar{p}_p R_0) - J_2(\bar{p}_p R_0) J_0(\bar{p}_p R_1)] + \frac{k_z^2}{\varepsilon_0^2 \bar{\varepsilon}_p^2} \tilde{D}_2^2 [J_0^2(\bar{p}_p R_0) + J_1^2(\bar{p}_p R_0)] \} \frac{(\bar{p}_p^2 R_0)^2}{2} \tag{16}$$

$$W_p = \frac{k_z \varepsilon_r^2}{\varepsilon_0^2 (\varepsilon_r^2 + \varepsilon_i^2)} D_2^2 \frac{(p_p R_0)^2}{2} [J_1^2(p_p R_0) - J_2(p_p R_0) J_0(p_p R_0)] L \tag{17}$$

where:

$$\bar{\varepsilon}_p = \varepsilon_1 ; \qquad \bar{p}_p = p_{p|\varepsilon_p = \varepsilon_1} ; \qquad \bar{D}_2 = D_{2|\varepsilon_p = \varepsilon_1} \tag{18}$$

The loss also consists of two parts: the loss on the cavity wall and the loss in the plasma column. The loss on the wall can be obtained by:

$$P_L = \frac{R_s}{2} \oiint | H_z | \, ds \tag{19}$$

and the loss in the plasma column due to the collision can be found by:

$$P_{col} = \frac{k_z^2 \varepsilon_i^2}{\varepsilon_0^2 (\varepsilon_r^2 + \varepsilon_i^2)} D_2^2 \frac{(p_p R_0)^2}{2} [J_1^2(p_p R_0) - J_0(p_p R_0) J_2(p_p R_0)] + \frac{p_p^2 \varepsilon_i^2}{\varepsilon_0^2 (\varepsilon_r^2 + \varepsilon_i^2)} D_2^2 \frac{(p_p R_0)^2}{2} [J_0^2(p_p R_0) + J_1^2(p_p R_0)] \tag{20}$$

In eq.(17) and (20), we are use:

$$\varepsilon_1 = \varepsilon_r + j\varepsilon_1 \tag{21}$$

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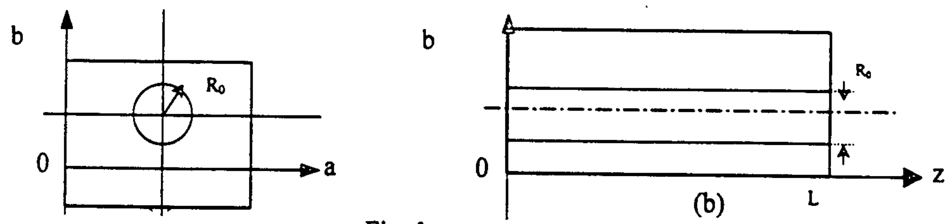

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$$E_y = -2\frac{jk_z}{\varepsilon_0}(\frac{n\pi}{b})\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)\sin(k_z z) \tag{4}$$

$$H_x = j\pi\omega(\frac{n\pi}{b})\sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)\cos(k_z z) \tag{5}$$

$$H_y = -j\pi\omega(\frac{m\pi}{a})\cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)\cos(k_z z) \tag{6}$$

$$H_z = 0 \tag{7}$$

for TM mode.

Inside the plasma column:

$$W_v = \frac{abL}{\varepsilon_0^2}\{\frac{1}{2}k_z^2[(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2] + \frac{1}{2}(\frac{m\pi}{a})^4 + \frac{1}{2}(\frac{n\pi}{b})^4 + \pi^2 \cdot (\frac{m\pi}{a})^2(\frac{n\pi}{b})^2\} = \frac{1}{8}\varepsilon_0 abLE_0^2 \tag{35}$$

where we has been made for.

$$k^2 - k_z^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 \tag{36}$$

$E_0 = 1$.

It is the formula for vacuum rectangular cavity. We can also get the other formulas for vacuum case.

IV. Computer calculations: (to be carried on)

V. Conclusion

Based on the theory given in paper[1], all formulas for calculating the parameters and design of the resonant cavity of a rectangular waveguide cavity with a cylindrical plasma column for the microwave plasma excited excimer laser have been obtained in the paper. These formulations are very valuable and useful.

It is a draft, all formulas should be checked carefully, and will be done later.

References

1. Theory of waveguide system for microwave system for plasma excited excimer laser (draft);
2. Panl Larrain and Dale R. Corson, "Electromagnetic field and waves", W.H. Frecman and Company, San Francisco, 1970.

$$\varepsilon_1 = \frac{1-\xi^2-\delta^2}{(1+\delta^2)}; \qquad \varepsilon_i = -\frac{(2-\xi^2)}{(1+\delta^2)} \tag{22}$$

$$\xi^2 = \frac{\omega_{pe}^2}{\omega^2}; \qquad \delta = \frac{\nu_{eff}}{\omega} \tag{23}$$

ω_{pe} and ν_{eff} is the plasma frequency and the effective collision frequency, respectively.

Then, we can calculate the Q factor of the cavity:

$$Q = \omega_0 \frac{W_T}{P_T} \tag{24}$$

$$P_T = P_L + P_{col} \tag{25}$$

Actually, knowing P_T, we can also calculate the

$$\begin{aligned} P_L^c = & \frac{k_z^2\varepsilon_i^2}{\varepsilon_0^2(\varepsilon_1^2+\varepsilon_i^2)} p_p^2 D_2^2 \frac{R_0}{2}[J_1^2(p_pR_0) - J_0(p_pR_0)J_2(p_pR_0)] \\ & + \frac{p_p^2\varepsilon_i^2(p_pR_0)^2}{\varepsilon_0^2(\varepsilon_1^2+\varepsilon_i^2)} D_2^2[J_0^2(p_pR_0)+J_1^2(p_pR_0)] \end{aligned} \tag{26}$$

Then, we can calculate the Q factor of the cavity:

$$Q = \omega_0 \frac{W}{P_L} \tag{27}$$

$$W = W^v + W^p \tag{28}$$

$$P_L = P_L^W + P_c^p \tag{29}$$

we can also calculate the attenuation constant of the wave propagation: $e^{j\omega t - jk_z z - \alpha z}$.

$$\alpha = \frac{P_L}{2P} \tag{30}$$

where P is the propagating power: $P = P_1 + P_2$.

$$P_1 = \frac{1}{2}\oiint (\bar{E}\times\bar{H}^*)ds = \frac{1}{2}\int_0^{R_0}\int_0^{2\pi}(\bar{E}\times\bar{H}^*)rdrd\varphi \tag{31}$$

and

$$P_2 = \frac{1}{2}\int_0^a dx \int_0^b dy(\bar{E}\times\bar{H}^*) \tag{32}$$

P_1 and P_2 is the propagating power in the vacuum area and in the plasma column, respectively.

$$\begin{aligned} P_2 = & \frac{ab}{2}\frac{k_z^2\omega}{\varepsilon_0}\{((\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 - \frac{k_z\omega}{\varepsilon_p\varepsilon_1}\pi D_2^2 \frac{(\bar{p}_pR_0)^2}{2} \\ & \times [J_1^2(\bar{p}_pR_0) - J_0(\bar{p}_pR_0)J_2(\bar{p}_pR_0)]\} \end{aligned} \tag{33}$$

$$P_1 = \pi\frac{k_z\omega}{\varepsilon_p\varepsilon_0}\frac{(p_pR_0)^2}{2}D_2[J_1^2(p_pR_0) - J_0(p_pR_0)J_2(p_pR_0)] \tag{34}$$

Substituting eq.(31)-(34) and (16), (17) and (20) into eq.(30), the attenuation constant α can be obtained.

Therefore, we obtained all formulas that we need for calculating and design the resonant cavity for microwave excited excimer laser.

III. Discussion and analysis

To simply our formulas obtained in previous section: let $D_2 = 0$, we get from eq.(16):

13. Shenggang Liu, et al, "Theoretical Study on Micro-Hollow Cathode Discharge".

Theoretical Study on Micro-Hollow Cathode Discharge
(First Draft)

1.Introduction

The Micro-Hollow Cathode Discharge(MHCD) is one of the most attractive topics in science and technology for a long time because of its interesting characteristics and good practical/potential applications. However, although the experimental study has been carried on very well, the theoretical study, both the analytical as well as the computer simulation, seems far from enough[1]-[4], for the physical processes happened in MHCD is very complicated. Based on the results obtained by experiments[1]-[3],the paper attempts to give a theoretical analysis of MHCD, and on this basis a modeling for computer calculation is presented as well.

2. Modeling

The following facts are essential for the theoretical modeling:

1.The glow discharge occurs mainly in the region between the hollow cathode surface and the virtual anode which is actually the axis of the structure. The discharge forms a radial glow;

2.The cathode fall is formed in the vicinity of the cathode surface of about few microns;

3.The thermionic emission of cathode is of significant, since the temperature of cathode is high due to the ion bombardment;

4.The Negative Glow Discharge follows the cathode fall and lasts to the axis, the virtual anode. So the negative glow occupies almost the entire space of the cathode region;

5.most of the potential drop happens in the cathode fall, a good potential well forms between the cathode fall and the axis ,the virtual anode, that enable a large number of electrons oscillating in the potential well, these electrons are known as the pendular electrons;

6There are varieties of different kinds of ionization to support the discharge. The pendular electron that exist in the cathode region play very important role. The large number of such electrons oscillating between the edges of opposite cathode fall may make ionization many times on their path;

7. Besides the ionization, strong excitation happens in the cathode region due to the collisions of electrons with atoms/molecules. This excitation produces radiation and makes the cathode region bright;

8.A cylindrical positive column forms in the axis area from anode to one of the cathode surface opening to the anode area. It is a cylindrical positive glow discharge followed by a anode fall to the anode as usual case. By means of the Ambipolar Diffusion process, a large number of electrons come into this area from cathode region through the opening cathode surface. That is the connection of the two regions.

The above facts are assumed as the basis of the theory presented in the paper.
According to the above facts, a modeling is proposed as it is shown in Fig. 1.

As it is see4n from Fig.1, that we can divide the whole MHCD into two regions. Region1 is the cathode region, a radial negative glow discharge occurs with intensive excitation radiation, while region2 is the anode region, a cylindrical positive glow discharge takes place from cathode hole surface opening to the area to the anode. Therefore, the theoretical analysis of the MHCD may be splited into two parts: the theory for the region 1 and that for region 2. By using a proper Matching Condition on the boarder between the two regions, we are able to work out a complete theory for the entire MHCD.

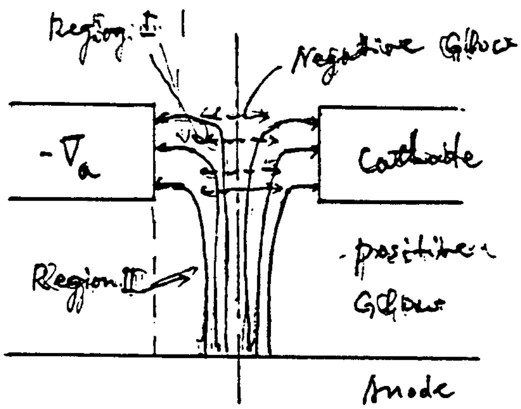


Fig.1

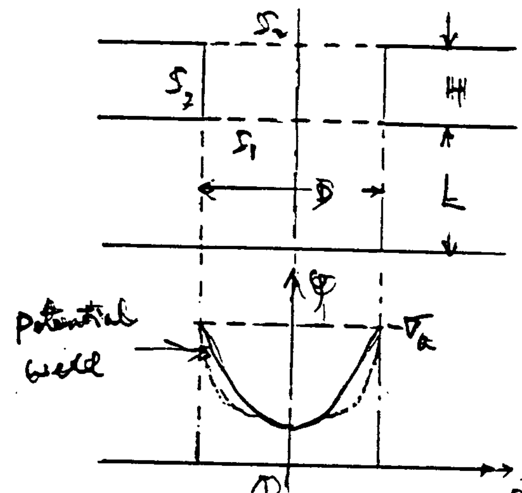


Fig.2

3. Theory for Region1

As it is mentioned above that the negative glow discharge occurs in the region. Since the both sides of the region are open, and there is electric field in the z-direction, charged particles' flow through these open sides hole is the main loss of particles in the region. The recombination of charged particles on the cathode surface can be neglected. As it is shown in Fig.2. Fig.2(a) is the geometry of the MHCD, and Fig.2(b) is the potential curve in the region. It can be seen that there is a good potential well in the region that keeps the pendular electrons.

A. We can find the potential function in the region before and after the discharge takes place. Before discharge, we have:

$$\varphi = \frac{V_a}{I_0(k_z D/2)} I_0(k_z R)\, ch\, k_z z \qquad (1)$$

Where k_z is a constant, $I_0(x)$and $I_1(x)$ is the modified Bessel function of zero and first order.

At the axis the potential is:

$$\varphi(R=0) = \frac{V_a}{I_0(k_z D/2)}\, ch\, k_z z \qquad (2)$$

When the discharge takes place, to find the potential distribution is rather complicated. For the cathode fall region, it is known that ion space-charge exists, then we have the Poisson Equation:

$$\nabla^2 \varphi = |e| n_i / \varepsilon_0 \qquad (3)$$

In order to simplify the solution, we assume that the ion space-charge is uniform, the we can find:

$$\varphi = \frac{V_a}{I_0(K\,D/2)} I_0(KR)\, ch\, k_z z \qquad (4)$$

Where

$$K^2 = k_z^2 + b_i^2 \,; \qquad b_i^2 = \frac{|e|\, n_i}{\varepsilon_0} \qquad (5)$$

We can also get the electric field:

$$E_R = \frac{K\, V_a}{I_0(K\, D/2)} I_1(KR)\, ch\, k_z z \qquad (6)$$

By using the above equations we obtain the cathode fall potential drop:

$$V_c = \frac{V_a \Delta}{I_0(K D/2)} I_1(K\, D/2) \qquad (7)$$

Where Δ is the thickness of the cathode fall.

Now we can calculate the electric potential in the negative glow region that is followed the cathode fall. Since in the negative glow the space-charge is negative, we get:

$$\varphi = \frac{V_a}{I_0(K D/2)} \cdot \frac{I_0[K(D/2 - \Delta)]}{I_0[\bar{K}(D/2-\Delta)]} I_0(\bar{K}R)\, ch\, k_z z \qquad (8)$$

$$\bar{K}^2 = k_z^2 - b_e^2 \,; \qquad b_e^2 = \frac{|e|\, n_e}{\varepsilon_0} \qquad (9)$$

B. The Motion of the pendular electrons.

Now we can study the motion of the pendular electrons in the cathode region. It can be seen from Fig. 2(b) that a good potential well ,ay keep the pendular electrons oscillate with the region. Since electrons are able to collide with other particles many times on their oscillating path, and each time they loss energy and get energy again from the electric field, the minimum time for electrons to complete a round trip can be found as:

$$T_{min} = \left(\frac{m_0}{2\pi V_c}\right) D = 2\, \frac{D/2}{v_{ec}} \qquad (10)$$

Where v_{ec} is the speed of the electrons gained from the cathode fall:

$$v_{ec} = \sqrt{\frac{2|e|\, V_c}{m_0}} \qquad (11)$$

C.Charged Particles' Balance.

It is assumed that the following equation can be used to calculate the balance of the charged particles in the negative glow discharge.[1]-[4]:

$$D_a \nabla^2 n_e + \nu n_e + k_c n_e^2 = 0 \qquad (12)$$

Where D_a is the Ambipolar diffusion coefficient. And k_e, ν is the one step and two step ionization rate due to the collision of electrons with neutral gas in the region. Taking integration of Eq.(12) over the whole region, we obtain:

$$-D_a \oint \nabla n_e \cdot d\vec{s} = \nu N_e + k_c \int n_e^2 \, dV \qquad (13)$$

Here n_e is the electron density. Let

$$n_e = n_0 \, g(R) \, f(z) \qquad (14)$$

Equation(13) can be written as:

$$2\pi D_a n_0 \left\{ \frac{D}{2} \int_{-H/2}^{H/2} \frac{\partial g(R)}{\partial R}\bigg|_{R=D/2} f(z)\, dz + \int_0^{D/2} \frac{\partial f(z)}{\partial z}\bigg|_{z=-H/2} g(R) R\, dR - \right.$$

$$\left. - \int_0^{D/2} \frac{\partial f(z)}{\partial z}\bigg|_{z=\frac{H}{2}} g(R) R\, dR \right\} + \nu N_e + k_c \int_V n_e^2 \, dV = 0 \qquad (15)$$

Where N_e is the total number of electrons in the region.
Equation (15) is the main equation describe the particles' balance in the cathode region.

C. Intensity and Spectrum of the Radiation.

Now we deal with the radiation excited by electrons in the cathode region. We assume that in the region there is only one specie of gas and the radiation is mainly excited by the collision of electrons with atoms. Then the generation of excited atoms a j excited state comes from:

1)The collisions of electrons with atoms may excite atoms at ground state up to j state, the number is: $\alpha_j n_a n_e$ where n_a is the density of the atoms;

2). The collisions of electrons with atoms at lower state k to state j . the number is: $\alpha_{kj} n_k n_e$

3). The non-ballistic collisions of atoms at higher state I and other particles result in radiation to lower state j , the number is: δ_j . So in unite time, unite volume we have following number of atoms at j state : $\alpha_j n_a n_e + \alpha_{kj} n_k n_e + \delta_j$.

During the same period of time and the same volume we have the loss of the atoms at the j state :

1). Spontaneous radiation from j state to the ground state, the number is: $A_{j0} n_j$;

2). The collisions of atoms at j state with electrons result in a jump of the atoms to higher state, the number is: $\sum \alpha_{ji} n_e n_j$;

3). The collisions of atoms at j state with low energy electrons result in increasing of the kinetic energy of electrons, the number is: $\beta_j n_j n_e$;

4). The collisions of atoms at j state with other particles of other species result the change of their state, the number is: $\gamma_j n_j n$

Therefore, the total balance of the atoms at j state gives:

$$\alpha_j n_a n_e + \alpha_{kj} n_k n_e + \delta_j = A_{j0} n_j + \sum_i \alpha_{ji} n_j n_e + \beta_j n_j n_e$$

$$+ \gamma_j n_j n \qquad (16)$$

$$j = j_{ic}\left(\frac{1-D+\gamma_i}{1-D-G}\right)+j_{tc} \tag{25}$$

Where j_{tc} is the current of the cathode thermionic emission, and

$$D=\gamma_p f_d n_d, \quad G=\gamma_p f_g n_g \tag{26}$$

It is obvious that when $D=0$, $G=0$ and $j_{tc}=0$, we have

$$j = j_{ic}(1+\gamma_i) \tag{27}$$

By using the potential function we can calculate the ion density in the cathode fall region.

4. Theory of MHCD

Before move on to the theory of MHCD, we first deal with the physical processes happened in region II, the positive column region, the anode region. We assume that from the open hole of the cathode, the surface S_1, to the anode there is the positive glow discharge and followed by the anode fall. Then by using a proper matching condition on the boarder of the region I and region II, we are able to work out the theory of the entire MHCD.

Assume that Eq.(12) can be used also for the region II, the positive glow discharge occurs in the anode region. Similar to that in section 3, we get:

$$2\pi D_a \bar{n}_e\left\{\frac{D}{2}\int_0^L \frac{\partial \bar{g}}{\partial R}\Big|_{R=\frac{D}{2}} \bar{f}(z')\,dz' + \int_0^{D/2}\frac{\partial \bar{f}(z')}{\partial z'}\Big|_{z'=0} \bar{g}(R)R\,dR\right. + \left. \mu \bar{N}_e + k_i\int \bar{n}_0^2\bar{f}^2(z')\,dz'\,\bar{g}^2(R)R\,dR\right\} = -2\pi \bar{n}_0\int_0^{D/2}\frac{\partial f(z)}{\partial z}\Big|_{z=-H/2} g(R)R\,dR \tag{28}$$

Where: $\bar{n}_e = \bar{n}_0\bar{f}(z')\bar{g}(R)$ (29)

And $\bar{n}_e$ is the electron density in this region:

$$\bar{N}_e = \bar{n}_0\int_0^L dz'\int_0^{D/2} R\,dR\,\bar{f}(z')\bar{g}(R); \tag{30}$$

N_e is the total number of electrons in region II:

The physical meaning of Eq.(28) is that the particles loss in the anode region is compensated by the flux flowing from the cathode region through the surface S_1 into the region II.

Equations (15) and (28) are two important equations describing the physical processes and the particles balance in MHCD.

The matching condition on the boarder between the two regions is:

$$n_e(z,R)\big|_{z=-\frac{H}{2}} = \bar{n}_e(z',R)\big|_{z'=L} \tag{31}$$

Solving equation (15), equation (28) and equation (30), we will be able to get the information of the MHCD.

5. Modeling for Computer Calculation

For a low pressure and pure gas filling case, we can assume that:

$$\alpha_j n_a n_e = A_{j0} n_j \quad (n_a >> n_e, n_k, n_j).$$

And the δ_j is negligible. Then we get: (17)

$$I_{j0} = A_{j0} n_j h\nu_{j0} = \alpha_j n_a n_e h\nu_{j0} \qquad (18)$$

Eq.(17) means that only the collisions of the atoms of one specie and electrons are considered. Then the intensity of the radiation can be calculated by the following equation:

$$I_{j0} = \int_{\varepsilon_j}^{\infty} f(\varepsilon)\sigma_{0j}(\varepsilon)\, n_a n_e h\nu_{j0} \sqrt{\frac{2\varepsilon}{m}}\, d\varepsilon \qquad (19)$$

Here the h is the Plank constant and ν_{j0} is the radiation frequency. Since α_j should be:

$$\alpha_j = \int_{\varepsilon_j}^{\infty} f(\varepsilon)\sigma_{0j}(\varepsilon)\sqrt{\frac{2\varepsilon}{m}}\, d\varepsilon \qquad (20)$$

$f(\varepsilon)$ is the distribution function of electrons, σ_{0j} is the differential cross-section of collision, ε is the energy of electrons and ε_j is the energy required for exciting atoms to j state from ground state. Assume that $f(\varepsilon)$ is Maxwellian distribution function:

$$f(\varepsilon) = \frac{2}{\sqrt{\pi}} \frac{\sqrt{\varepsilon}}{(kT_e)^{3/2}} \exp(-\varepsilon/kT_e) \qquad (21)$$

Where k is the Boltzmann constant. Although the equilibrium state may not be realistic. Thus we can obtain the following equation:

$$I_{j0} = 1.75\times 10^{8}\, n_a n_e h\nu_{j0}\, \sigma_{0j}(\varepsilon_m)\sqrt{kT_e}\, \frac{(b-1)}{b^3}\left(2 + b\frac{\varepsilon_j}{kT_e}\right) e^{-\frac{\varepsilon_j}{kT_e}} \qquad (22)$$

Where ε_m is the value that makes σ_{0j} maximum, and

$$b = 1 - \frac{kT_e}{(\varepsilon_m - \varepsilon_j)} \qquad (23)$$

D. The Current Density of MHC.

The total current of MHC consists of four parts: the cathode emission due to the ion bombardment; the cathode current due to the bombardment of photons from the cathode dark region; the cathode current due to the bombardment of photons from the negative glow discharge region and the cathode thermionic emission current. It should be noticed that the cathode thermionic emission is another special features of Micro-Hollow Cathode Discharge. Then we have:

$$j_{ec} = \gamma_i j_{ic} + \gamma_p f_d n_d j_{ec} + \gamma_p f_g n_g j_{eg} + j_{tc} \qquad (24)$$

Where: j_{ec} and j_{ic} ion current density and electron current density at cathode surface, respectively. n_d and n_g is the number of photons generated in cathode dark and negative glow discharge region, respectively, the energy of the photons should be larger than the work function of the cathode. f_d and f_g is the percentage of the related photons that bombard the cathode surface. The γ_i and γ_p is the second emission coefficient of ion and photon. Then we get:

A. According to the theory and formulations given above, we can work out the modeling for computer calculation of the MHCD. The working equations are (15), (28)
And the Matching condition is: Eq. (31)
The cathode current density can be used as an initial condtion:

$$j = j_{si}\left(\frac{1-D+\gamma_i}{1-D-G}\right) + j_{tc} \qquad (2?)$$

The following parameters are known: ?
For starting the calculation, the initial function of g(R) and g(R) can be given approximately, for example, we can use:

$$g(R) = I_0(K'R), \quad K' = \sqrt{\frac{D_a}{L}} \qquad (32)$$

And the f(z) can be calculated by using computer to solve the boundary problem before the discharge takes place.

B. The spectrum and the intensity of the radiation in both regionI and region II may be calculated by using equation(19) and equation (22), provided the parameters are given.

6. Discussion and Conclusion

The most interesting and important features of MHCD are: 1. The negative glow discharge dominates the cathode region; 2. The pendular electrons that exist in the cathode region play very important role , and 3. The thermionic emission of cathode is of significant,since the temperature of cathode is high enough due to ion bombardment. In order to work out a theory that can reflect the above essential facts, we split the MHCD into two regions. The working equation for each region has been formulated and the matching condition on the boarder between the two regions is presented. Solving the two working equations with the boundary condition and the cathode current density as the initial condition, we can get the information of the discharge. The equations for calculation of the spectrum and intensity of the radiation in both regions have also given in the paper.

References

1. Karl H. Schoenbach et al.; "High –pressure Hollow Cathode Discharge", Plasma Sources Sci. Techn. 6.(1997), 468-477;
2. Karl H. Schoenbach et al.; Appl. Phys. Letters. 68(1), (1996), 13-15.;
3. G. Schaefer and K.H.Shoenbach, " Basic Mechanism Contribution to the Hollow Cathode Effect", Physcs and Applications of Pseudosparks. Eddited by M.A.Gunderson and G. Schaefer. Plenum Press. N.Y. 1990;
4. G. Rozoff, J.of Appli. Phys. 50(11), (1979), 6806-6810;
   51(6), (1980) , 3144-3148;
   52(11),(1981), 6601-6606);
   59(4), (1988), 1048-1051.;

14. Shenggang Liu, et al, "Microwave Cavity Excitation".

$$E_x^T = E_x^A + E_x^B; \qquad E_y^T = E_y^A + E_y^B \qquad (5)$$

Substituting eq. (3)–(4) into (5) we get:

$$E_x^T = \sum \left[A_{mn} \sin \frac{p\pi}{L} z - B_{mn} \sin\left(\frac{p\pi}{L} z + \varphi\right)\right] \left(\frac{\omega\mu}{k_c^2}\right)_{mn} \cos \frac{m\pi}{a} x \sin \frac{n\pi}{b} y$$

(m, n odd) (7)

$$E_y^T = -\sum \left[\left[A_{mn} \sin \frac{p\pi}{L} z - B_{mn} \sin\left(\frac{p\pi}{L} z + \varphi\right)\right] \left(\frac{\omega\mu_0}{k_c^2}\right)_{mn} \sin \frac{m\pi}{a} x \cos \frac{n\pi}{b} y \qquad (8$$

For simplicity, at first, we only take the dominate mode $m=1$, $n=1$, into account, ~~there~~ and also we assume that $A_{mn} = A_{11} = B_{11}$. Therefore, we obtained:

$$E_x^T = A_{11} \left(\frac{\omega\mu_0}{k_c^2}\right)_{11} \left(\sin \frac{p\pi}{L} z - \sin\left(\frac{p\pi}{L} z - \varphi\right)\right) \cos \frac{\pi}{a} x \sin \frac{\pi}{b} y \qquad (9)$$

$$E_y^T = A_{11} \left(\frac{\omega\mu_0}{k_c^2}\right)_{11} \left[\sin \frac{p\pi}{L} z - \sin\left(\frac{p\pi}{L} z + \varphi\right)\right] \sin \frac{\pi}{a} x \cos \frac{\pi}{b} y. \qquad (10)$$

Now we can calculate $E^2 = (E_x^T)^2 + (E_y^T)^2$

$$E_T^2 = (E_x^T)^2 + (E_y^T)^2 \qquad (11)$$

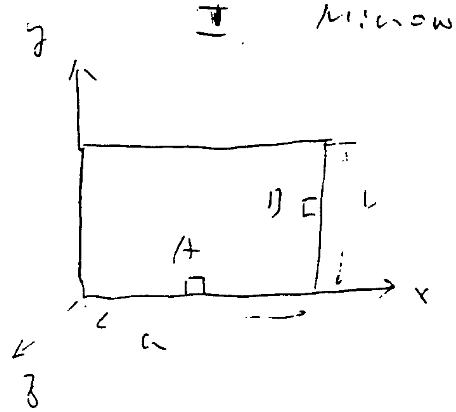
$$\left[\sin \frac{p\pi}{L} z - \sin\left(\frac{p\pi}{L} z + \varphi\right)\right] = -2 \cos\left(\frac{p\pi}{L} z + \varphi/2\right) \sin \frac{\varphi}{2}$$

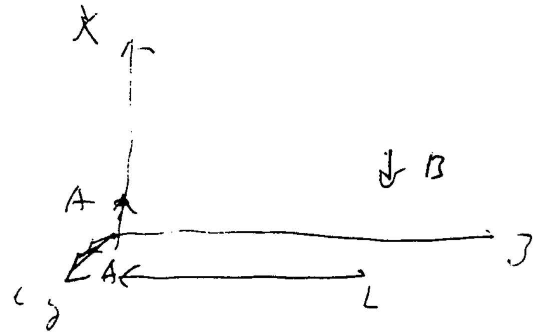
$$E_T^2 = A_{11}^2 \left(\frac{\omega\mu_0}{k_c^2}\right)^2_{11} \cdot 4 \cos^2\left(\frac{p\pi}{L} z + \varphi/2\right) \sin^2 \varphi/2 \left[\cos^2 \frac{\pi}{a} x \sin^2 \frac{\pi}{b} y + \right.$$

$$\left. + \sin^2 \frac{\pi}{a} x \cos^2 \frac{\pi}{b} y \right] \qquad (12)$$

II. Microwave Cavity Excitation

1.





Assume that: ① there is no coupling between attenna A and B. ② only TE_{mn} can be excited. ③ the phase shift between A and B is $\varphi = k_z L$

A: TE_{mn} mode may be excited, where m should be odd.

B: TE_{mn} modes may be excited, n should be odd.

For A.

$$E_x^A = \sum_{\substack{m\ \text{odd} \\ n}}^{\infty} A_{mn}\left(\frac{j\omega\mu_0}{k_c^2}\right)_{mn} \frac{n\pi}{b}\cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \sin\frac{p\pi}{L}z \quad (1)$$

$$E_y^A = -\sum_{\substack{m\ \text{odd} \\ n}}^{\infty} A_{mn}\left(\frac{\omega\mu_0}{k_c^2}\right)_{mn} \frac{m\pi}{a}\sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \sin\frac{p\pi}{L}z \quad (2)$$

For B.

$$E_x^B = -\sum_{\substack{m \\ n\ \text{odd}}}^{\infty} B_{mn}\left(\frac{\omega\mu_0}{k_c^2}\right)_{mn} \frac{n\pi}{b}\cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \sin\left(\frac{p\pi}{L}z+\varphi\right) \quad (3)$$

$$E_y^B = \sum_{\substack{m \\ n\ \text{odd}}} B_{mn}\left(\frac{\omega\mu_0}{k_c^2}\right)_{mn} \frac{m\pi}{a}\sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \sin\left(\frac{p\pi}{L}z+\varphi\right) \quad (4)$$

A_{mn} and B_{mn} are the coefficients.

II. Gas Discharge

1. Mainly transverse electric field. $\boxed{\text{Electric field Discharge}}$

2. High pressure: 2 [illegible]

Diffusion type discharge. Glow Discharge

The following equations can be used:

$$\nabla^2 (D n_e) + \nu n_e + k_e n_e^2 = 0. \qquad (15)$$

D ambipolar diffusion coefficient;

ν — one step ionization rate due to the collision of electrons with neutral gas

k_e — two step ionization rate due to the collision of electrons with neutral gas or ionized atoms/molecules

[illegible] μ_i.

Rf field. $E = E_0 \cos\omega t$.

[illegible]

$$x = \int_0^{\omega t} \frac{\mu_i}{\omega} \cos\omega t \, d(\omega t) = \frac{\mu_i E_0}{\omega} \sin\omega t.$$

$\therefore$ $x = d$, [illegible]

$$f_{max} = \frac{\mu_i E_0}{2\pi d} \quad \text{or} \quad d_{max} = \frac{\mu_i E_0}{2\pi f}$$

[illegible]

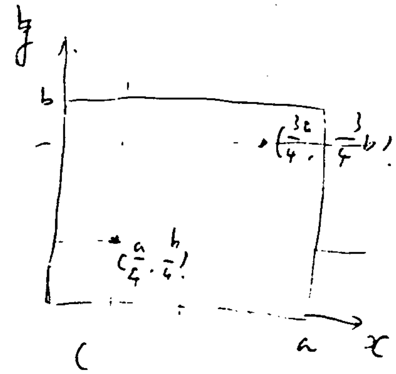
$$f_c = \frac{\mu_i E_0}{\pi d} = 2 f_{max}$$

$f_{max} < f < f_c$. [illegible]) (16)

[illegible] [illegible]

Based on (12), we obtain:

At points $(\frac{a}{4}, \frac{b}{4})$ and $(\frac{3}{4}a, \frac{3}{4}b)$ the electric field is strongest:



$$E_T^2 = 4 A_{1,1}^2 \left(\frac{\omega\mu_0}{k_c^2}\right)_{1,1}^2 \cos^2\left(\frac{p\pi}{L}z + \varphi/2\right) \sin^2 \varphi/2 \qquad (13)$$

Fig. 2.

If we adjust: $\varphi = \pi$; $\frac{p\pi}{L}z = p\pi$

we then get: At: $(\frac{a}{4}, \frac{b}{4})$, and $(\frac{3a}{4}, \frac{3}{4}b)$:

$$E_T^2 = 4 A_{1,1}^2 \left(\frac{\omega\mu_0}{k_c^2}\right)_{1,1}^2 \qquad (14)$$

(18) — particles' balance.

Energy balance.

[illegible] : $P = \vec{J}\cdot\vec{E}$;

$$P = \frac{n_e e^2}{m(\nu_m - j\omega)} \vec{E}\cdot\vec{E} = \frac{n_e e^2}{m(\nu_m - j\omega)} E_0^2 e^{2j\omega t}$$

$$= \frac{n_e e^2 E_0^2}{m(\nu_m^2 + \omega^2)} \nu_m e^{i(2\omega t - \phi)}$$

[illegible]: $$\langle P \rangle = \frac{1}{T}\int_{-T/2}^{T/2} dt\ P = \frac{n_e e^2 E_0^2}{m}\left(\frac{\nu_m}{\nu_m^2 + \omega^2}\right) \quad (19)$$

[illegible]: $\nu_m > \frac{\omega}{2\pi}$,

$$\langle P \rangle = \frac{n_e e^2 E_0^2}{m \nu_m}$$

[illegible]

$$E_b = \nu_m \left(\frac{m^3 v^2}{e^2 M}\right)^{1/2} \quad m \ [illegible]\ M\ [illegible]$$

$\left(\frac{m^3 v^2}{e^2 M}\right)^{1/2}$ [illegible], $E_b \propto P$ [illegible] $\boxed{E_b \propto P}$

[illegible], [illegible] recombination, [illegible]
linear momentum, angular momentum
& kinetic energy [illegible].

$$\frac{\partial n_e}{\partial t} = \nu_i n_e + k n_e^2 - \nu_e n_e + D\nabla^2 n_e$$

[illegible]

[illegible] (photo absorption and recombination)

Diffusion Discharge: $p > 1$ Pa; $f > 100$ MHz

now: $p > 1$ atmosph. $f = 2450$ MHz

Brown microwave Discharge theory.

[illegible]

$$\vec{J} = -\nabla(Dn)$$

$$\frac{\partial n}{\partial t} = -\nabla\cdot\vec{J} = \nabla^2 n$$

n_e 電子密度; D_e [illegible], $\nu_i n_e$.

$$\frac{\partial n_e}{\partial t} = \nabla^2(D_e n_e) + \nu_i n_e = 0.$$

[illegible]

$$\nu_i n_e + D_e \frac{d^2 n}{dx^2} = 0. \quad (17)$$

$\nu_i = \alpha v_d$ α Townsend [illegible],

v_d [illegible].

$\nu_i = \alpha \mu_e E$.

[illegible]: $\eta = \alpha/E$ $\rightarrow$ $\nu_i = \eta \mu_e E^2$

[illegible] ζ: $\zeta = \frac{\nu_i}{D_e E^2}$ $\rightarrow$ [illegible]

[illegible]

$$\frac{\nu_i}{D_e} = \zeta E^2.$$

[illegible] n_e [illegible] $n_e = \hat{n}_e(x,y,z,t) = n_e(\vec{r})\, e^{t/\tau}$.

$$\boxed{\frac{\nu_i}{D}\hat{n}_e + \frac{d^2 \hat{n}_e}{dx^2} = 0} \quad (18).$$

一方 [illegible]

$$\frac{\langle P\rangle}{n_e \ldots} = \frac{e^2 E_0^2}{m(\nu_m^2+\omega^2)}$$

[illegible] $\frac{1}{2}mv^2$。 [illegible]

$\frac{2m}{M}$ — [illegible]

$$\frac{1}{2}mv^2\,\frac{2m}{M} = \frac{m^2v^2}{M}$$

ν_m [illegible]

$$\frac{m^2v^2}{M} = \frac{e^2E_0^2}{m(\nu_m^2+\omega^2)} \rightarrow E_b = (\nu_m^2+\omega^2)^{1/2}\left(\frac{m^3v^2}{e^2M}\right)^{1/2}$$

[illegible]

[illegible] $\langle p\rangle = eV_i\nu_i$

$$\frac{\langle p\rangle}{n_e} = \frac{e^2E_0^2\nu_m}{m(\nu_m^2+\omega^2)}$$

$$\langle p\rangle = eV_i\nu_i = \frac{e^2E_0^2\nu_m}{m(\nu_m^2+\omega^2)}$$

$$E_0^2 = \frac{m e V_i\nu_i(\nu_m^2+\omega^2)}{e^2\nu_m}$$

[illegible] $\boxed{E_b^2 = \frac{m e V_i\nu_i(\nu_m^2+\omega^2)}{e\nu_m}}$

Total power.

$$P = \int p\,d\tau = \frac{n_e e^2}{m(\nu^2+\omega^2)}\,\nu\int E_0^2(\vec{r})\,d\vec{r}.$$

The particle balance of discharge in high pressure. 7

$$\frac{\partial n_e}{\partial t} = \nu_i n_e + k n_e^2 - \nu_e n_e + D \nabla^2 n_e = 0.$$

ν_i [illegible]

$\nu_i = \alpha v_d$ α [illegible] v_d [illegible]

$\nu_i = \alpha \mu_e E$ $v_d = \mu_e E$. μ_e [illegible]

define: efficiency of ionization:

$$\eta = \alpha / E$$

$$\boxed{\mu_e = \frac{v}{E} = \frac{e/m}{\nu_m + j\omega}}$$

$$\nu_i = \eta \mu_e E^2$$

Let ζ — high frequency ionization coefficient.

$$\zeta = \frac{\nu_i}{D_e E^2} \qquad \frac{\nu_i}{D} = \zeta E^2$$

Energy balance.

The power done by electric field (~~per volume~~ unit of discharge)

$$p = \vec{J} \cdot \vec{E}$$

$$\vec{J}_- = n_e e v = \frac{n_e e^2/m}{\nu_m + j\omega} E$$

$$p = \vec{J}\,\vec{E} = \frac{n_e e^2}{m(\nu_m + j\omega)} E_0^2 e^{j2\omega t} = \frac{n_e e^2 E_0^2}{m(\nu_m^2 + \omega^2)} \nu_m e^{j(2\omega t - \phi)}$$

$$\boxed{\langle p \rangle = \frac{n_e e^2 E_0^2}{m} \cdot \frac{\nu_m}{(\nu_m^2 + \omega^2)}}$$

The Dispersion Equations and

E_z in plasma Region

For the case that the dielectric tube wall is neglected, the dispersion equation is

$$\frac{\pi \,\delta\, \varepsilon_p}{p_p^2 J_0(p R_0)} J_0'(p R_0) \cdot \sum_{m,n,s} \left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right) \overbrace{J_s(K_0')}^{J_s(K_0)} e^{js\frac{\pi}{2}} =$$

$$= \sum_{m,n,s} e^{js\frac{\pi}{2}} \left[\left(\frac{n}{b}\right) J_s(K_0') + j\left(\frac{m}{a}\right) J_s'(K_0') \right] J_s(K_0) \quad (1)$$

$$\zeta = \frac{\pi^2 \varepsilon_p}{p_p^2 J_0(p R_0)} J_0(p R) \sum_{m,n,s} \left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right) J_s(K_0) J_s(K_0') e^{js\frac{\pi}{2}} \quad (2)$$

where: $K_0 = \frac{m\pi}{a} R_0$; $K_0' = \frac{n\pi}{b} R_0$ (3)

$m = 0, 1, 2, \ldots$; $n = 0, 1, 2, \ldots$, $s = \ldots -2, -1, 0, 1, 2 \ldots$. m and n can not be zero simultaneously.

The dispersion equation for the case where the dielectric tube wall is taken into account is:

$$p\left[B_1 J_0'(p R_0) + B_2 Y_0'(p R_0)\right] = p_p D_2 J_0'(p_p R_0) \quad (4)$$

where,

III. Radiation,

Intensity and Spectrum of Radiation.

$$D_2 = \frac{p^2 \varepsilon_p}{p_p^2 J_0(p_p R_0) \varepsilon_D} \left[B_1 J_0(pR_0) + B_2 N_0(pR_0) \right]. \qquad (5)$$

$$B_1 = \frac{\Delta_1}{\Delta}; \qquad B_2 = \frac{\Delta_2}{\Delta} \qquad (6)$$

$$\Delta = \frac{p^3}{\varepsilon_D} \left[J_0(pR_1) N_0'(pR_1) - N_0(pR_1) J_0'(pR_1) \right] \qquad (7)$$

$$\Delta_1 = p N_0'(pR_1) \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \sum_s J_s(K_1) J_s(K_1') e^{js\frac{\pi}{2}} -$$
$$- \frac{p^2}{\varepsilon_D} N_0(pR_1) \sum_s J_s(K_1) e^{js\frac{\pi}{2}} \left(\left(\frac{n\pi}{b}\right) J_s(K_1') - j \left(\frac{m\pi}{a}\right) J_s'(K_1') \right] \qquad (8)$$

$$\cancel{\Delta_2 = \frac{p^2 \varepsilon_1}{p_p^2 \varepsilon_D J_0(p_p R_0)} \left[B_1 J_0(pR_0) + B_2 N_0(pR_1) \right]}$$

$$\Delta_2 = \frac{p^2 \pi}{\varepsilon_D} J_0(pR_1) \sum_s J_s(K_1) e^{js\frac{\pi}{2}} \left[\left(\frac{n\pi}{b}\right) J_s(K_1') + j \frac{m\pi}{a} J_s'(K_1') \right] -$$
$$- p J_0'(pR_1) \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \sum_s J_s(K_1) J_s(K_1') e^{js\frac{\pi}{2}} \qquad (9)$$

where:

$$K_1 = \frac{m\pi}{a} R_1, \qquad K_1' = \frac{n\pi}{b} R_1 \qquad (10)$$

and:

$$\xi = D_2 \frac{p_p^2}{\varepsilon_0 \varepsilon_p} J_0(p_p r) =$$
$$= \frac{p^2}{\varepsilon_p J_0(p_p R_0)} \left[B_1 J_0(pR_0) + B_2 N_0(pR_0) \right] J_0(p_p r) \qquad (11)$$

The m, n depend on the operation frequency and the a and b. The convergence of Bessel functions $J_s(K)$ and $J_s(K')$, $J'(K')$ is very good. The coupling between modes should be considered

15. Shenggang Liu, "Higher Education in P. R. China," presentation to graduate students and faculty in the Electrical and Computer Engineering Department of ODU (viewgraphs).

1. Introduction
2. Institutions of Higher Learning
   a. Category
      - Comprehensive University
      - University of Science and Technology
      - Normal University
      - Medical University
      - Agriculture University
      - TV University and others
   b. Affiliation
      - Central government affiliation
      - Local Government Affiliation
      - Private (very few)
      - TV universities
   c. Key University

      Among 1060 universities, there are about 60 are National Key Universities.
   d. 211 National Project

In order to meet the needs in the next century, Chinese government decided to pay more attention to some national key universities (about 100). UESTC has been included in the project.

15
7

Higher Education in P.R.China

Prof. Liu Shenggang

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Academician, Chinese Academy of Sciences

Distinguished visiting professor of ODU and CWM

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Abstract

The historical and recent situations of the Higher Education in P.R.China are reported in the talk, it consists of the following 10 sections.

presented in ODU for graduate students and faculties

4.Students

a. Admittance, Entrance Examinations,

- National wide entrance examination
- Time
- Courses to be taken
- Percent to be admitted
- Similar entrance examinations for graduate students of both doctor and master degree admission

b. Tuition Fee

About 2,000.00 to 3,500.00 Yuan per year for under-graduate students

Some stipends are given to graduate students monthly.

c. Scholarships

A number of different scholarships are available for outstanding students, some of them are for students from poor families.

d. Awards

A number of awards are for outstanding students, the competition is very strong.

e. Students' association

Students have their own organization:

- University association
- Department association
- Class Sub division

f. Foreign students

3.Faculty and Stuff Members

a. Faculty Members

It is required that the standard ratio of the number of teachers to that of students is 1:11.

The rank system of faculty member is professor, associate professor, lecturer and assistant professor.

b. Stuff Members

It is required that the total number of stuffs should be less than that of the faculty members.

c. Colleges, Departments and Specialties.

Each one of faculty members should work in a college or a department.

d. Foreign teachers

Some universities invite foreign experts and teachers to give courses for short or long terms.

7. Research Activities

a. Funding Resources:

- National Science Foundation
- Foundation from different ministries and from local governments
- enterprises

b. Research Institutes

Research institutes exist in some key universities.

c. National Labs

Some National Labs are established in some key universities.

d. Research Centers

e. Publications

Some good universities may have their own Journals. Faculty members and students can publish their papers in these journals.

Some universities in China may admit foreign students both for undergraduate and graduate students. In general, free accommodation is provided for foreign students.

More than 95% students, teachers and stuffs are living in campus. So there a lot of dometories for students and houses for teachers and stuffs.

5. Degree Systems
   a. Doctors: doctor for Science, Engineering and Humanity Science, respectively.
   b. Masters: the same as that for doctors
   c. Bachelor
   d. Associate.

6. Teaching,
   a. Credit System, it is really a mixed one. Semester (two per year) and credit system.
   b. Regular and optional courses, about 10-20 percent of total courses are optional by students.
   c. Laboratory works: Regular and free works in labs.
   d. Out Class and Social Activities: quite a lot of interesting out class and social activities are organized for students.

encourage themselves constantly and gain new knowledge by themselves, while any new knowledge may out of date sooner or later, as science and technology are developing so rapidly.

To conclude my talk, I would say that education is a lifelong issue.

11. Shenggang Liu and Robert J. Barker, “Electromagnetic Wave Field Excited by a Single Moving Charged Particle in Plasma”.

Electromagnetic Wave Field Excited by a Single Moving Charged particle in Plasma*

Liu Shenggang** R.J.Barker***

Abstract-The electromagnetic wave field excited by a single moving charged particle in plasma is studied. It shows that there is no Cherenkov radiation wave but decay wave in plasma, both free plasma and magnetized plasma. Besides, the deflect coefficient for the wave, ($n'^2 = \varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2}$) is the same for both free plasma and magnetized plasma. The main important conclusion shows that the plasma, both free plasma and weakly magnetized plasma can never be considered as a slow wave media. From the point view of physical concept, this conclusion is of significance for the plasma Cherenkov maser.

I. Introduction

The electromagnetic wave field excited by a single moving particle was studied long ago. It was discovered that in a dense media, ($\varepsilon>1$), we have the Cherenkov radiation wave. This kind of radiation now becomes more interesting because of the applications, especially in coherent electromagnetic wave generation, such as Cherenkov maser and Cherenkov FEL. Recently, plasma microwave Cherenkov maser is one of most attractive figures in this field. To study the electromagnetic wave field excited by a single moving charged particle in plasma, therefore, is a basic research both from the point of view of physics and practice.

This paper is organized as follows: Section I is introduction. In section II, the formulation of the electromagnetic wave field excited by a single charged particle in plasma is studied. A detailed discussion is given in section III. Section IV is conclusion.

II. Formulation of the electromagnetic field excited by a single moving charged particle in plasma

The electromagnetic field excited by a single moving charged particle in plasma is studied in this section. For an infinite plasma with an external magnetic field $\bar{B} = B_0\bar{e}_z$ in +z direction, if the electromagnetic wave is propagating in +z direction with propagation factor $e^{i\omega t - \gamma z}$, we can treat it by using a cold fluid model, then the electric permittivity tensor of this magnetized plasma may be written as:

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \varepsilon_1 & -\varepsilon_2 & 0 \\ \varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \qquad (1)$$

Where:

*Supported partly by Chinese National Science Foundation and partly by AFOS,USA

**University of Electronic Science and Technology of China,

*** AFOSR, USA

$$\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} \tag{2}$$

$$\varepsilon_2 = i \frac{\omega_c \omega_p^2}{\omega\left[\omega^2 - \omega_c^2\right]} \tag{3}$$

$$\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2} \tag{4}$$

$\omega_p = \left(\frac{Ne^2}{m_e \varepsilon_0}\right)^{\frac{1}{2}}$ is the electron plasma frequency, $\omega_c = \frac{e}{m_e} B_0$ is the electron cyclotron frequency, and **i** is the sign of imaginary number. The collision effect is neglected.

The Maxwell's equations can be written as:

$$\nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \tag{5}$$

$$\nabla \times \bar{H} = \varepsilon_0 \frac{\partial}{\partial t}\left(\bar{\varepsilon} \cdot \bar{E}\right) + \bar{J} \tag{6}$$

and the Poisson's equation is:

$$\nabla \cdot \left(\bar{\varepsilon} \cdot \bar{E}\right) = \rho / \varepsilon_0 \tag{7}$$

ρ and $\bar{J}$ are the space charge density and current density of particle, respectively.

In the free plasma and weakly magnetized plasma conditions, the follow wave equation for E_z can be obtained:

$$\nabla_\perp^2 E_z + \frac{\varepsilon_3}{\varepsilon_r} \frac{\partial^2 E_z}{\partial z^2} - \varepsilon_0 \varepsilon_3 \mu_0 \frac{\partial^2 E_z}{\partial t^2} = \mu_0 \frac{\partial J_z}{\partial t} + \frac{1}{\varepsilon_0 \varepsilon_r} \frac{\partial \rho}{\partial z} \tag{8}$$

Where:

$$\varepsilon_r^2 = \varepsilon_1^2 + \varepsilon_2^2 \tag{9}$$

Let:

$$z' = \sqrt{\frac{\varepsilon_r}{\varepsilon_3}} z \tag{10}$$

Eq. (8) becomes:

$$\nabla_\perp^2 E_z + \frac{\varepsilon_3}{\varepsilon_r} \frac{\partial^2 E_z}{\partial z'^2} - \varepsilon_0 \varepsilon_3 \mu_0 \frac{\partial^2 E_z}{\partial t^2} = \mu_0 \frac{\partial J_z}{\partial t} + \frac{1}{\varepsilon_0 \sqrt{\varepsilon_3 \varepsilon_r}} \frac{\partial \rho}{\partial z'} \tag{11}$$

Eq. (8) and (11) are basic equations for studying the electromagnetic field excited by moving particle in free plasma and weakly magnetized plasma. Neglecting the sources terms in the right side of the equations, we can get the same as that given in reference [5] eq. (1) in reference [1].

Assuming a single charged particle (electron, for example) is moving in the plasma with an uniform straightforward velocity **u**, then we have:

$$\rho = e \frac{\delta(r)}{r} \delta(z' - ut) \tag{12}$$

$$J_z = eu \frac{\delta(r)}{r} \delta(z' - ut) \tag{13}$$

$$\delta\left(z' - ut\right) = \left|\sqrt{\frac{\varepsilon_3}{\varepsilon_r}}\right| \delta\left(z - \sqrt{\frac{\varepsilon_3}{\varepsilon_r}} ut\right) \tag{14}$$

Where $\delta(x)$ is the Dirac Delta function.

Making use of the expansion expressions of the Delta function, we get:

$$\rho = \frac{e}{2\pi}\int_{-\infty}^{+\infty} dk_{//} \int_0^{+\infty} J_0(k_\perp r) e^{ik_{//}(z'-ut)} k_\perp dk_\perp \tag{15}$$

$$J_z = \frac{eu}{2\pi}\int_{-\infty}^{+\infty} dk_{//} \int_0^{+\infty} J_0(k_\perp r) e^{ik_{//}(z'-ut)} k_\perp dk_\perp \tag{16}$$

Substituting equations (15) and (16) into eq. (11) and solving this wave equation, we can get the field expression of E_z as follow:

$$\begin{aligned} E_z = -\frac{ie}{2\pi\varepsilon_0}\int_0^{+\infty} dk_{//} \int_0^{+\infty} \frac{\left(1-\beta^2\sqrt{\varepsilon_r\varepsilon_3}\right)}{\sqrt{\varepsilon_r\varepsilon_3}} J_0(k_\perp r) e^{ik_{//}(z'-ut)} k_{//} k_\perp \cdot \\ \left[\frac{1}{k_\perp^2 + k_{//}'^2(1-\varepsilon_r\beta^2)} + i\pi\frac{k_{//}}{|k_{//}|}\delta\left(k_\perp^2 + k_{//}'^2(1-\varepsilon_r\beta^2)\right)\right] dk_\perp \end{aligned} \tag{17}$$

Where $k_{//}' = k_{//}\sqrt{\frac{\varepsilon_3}{\varepsilon_r}}$.

Making use of the following formulas:

$$\int_0^{+\infty} \frac{J_0(k_\perp r)}{k_\perp^2 - a^2} k_\perp dk_\perp = -\frac{\pi}{2} N_0(ar) \tag{18}$$

$$\int_0^{+\infty} J_0(k_\perp r)\delta(k_\perp^2 - a^2) k_\perp dk_\perp = \frac{1}{2} J_0(ar) \tag{19}$$

We then obtain:

$$\begin{aligned} E_z = \frac{e}{4\varepsilon_0}\int_0^{+\infty} \frac{\left(1-\beta^2\sqrt{\varepsilon_r\varepsilon_3}\right)}{\sqrt{\varepsilon_r\varepsilon_3}} e^{ik_{//}(z'-ut)} \frac{k_{//}}{|k_{//}|}\left\{J_0\left[|k_{//}|\left(n'^2\beta^2 - 1\right)^{1/2} r\right] + \right. \\ \left. iN_0\left[|k_{//}|\left(n'^2\beta^2 - 1\right)^{1/2} r\right]\right\} k_{//} dk_{//} \end{aligned} \tag{20}$$

Where $n'^2 = \varepsilon_r$. And we can also obtain E_r and H_θ in terms of E_z.

III. Discussion

1. For uniformly isotropic media, we have: $n'^2 = \varepsilon$. If $\varepsilon > 1$, the dense medium, eq. (20) shows the existence of the Cherenkov radiation.

2. For free plasma, ($B_0 = 0$), we have: $n'^2 = \varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2} \le 1$, then we have $n'^2\beta^2 - 1 = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\beta^2 - 1 < 0$, Eq. (20) becomes:

$$E_z = \frac{e}{4\varepsilon_0}\int_0^{+\infty} \frac{\left(1-\beta^2\varepsilon_1\right)}{\varepsilon_1} e^{ik_{//}(z'-ut)} \frac{k_{//}}{|k_{//}|} H_0^{(1)}\left[|k_{//}|\left(n'^2\beta^2 - 1\right)^{1/2} r\right] k_{//} dk_{//} \tag{21}$$

Eq. (21) shows that there is no Cherenkov radiation.

3. For weakly magnetized plasma, Eq. (11), (17), (20) and (21) show that we can get the similar results for the wave field:

$$E_z = \frac{e}{4\varepsilon_0}\int_0^{+\infty}\frac{\left(1-\beta^2\varepsilon_1\right)}{\varepsilon_1}e^{ik_{//}(z'-ut)}\frac{k_{//}}{|k_{//}|}H_0^{(1)}\left[|k_{//}|\left(n'^2\beta^2-1\right)^{1/2}r\right]k_{//}dk_{//} \tag{22}$$

We only can have decay wave, there is no Cherenkov radiation.

IV. Conclusion

1. The deflection coefficient for the electromagnetic wave excited by a single moving charged particle in weakly magnetized plasma is the same as it is in free plasma: $n'^2 = \varepsilon_3 = 1-\frac{\omega_p^2}{\omega^2}$. We have $n'^2 < 1$, even $n'^2 < 0$, when $\omega_p > \omega$.

2. Therefore, in the plasma, both free plasma and weakly magnetized plasma, there is no Cherenkov radiation.

3. We believe that this conclusion is also valid for strong magnetized plasma. To prove it, the mathematical manipulation becomes more complicated.

So for plasma Cherenkov device, it is necessary to have dielectric load, besides the plasma filling.

References

1. Liu Shenggang, "Relativistic Electronics", Chapt. 12, 1987, Chinese Academy of Science Publishing House.
2. Jackson J. D., "Classical Electrodynamics", 1976, Chinese version.
3. Liu Shenggang, et al., Science in China (E), 39(5), 1996, p485.
4. J. S. De. Groot, et al., IEEE Trans. PS-16, 2, 206, 1988.

12. Shenggang Liu, et al, "Theory of Waveguide System for Microwave Plasma Excited Excimer Laser".

Theory of Waveguide System for Microwave Plasma Excited Excimer Laser

Abstract: Theory of wave propagation along a rectangular waveguide filled with an annular plasma column in the center is presented in the paper. This waveguide system might be a good candidate for microwave plasma excited excimer laser.

I. Introduction

Microwave plasma excited excimer lasers may form a new area of Microwave Plasma Electronics. There are a lot of advantages when the microwave plasma is used for excitation of the excimer lasers.[]-[] Variety of waveguide systems have been tested to excite excimer laser, cylindrical and elliptical waveguides, for example. The rectangular waveguide may another or even better choice, for rectangular waveguide is a commonly used one, it is very convenient to be coupled or connected with the whole waveguide system and with the input/output. The efficiency and the costs of the excimer laser by means of rectangular waveguide, therefore, should be better than the others.

However, theoretical analysis that may be helpful and useful for the design and better understanding of the rectangular waveguide microwave plasma excimer laser did not appear in published papers. It is on purpose to work out a theory for the rectangular waveguide system for microwave plasma excited excimer laser, as shown in Fig.1. A dielectric tube filled with plasma is in the center of a rectangular waveguide. R0, R1 is the inner and outer radius of the tube, respectively. ε_0, ε_D and ε_p is the dielectric constant of the vacuum, dielectric and plasma, respectively. If there is axial magnetic field, ε_p should be a tensor: $\bar{\bar{\varepsilon}}_p$.

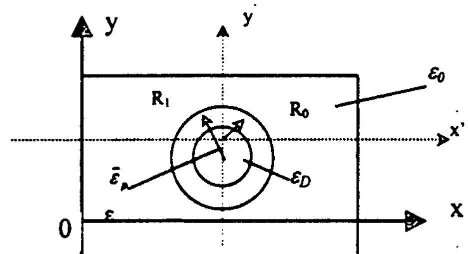

Fig.1

An analytical theory of the waveguide system shown in Fig.1 has been worked out. The dispersion equation, the field components in the interaction (excitation) region, ($0 \le R \le R_0$) are calculated. The theory can be used in the design of the excimer laser. The paper is organized as follows: section 1 is introduction, section 2, and section 3 deel with the theory of the waveguide without axial magnetic field ($B_0 = 0$). in section 4 ,the theory with the axial magnetic field taken into consideration is given. The numerical calculation is given in section 5, and the section 6 is conclusion.

II. Theory of Rectangular waveguide system for microwave plasma excitation of excimer laser (without magnetic field)

Theoretical study of the rectangular waveguide system shown in Fig1 is presented in this section. At moment we assume that there is as magnetic field $B_0 = 0$ then in the plasma, as well as that in vacuum region, the TE made and TM mode are independent.

In the vacuum region of the waveguide, i.e. the region outside the dielectric tube, we have:

$$
\begin{aligned}
E_x &= -j\omega\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \\
E_y &= j\omega\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \\
E_z &= 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1) \\
H_x &= -\frac{jk_z}{\mu_0}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)
\end{aligned}
$$

$$H_y = -\frac{jk_z}{\mu_0}\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$H_z = -\frac{\pi^2}{\mu_0}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

for TE modes,

$$E_x = -\frac{jk_z}{\varepsilon_0}\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_y = -\frac{jk_z}{\varepsilon_0}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$E_z = \frac{\pi^2}{\varepsilon_0}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \tag{2}$$

$$H_x = j\omega\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_y = -j\omega\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$H_z = 0$$

for TM modes.

In the dielectric tube region, including the plasma region, the field components should be expressed in cylindrical coordinate system (r, θ, z). Then, within the dielectric tube wall, we have: ($R_0 \le r \le R_1$)

$$E_r = -j\frac{l\omega}{r}\left[A_1 J_e(pr) + A_2 N_e(pr)\right]\sin l\varphi$$

$$E_y = -j\omega p\left[A_1 J_e^{'}(pr) + A_2 N_e^{'}(pr)\right]\cos l\varphi$$

$$E_y = 0$$

$$H_r = \frac{jk_z}{\mu_0} p\left[A_1 J_e^{'}(pr) + A_2 N_e^{'}(pr)\right]\cos l\varphi \tag{3}$$

$$H_\varphi = -\frac{jk_z}{\mu_0}\left(\frac{l}{r}\right)\left[A_1 J_e(pr) + A_2 N_e(pr)\right]\sin l\varphi$$

$$H_z = -\frac{p^2}{\mu_0}\left[A_1 J_e(pr) + A_2 N_e(pr)\right]\cos l\varphi$$

for TE modes, and

$$E_r = -\frac{jk_z}{\varepsilon_0\varepsilon_0} p\left[B_1 J_e^{'}(pr) + B_2 N_e^{'}(pr)\right]\cos l\varphi$$

$$E_\varphi = \frac{jk_z}{\varepsilon_0\varepsilon_0}\left(\frac{l}{r}\right)\left[B_1 J_e(pr) + B_2 N_e(pr)\right]\sin l\varphi$$

$$E_z = \frac{p^2}{\varepsilon_0\varepsilon_0}\left[B_1 J_e(pr) + B_2 N_e(pr)\right]\cos l\varphi \tag{4}$$

$$H_r = j\omega\left(\frac{l}{r}\right)\left[B_1 J_e(pr) + B_2 N_e(pr)\right]\sin l\varphi$$

$$H_\varphi = -j\omega p\left[B_1 J_e^{'}(pr) + B_2 N_e^{'}(pr)\right]\cos l\varphi$$

$$H_z = 0$$

for TM modes. Where

$$p^2 = \left(k^2\varepsilon_D - k_z^{\ 2}\right) \tag{5}$$

In the plasma region, ($0 \le R \le R_0$), we have:

$$\begin{aligned}
E_\varphi &= -j\omega p_p D_1 J_l^{'}\left(p_p r\right)\\
E_z &= E_r = 0\\
H_r &= \frac{jk_z}{\mu_0} p_p D_1 J_l^{'}\left(p_p r\right)\\
H_Z &= -\frac{P_p^{\ 2}}{\mu_0} D_1 J_l\left(p_p r\right)\\
H_\varphi &= 0
\end{aligned} \tag{6}$$

for TE mode, and

$$\begin{aligned}
E_r &= -\frac{jk_z}{\varepsilon_0\varepsilon_0} p_p D_2 J_l^{'}\left(p_p r\right)\\
E_Z &= \frac{P_p^{\ 2}}{\varepsilon_0\varepsilon_0} D_2 J_l\left(p_p r\right)\\
H_\varphi &= -j\omega p_p D_2 J_l^{'}\left(p_p r\right)\\
E_\varphi &= H_Z = H_r = 0
\end{aligned} \tag{7}$$

for TM modes, where:

$$p_p^{\ 2} = \left(k^2\varepsilon_p - k_z^{\ 2}\right) \tag{8}$$

$$\varepsilon_p = \left(1 - \frac{\omega_{pl}^{\ 2}}{\omega^2}\right) \tag{9}$$

The boundary conditions are as follows:

$$r = R_1: \qquad E_z^{\ I} = E_z^{\ II}; \qquad H\varphi^I = H_\varphi^{\ II} \tag{10a}$$

$$r = R_0: \qquad E_z^{\ II} = E_z^{\ III}; \qquad H\varphi^{II} = H_\varphi^{\ III} \tag{10b}$$

For excimer laser application, we only need to study the TM modes, similar approach can be used for TE modes. The by using e.q. (2).(4) and e.q. (7), we get:

$$\pi^2\left[\left(\frac{m^2}{a^2}\right)+\left(\frac{n^2}{b^2}\right)\right]\sin(K\cos\varphi)\sin\left(K^{'}\sin\varphi\right) = \frac{p^2}{\varepsilon_D}\left[B_1 J_0(pR_1) + B_0 N_0(pR_1)\right] \tag{11}$$

$$\begin{aligned}
&\left(\frac{n\pi}{b}\right)\sin(K\cos\varphi)\cos\left(K^{'}\sin\varphi\right)\cos\varphi + \left(\frac{m\pi}{a}\right)\cos(K\cos\varphi)\sin\left(K^{'}\sin\varphi\right)\sin\varphi\\
&\qquad = p\left[B_1 J_0^{'}(pR_1) + B_2 N_0^{'}(pR_1)\right]
\end{aligned} \tag{12}$$

$$p[B_1 J_0^{'}(pR_0) + B_2 N_0^{'}(pR_0)] = p_p D_2 J_0^{'}(p_p R_0) \tag{13}$$

and

$$\frac{p^2}{\varepsilon_D}\left[B_1 J_0(pR_0) + B_2 N_0(pR_0)\right] = \frac{p_p^{\ 2}}{\varepsilon_p} D_2 J_0\left(p_p R_0\right) \tag{14}$$

After straightforward mathematics manipulations, we obtain the dispersion equation:

$$\sum_{m,n,s}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\frac{\pi}{2s} J_s(K_1) J_s\left(K_1^{'}\right) = B_1\frac{p^2}{\varepsilon_D}\left[J_0(pR_1) + \frac{B_2}{B_1} N_0(pR_1)\right] \tag{15}$$

where:

$$B_1 = \sum_s \frac{\frac{\pi}{2}\left(\frac{m}{a}+\frac{n}{b}\right)\sum_s\left(\frac{1}{s-1}\right)J_s(K_1)J_s(K_1')}{p\left[J_0'(pR_1)+\frac{B_2}{B_1}r_s'(pR_1)\right]} \tag{16}$$

$$\frac{B_2}{B_1} = \frac{\frac{p_p}{\varepsilon_p}J_0(p_pR_0)J_0'(p_pR_0)-\frac{p}{\varepsilon_D}J_0(pR_0)J_0'(p_pR_0)}{\left[\frac{p}{\varepsilon_D}N_0(p_pR_0)J_0'(p_pR_0)-\frac{p_p}{\varepsilon_p}J_0(p_pR_0)N_0'(p_pR_0)\right]} \tag{17}$$

The field within the plasma can also be found:

$$D_2 = B_1\frac{P^2}{\varepsilon_D}\frac{\left[J_0(pR_0)+\frac{B_2}{B_1}N_0(pR_0)\right]}{\frac{p_p^{\,2}}{\varepsilon_p^{\,2}}J_0(p_pR_0)} \tag{18}$$

Substituting of eq. (16)-(18) into (7), we obtain the most important field component: E_z in the plasma region:

$$E_y = B_1p^2\frac{\varepsilon_0}{\varepsilon_d}\frac{\left[J_0(pR_0)+\frac{B_2}{B_1}N_0(pR_0)\right]}{J_0(p_pR_0)}J_0(p_pr) \tag{19}$$

If we neglect the influence of the dielectric wall of the tube, the dispersion equation reduced to:

$$\sum_{m,n,s}\left(\frac{m}{a}+\frac{n}{b}\right)\frac{1}{(s-1)}J_s(K_0)J_s(K_0') = p_pD_2'J_0'(p_1R_0) \tag{20}$$

where:

$$D_2 = \frac{\pi^3}{2}\sum_{m,n,s}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)J_s(K_0)J_s(K_0') \tag{21}$$

It can be seen that since the Bessel function $J_s(x)$ is a converging function, so it may just take few terms for m, n, and s. Note, that K and K' also are dependent on m and n, respectively.

II. Theory of magnetized plasma filling

Now we can study the case where an axial magnetic field B_0 exists. In the magnetized plasma filling case, the dielectric constant ε_p becomes a tensor:[ ]

$$\bar{\varepsilon}_p = \begin{bmatrix} \varepsilon_1 & -\varepsilon_2 & 0 \\ \varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \tag{22}$$

where:

$$\begin{aligned} \varepsilon_1 &= 1-\frac{\xi^2(1-j\delta)}{(1-j\delta)^2-\tau^2} \\ \varepsilon_2 &= \frac{j\tau\xi^2}{(1-j\delta)^2-\tau^2} \\ \varepsilon_3 &= 1-\frac{\xi^2}{1-j\delta} \end{aligned} \tag{23}$$

$$\xi = \frac{\omega_{pe}}{\omega}; \qquad \tau = \frac{\omega_{ce}}{\omega}; \qquad \delta = \frac{\gamma_{eff}}{\omega} \tag{24}$$

ω_{pe} is the plasma frequency, ω_{ce} is the cyclotron frequency, γ_{eff} is the effective collision frequency.

In this case, the TE and TM mode are always coupled to give hybrid modes. Therefore, we have:

$$E_z = A_1 J_0(p_1 r) + A_2 J_0(p_2 r) \tag{25}$$

$$H_z = A_1 h_1 J_0(p_1 r) + A_2 h_2 J_0(p_2 r) \tag{26}$$

where p_1, p_2 are two eigen values:

$$p_{1,2} = \frac{1}{2}\left\{(a+c) \pm \left[(a+c)^2 - 4(ac-bd)\right]^{1/2}\right. \tag{27}$$

$$\begin{aligned} a &= \left(-k_z^2 + k^2\varepsilon_1\right)\varepsilon_s/\varepsilon_1 \\ b &= -k_z\omega\mu_0\,\varepsilon_2/\varepsilon_1 \\ c &= -k_z^2 + k^2\left(\varepsilon_1^2 + \varepsilon_2^2\right)/\varepsilon_1 \\ d &= k_z\omega\varepsilon_0\varepsilon_2\varepsilon_3/\varepsilon_1 \end{aligned} \tag{28}$$

and

$$h_{1,2} = \frac{\left(k^2\varepsilon_1 - k_z^2\right)\dfrac{\varepsilon_2}{\varepsilon_1} - p_{1,2}^2}{-\omega\mu_0 k_z\,\varepsilon_2\ \ \varepsilon_1} \tag{29}$$

A_1, A_2 are constants

In eq.(25), (26), we also limit our study on symmetrical case, it is the desired mode for excimer laser. By using E_z and H_z we can find all field components:

$$\begin{aligned} E_r &= \frac{1}{D}\left\{-jk_z K^2\left[A_1 p_1 J_0'(p_1 r) + A_2 p_2 J_0'(p_2 r)\right] + \right. \\ &\quad \left. \omega\mu_0 k_z^2\left[A_1 h_1 p_1 J_0'(p_1 r) + A_2 h_2 p_2 J_0'(p_2 r)\right]\right\} \\ E_\varphi &= \frac{1}{D}\left\{k_z K^2\left[A_1 p_1 J_0'(p_1 r) + A_2 p_2 J_0'(p_2 r)\right] + \right. \\ &\quad \left. j\omega\mu_0 K^2\left[A_1 h_1 p_1 J_0'(p_1 r) + A_2 h_2 p_2 J_0'(p_2 r)\right]\right\} \\ H_1 &= \frac{1}{D}\left\{-\omega\varepsilon_0\varepsilon_g k_z^2\left[A_1 p_1 J_0'(p_1 r) + A_2 p_2 J_0'(p_2 r)\right] - \right. \\ &\quad \left. jk_z K^2\left[A_1 h_1 p_1 J_0'(p_1 r) + A_2 h_2 p_2 J_0'(p_2 r)\right]\right\} \\ H_\varphi &= \frac{1}{D}\left\{-j\omega\varepsilon_0\left(\varepsilon_1 K^2 - \varepsilon_g k_g^2\right)\left[A_1 p_1 J_0'(p_1 r) + A_2 p_2 J_0'(p_2 r)\right] + \right. \\ &\quad \left. k_z k_g^2\left[A_1 h_1 p_1 J_0'(p_1 r) + A_2 h_2 p_2 J_0'(p_2 r)\right]\right\} \end{aligned} \tag{30}$$

where

$$\begin{aligned} D &= K^4 - k_g^4 \\ K^2 &= k^2\varepsilon_1 - k_z^2 \\ k_g^2 &= k^2\varepsilon_g, \qquad \varepsilon_g = |\varepsilon_2| \end{aligned} \tag{31}$$

By using the boundary conditions:

$$r = R_0: \qquad E_z = E_z; \qquad H_\varphi = H_\varphi \tag{32}$$

We can find:

$$A_1 = -\frac{\frac{\pi}{\varepsilon_0}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)}{(h_2-h_1)J_0(p_1R_0)}(h_2+1)\sum_s J_s(K)J_s(K^{'}) \tag{33}$$

$$A_2 = -\frac{\frac{\pi}{\varepsilon_0}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)}{(h_2-h_1)J_0(p_1R_0)}(h_1+1)\sum_s J_s(K)J_s(K^{'}) \tag{34}$$

Then, the most important field components E_z is:

$$E_z = \frac{\pi^3}{\varepsilon_0}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\frac{1}{(h_2-h_1)}\sum_s J_s(K)J_s(K^{'})\left\{-\frac{(h_2+1)}{J_0(p_1R_0)}J_0(p_1r)+\frac{(h_1+1)}{J_0(p_2R_0)}J_0(p_2r)\right\} \tag{35}$$

The existence of axial magnetic field may be helpful for ionization.

IV. Numerical calculation (to be carried on)

V. Conclusion

Rectangular waveguide might be a good candidate for microwave plasma excited excimer lasers. A theoretical study on a rectangular waveguide filled with an annular plasma column in the center has been worked out for both unmagnetized and magnetized plasma filling. The dispersion equations and field components, in particular the E_z at the plasma region where excimer laser excitation occurs, have been obtained. The theory may be of significance for understanding and design of a rectangular waveguide plasma excimer laser, and the basis of future theoretical study of the Microwave gas discharge processes in the tube located in the outer of the waveguide.

References

1. Lan W. Boyd, J.V. Thang, "New large one ultraviolet laser sources and their applications", *Nuclear instruments and Methods in Physics Research* B, 121, 349-356(1987)
2. Ziming He, Mark A, et al., "Microwave excitation and applications of an elliptical excimer laser", *Laser and particle Beams*, 16(3), 509-524(1998)
3. Liu Shenggang et al., "Theory of wave propagation along waveguide filled with plasma in finite magnetic field". Science in China, 39(5), 485-503(1996)
4. Liu Shenggang et al., "Theory of wave propagation along a Rectangular Magnetized plasma waveguide" (unpublished).

A proposal of a new construction of Rectangular waveguide Microwave plasma Excited Excimer Laser

Work plan (proposal)

1. Theoretical and experimental study on Excitation (input) of Microwave power aimed to microwave the efficiency; (for original mode and proposed mode)
2. Theory of Microwave Discharge.
3. Theory of laser excitation.